

1.9 LOI DES SIN ET LOI DES COS

cours 9

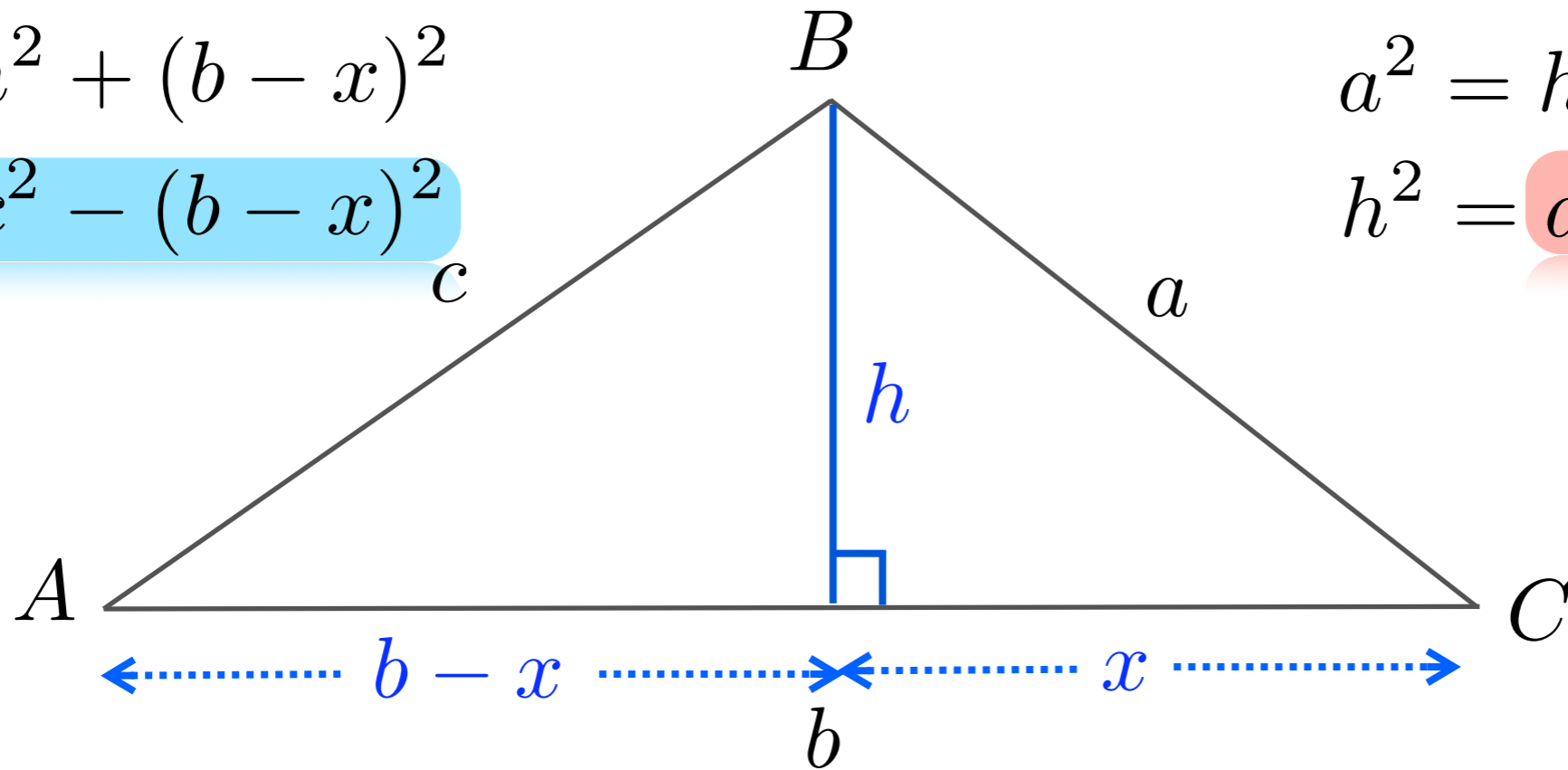
Loi des cosinus

$$c^2 = h^2 + (b - x)^2$$

$$h^2 = c^2 - (b - x)^2$$

$$a^2 = h^2 + x^2$$

$$h^2 = a^2 - x^2$$



$$a^2 - \cancel{x^2} = c^2 - (b^2 - 2bx + \cancel{x^2})$$

$$a^2 = c^2 - b^2 + 2bx$$

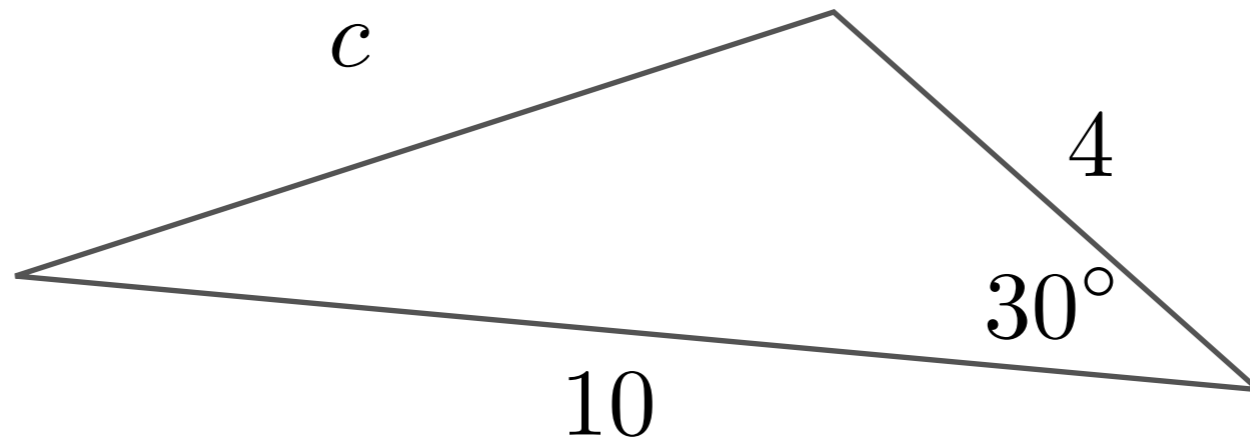
$$c^2 = a^2 + b^2 - 2bx$$

$$\frac{x}{a} = \cos C$$

$$x = a \cos C$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Exemple

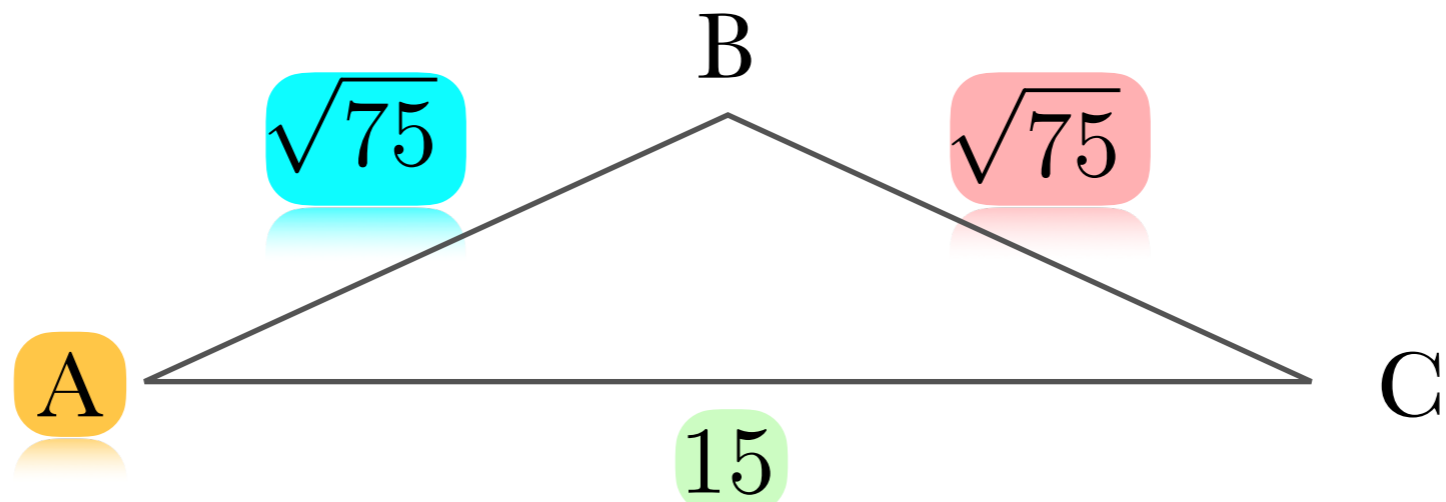


$$\begin{aligned}c^2 &= 10^2 + 4^2 - 10 \times 4 \times \cos 30^\circ \\&= 100 + 16 - 40 \times \frac{\sqrt{3}}{2} \\&= 116 - 20 \times \sqrt{3}\end{aligned}$$

Donc

$$c = \sqrt{116 - 20 \times \sqrt{3}}$$

Exemple



$$(\cancel{\sqrt{75}})^2 = (\cancel{\sqrt{75}})^2 + 15^2 - 2 \times \sqrt{75} \times 15 \cos A$$

$$0 = 15^2 - 2 \times \sqrt{75} \times 15 \cos A$$

$$15^2 = 2 \times \sqrt{75} \times 15 \cos A$$

$$15 = 2 \times \sqrt{75} \cos A$$

$$\cos A = \frac{15}{2 \times \sqrt{75}} = \frac{15}{2 \times \sqrt{3} \times \sqrt{25}} = \frac{15}{2 \times \sqrt{3} \times 5} = \frac{3}{2 \times \sqrt{3}}$$

$$= \frac{\sqrt{3} \times \sqrt{3}}{2 \times \sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$A = 30^\circ \quad C = 30^\circ$$

$$B = 180^\circ - 30^\circ - 30^\circ = 120^\circ$$

Loi des sinus

$$\sin A = \frac{h_1}{c}$$

$$h_1 = c \sin A$$

$$\sin A = \frac{h_2}{b}$$

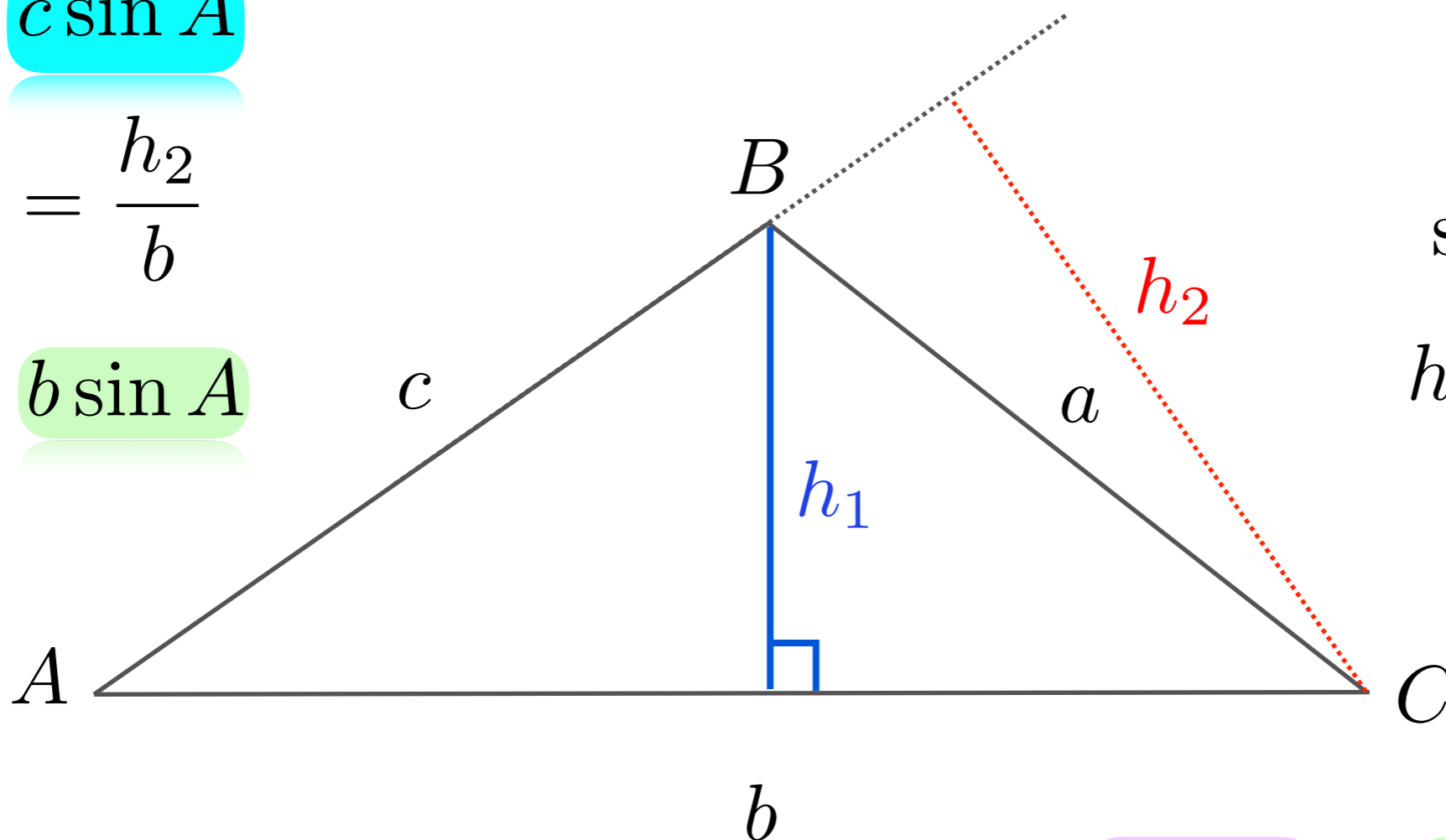
$$h_2 = b \sin A$$

$$\sin C = \frac{h_1}{a}$$

$$h_1 = a \sin C$$

$$\sin B = \frac{h_2}{a}$$

$$h_2 = a \sin B$$

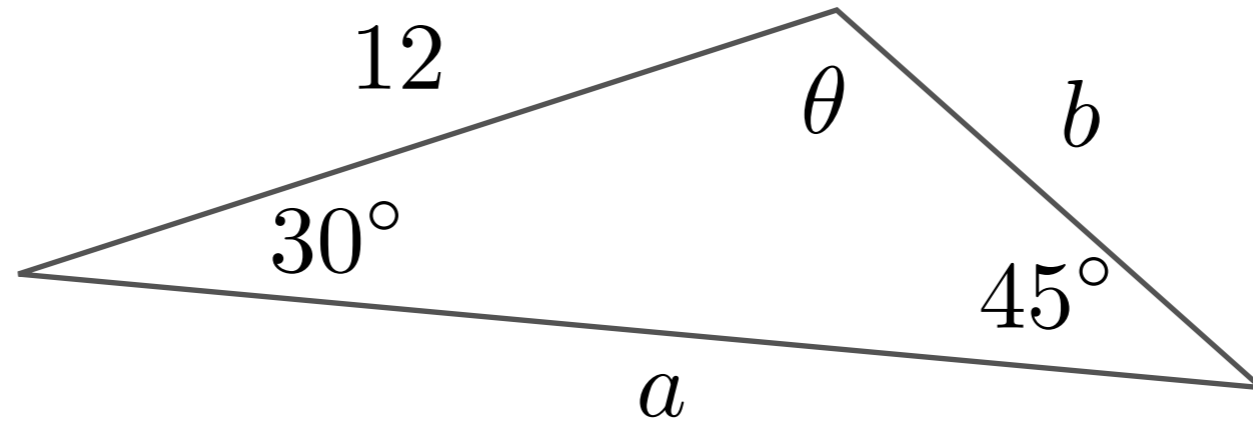


$$c \sin A = a \sin C$$

$$a \sin B = b \sin A$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$$

Example



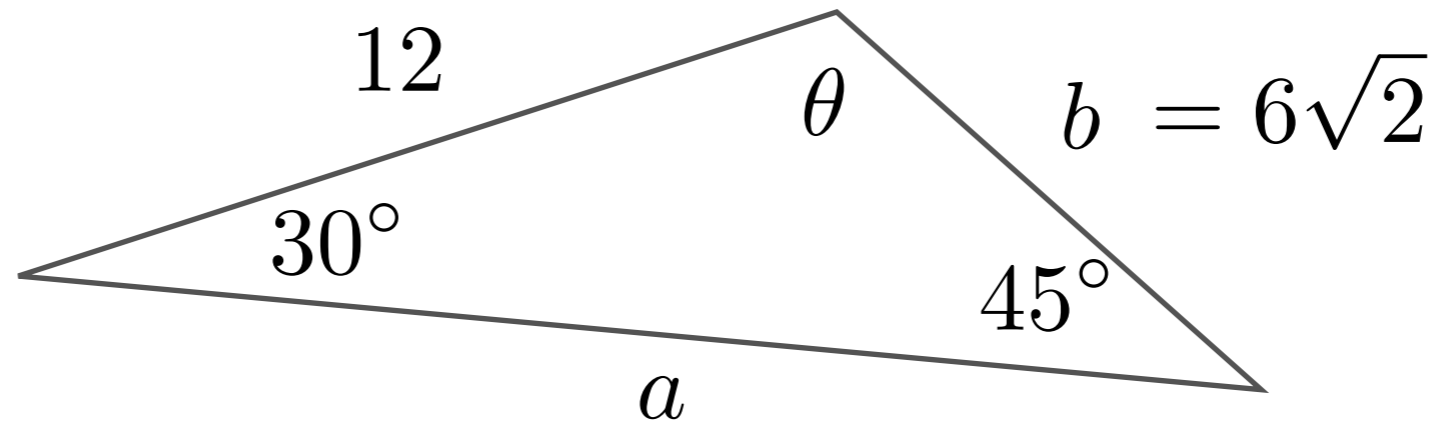
$$\theta = 180^\circ - 30^\circ - 45^\circ = 105^\circ$$

$$\frac{\sin 30^\circ}{b} = \frac{\sin 45^\circ}{12}$$

$$b = \frac{12 \times \sin 30^\circ}{\sin 45^\circ} = \frac{12 \times \frac{1}{2}}{\frac{\sqrt{2}}{2}}$$

$$= 12 \times \frac{1}{2} \times \frac{2}{\sqrt{2}} = \frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$$

Example



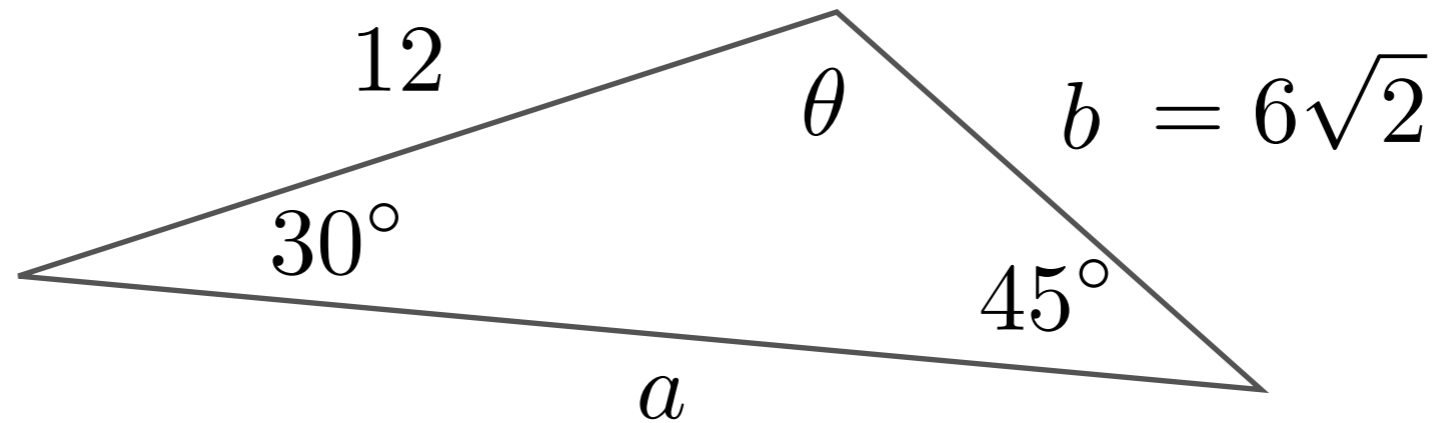
$$\theta = 105^\circ$$

$$\frac{\sin 105^\circ}{a} = \frac{\sin 45^\circ}{12}$$

$$a = \frac{12 \times \sin 105^\circ}{\sin 45^\circ} = \frac{12 \times \sin 105^\circ}{\frac{\sqrt{2}}{2}}$$

$$= \frac{24 \times \sin 105^\circ}{\sqrt{2}} = 12 \times \sqrt{2} \times \sin 105^\circ$$

Example



$$\theta = 105^\circ$$

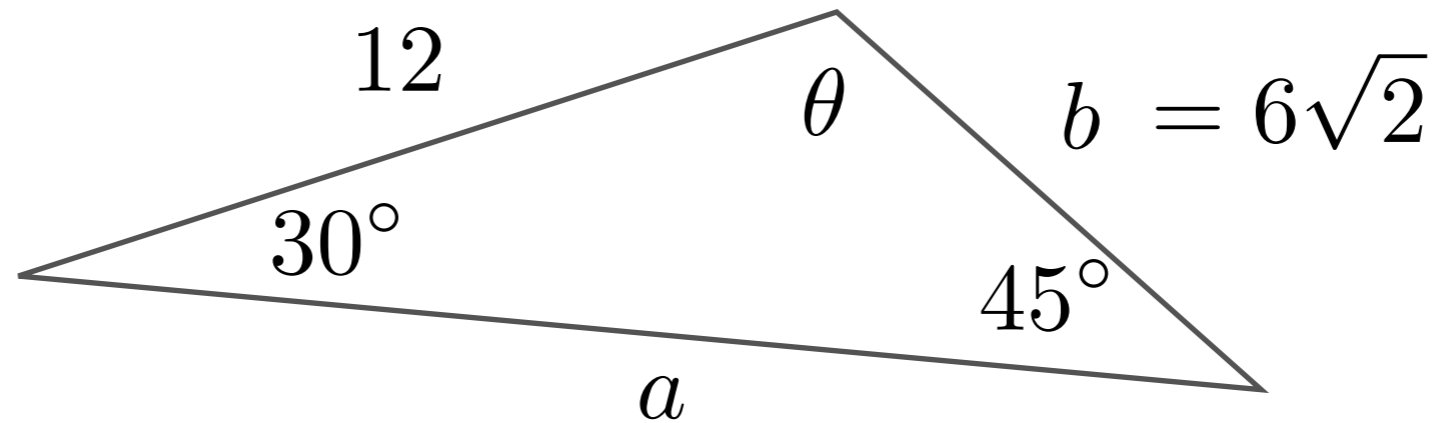
$$a = 12 \times \sqrt{2} \times \sin 105^\circ = 12 \times \sqrt{2} \times \frac{\sqrt{2}(1 + \sqrt{3})}{4}$$

$$\begin{aligned}\sin 105^\circ &= \sin(45^\circ + 60^\circ) \\ &= \sin(45^\circ) \cos(60^\circ) + \sin(60^\circ) \cos(45^\circ)\end{aligned}$$

$$= \frac{\sqrt{2}}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2} + \sqrt{3} \times \sqrt{2}}{4} = \frac{\sqrt{2}(1 + \sqrt{3})}{4}$$

Example



$$\theta = 105^\circ$$

$$\begin{aligned} a &= 12 \times \sqrt{2} \times \sin 105^\circ = 12 \times \sqrt{2} \times \frac{\sqrt{2}(1 + \sqrt{3})}{4} \\ &= 3 \times \sqrt{2} \times \sqrt{2}(1 + \sqrt{3}) = 6(1 + \sqrt{3}) \end{aligned}$$

$$\sin 105^\circ = \frac{\sqrt{2}(1 + \sqrt{3})}{4}$$

Faites les exercices suivants

p. 466 # 38

Devoir:

p.466 # 38 à 46