# 1.2 FONCTIONS VECTORIELLES, COURBES ET SURFACES

cours 2

# Au dernier cours, nous avons vu

- Survol des cours calcul différentiel, calcul intégral et algèbre linéaire.
- Introduction des idées que nous verrons cette session.

# Aujourd'hui, nous allons voir

- Fonctions vectorielles
- Comment les visualiser
- Limite
- Surfaces paramétrées

On a déjà vu les fonctions de la forme

$$\begin{aligned} f : \mathbb{R} &\longrightarrow \mathbb{R} \\ x &\longmapsto f(x) = y \end{aligned}$$

Aujourd'hui nous allons explorer les fonctions vectorielles.

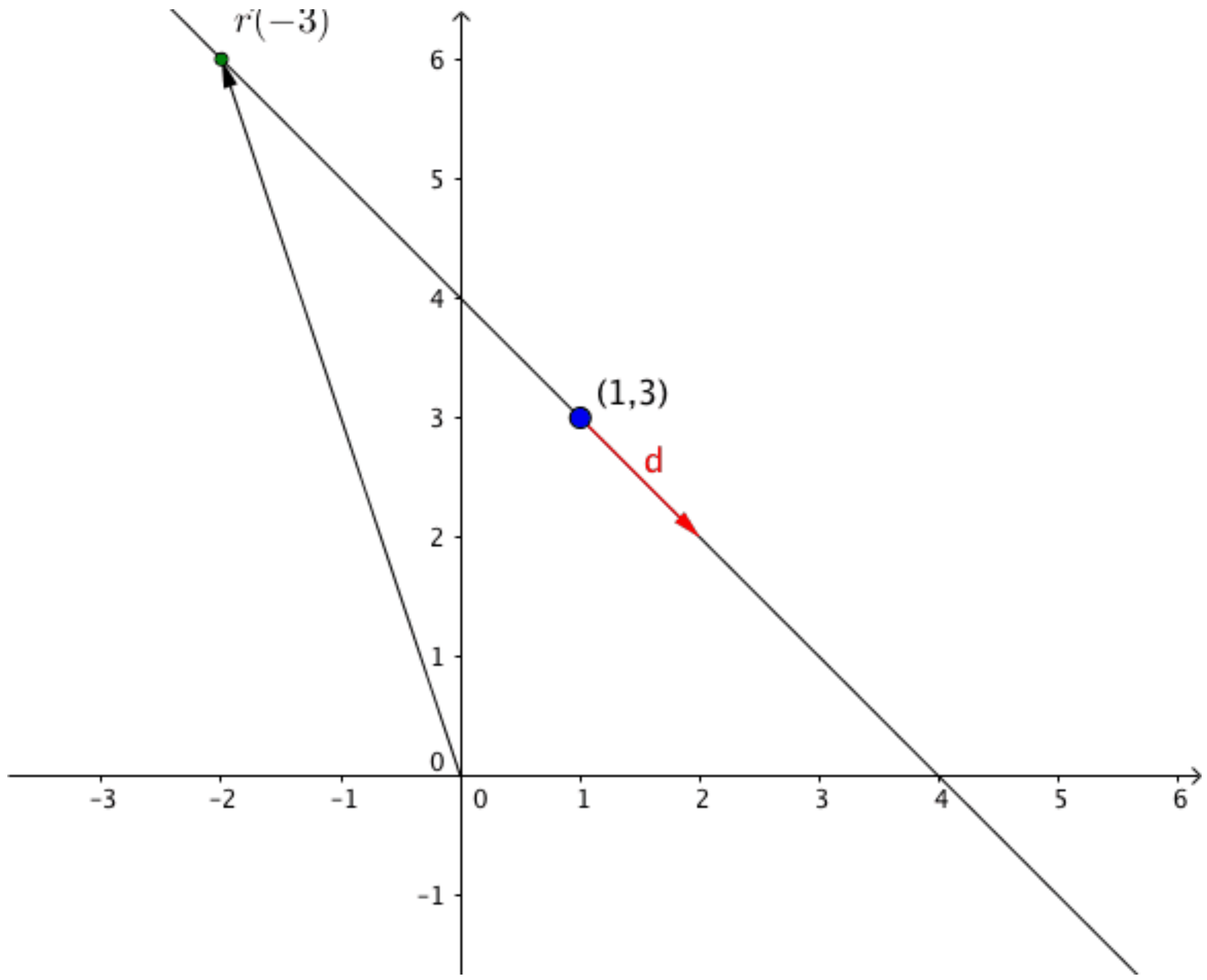
$$\begin{aligned} \vec{r} : \mathbb{R} &\longrightarrow \mathbb{R}^2 \\ t &\longmapsto \vec{r}(t) = (f(t), g(t)) \end{aligned}$$

$$\begin{aligned} \vec{r} : \mathbb{R} &\longrightarrow \mathbb{R}^3 \\ t &\longmapsto \vec{r}(t) = (f(t), g(t), h(t)) \end{aligned}$$

$$\begin{aligned} \vec{r} : \mathbb{R} &\longrightarrow \mathbb{R}^n \\ t &\longmapsto \vec{r}(t) = (f_1(t), f_2(t), \dots, f_n(t)) \end{aligned}$$

Exemple

$$\vec{r}(t) = (1, 3) + t(1, -1) \; = (1 + t, 3 - t)$$

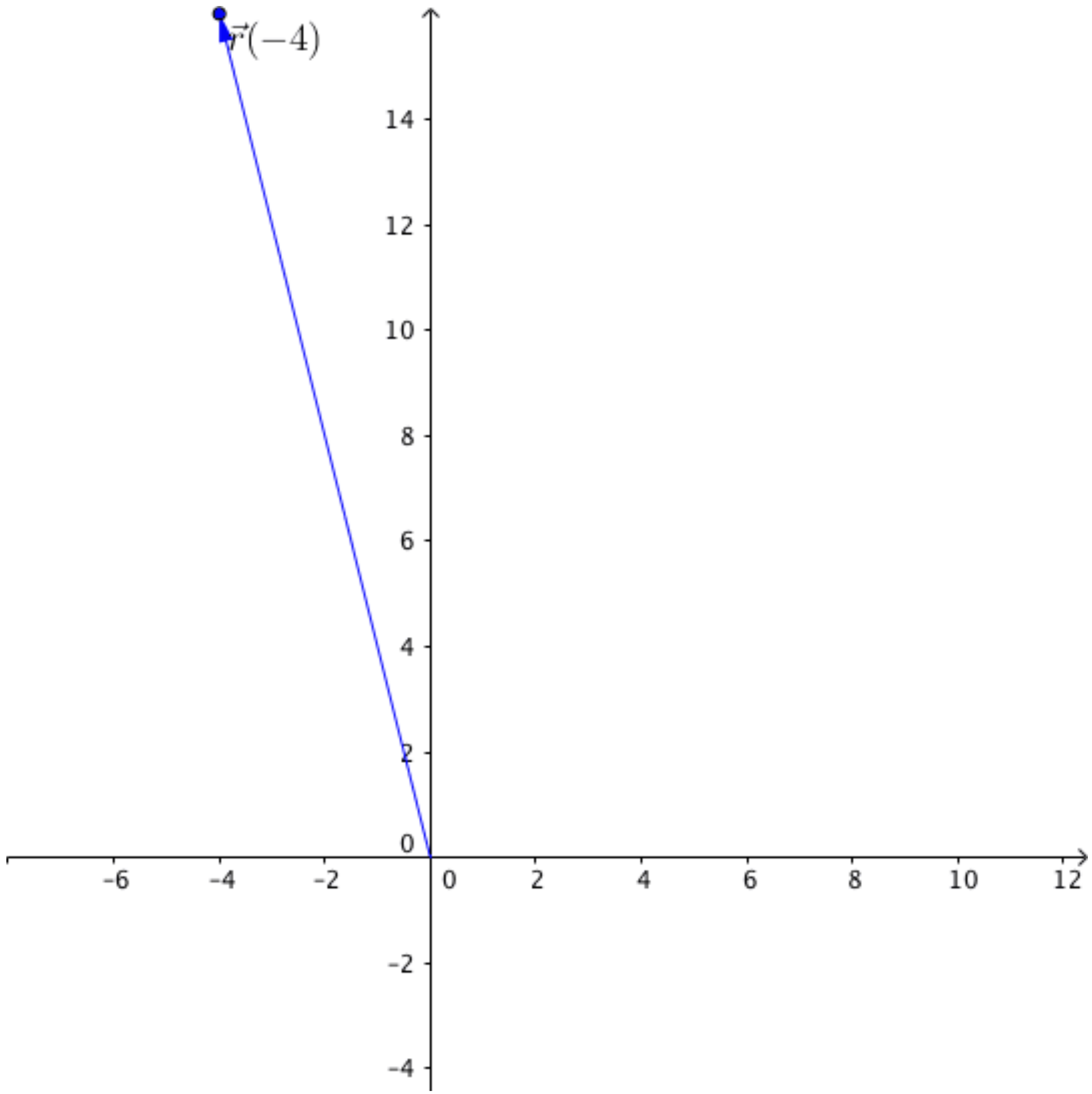


Exemple

$$\vec{r}(t) = (t, t^2)$$

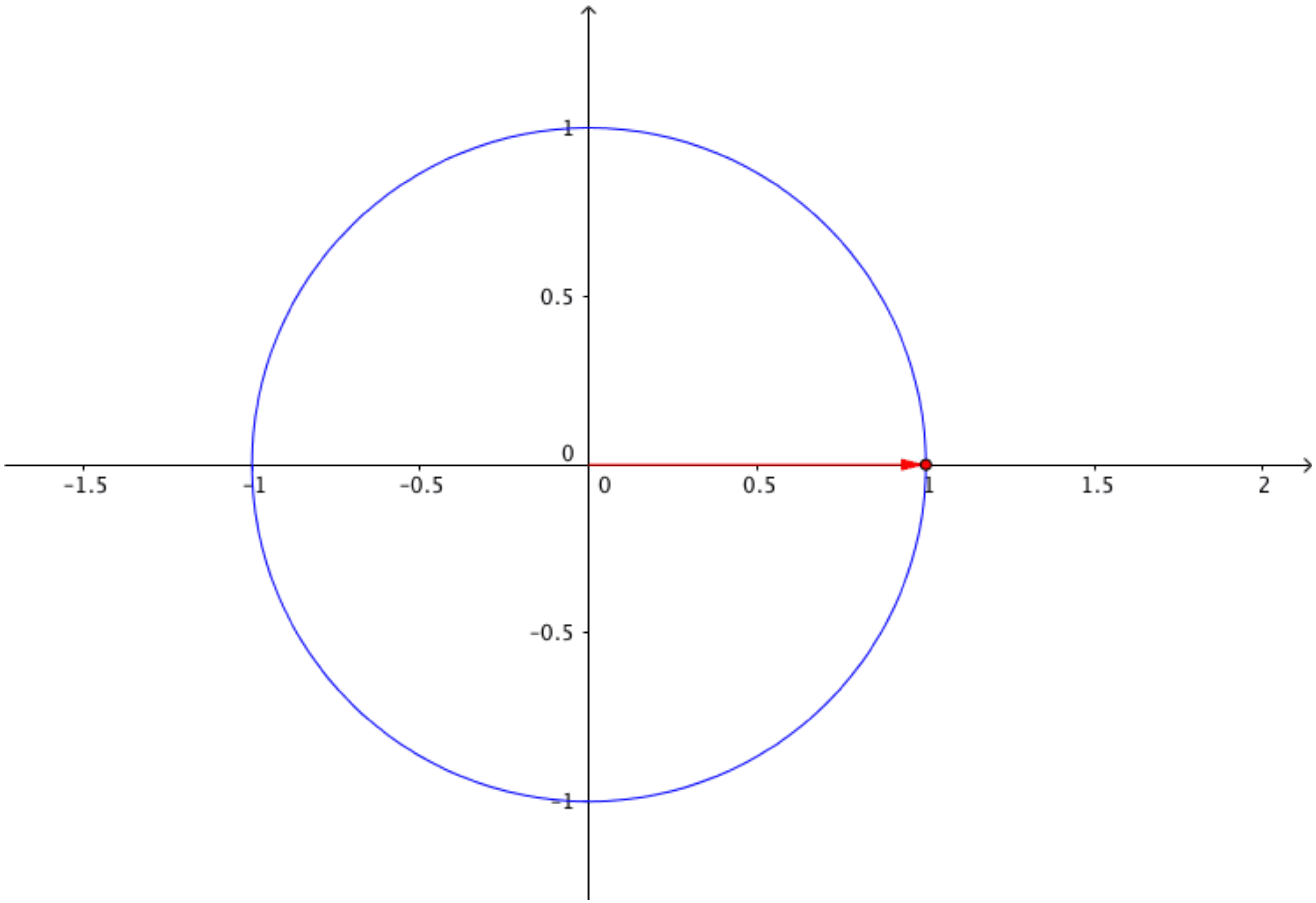
$$x = t$$

$$y = t^2 = x^2$$



Exemple Cercle $x^2 + y^2 = 1$

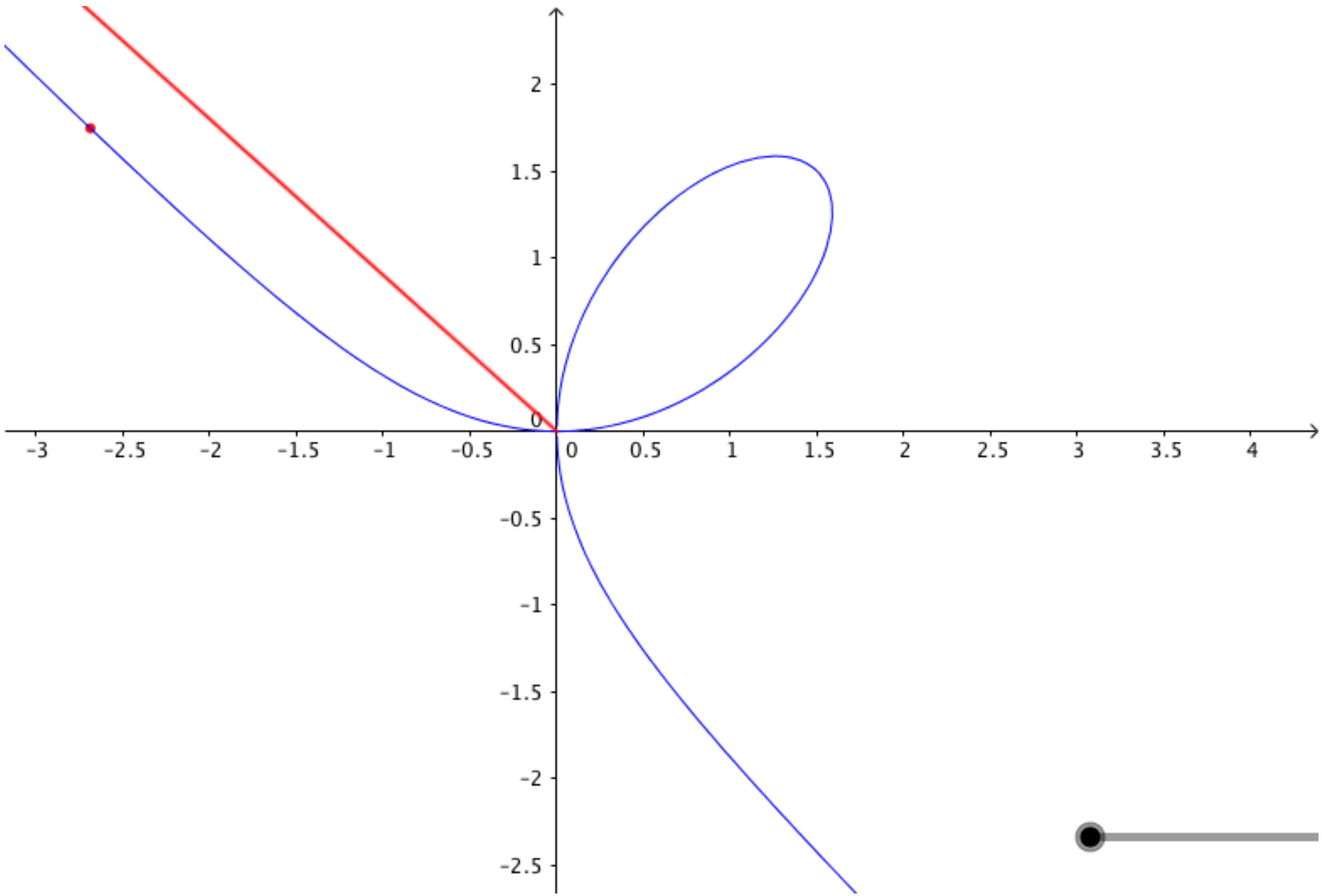
$$\vec{r}(t) = (\cos t, \sin t)$$



Exemple

Folium de Descartes $x^3 + y^3 = 3xy$

$$\vec{r}(t) = \left( \frac{3t}{1+t^3}, \frac{3t^2}{1+t^3} \right)$$

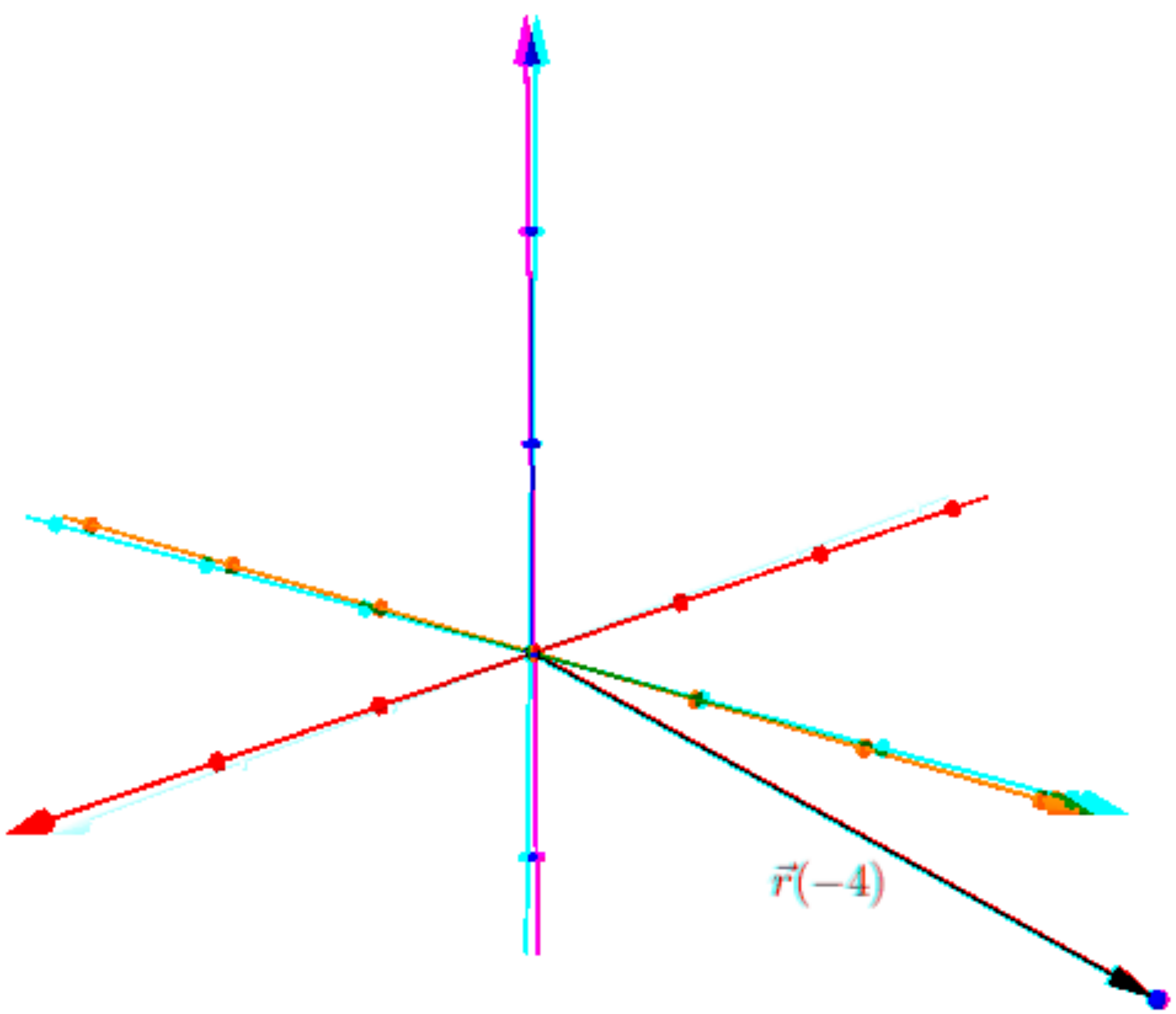


Exemple

$$\vec{r}(t) = (1+t, 1-t, 1+t)$$

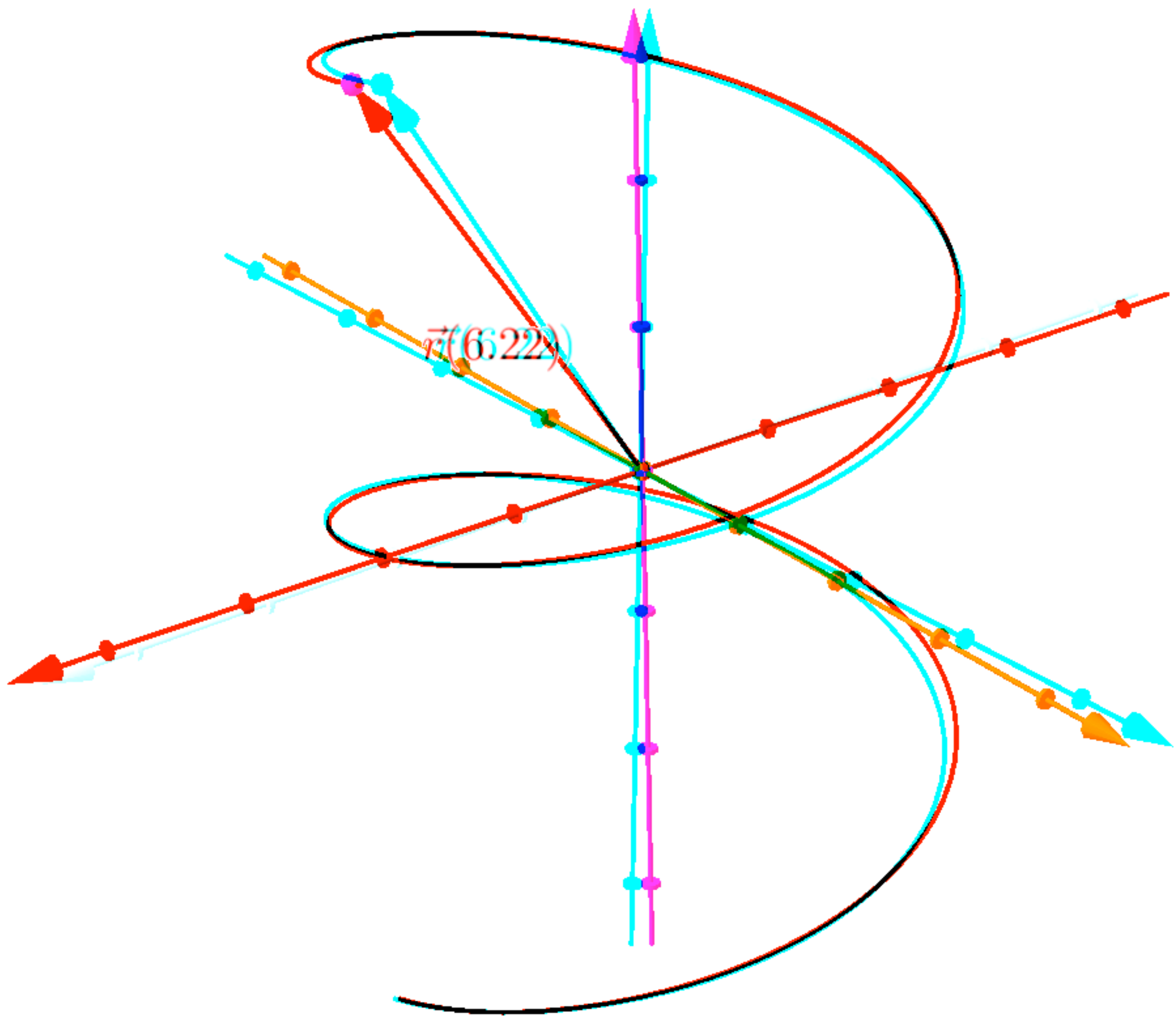
La droite de vecteur directeur $\vec{d} = (1, -1, 1)$

passant par le point $\vec{OA} = (1, 1, 1)$



Exemple $\vec{r}(t) = \left(\cos t, \sin t, \frac{t}{4}\right)$ puisque $\cos^2 t + \sin^2 t = 1$

la courbe est sur le cylindre $x^2 + y^2 = 1$

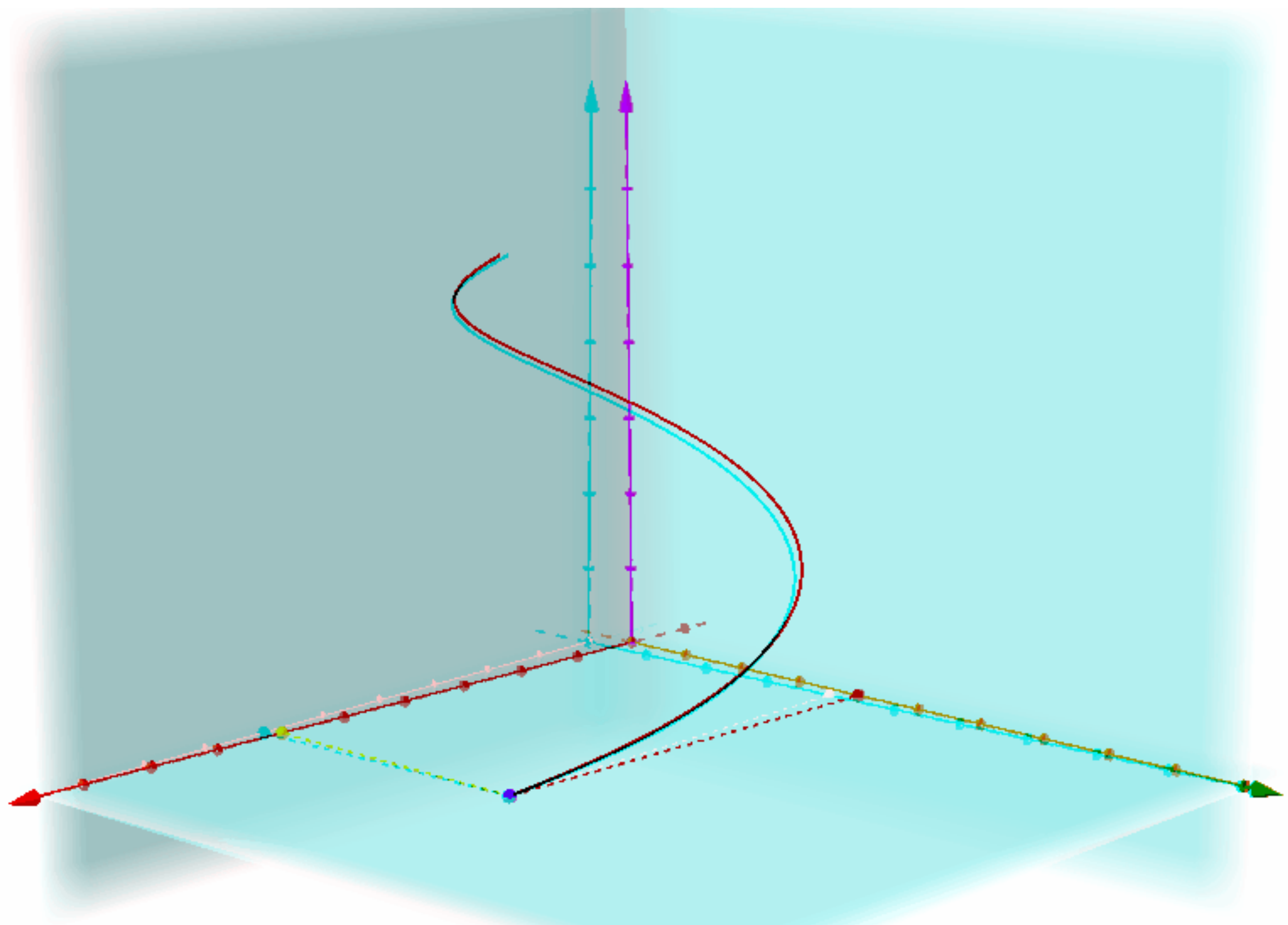


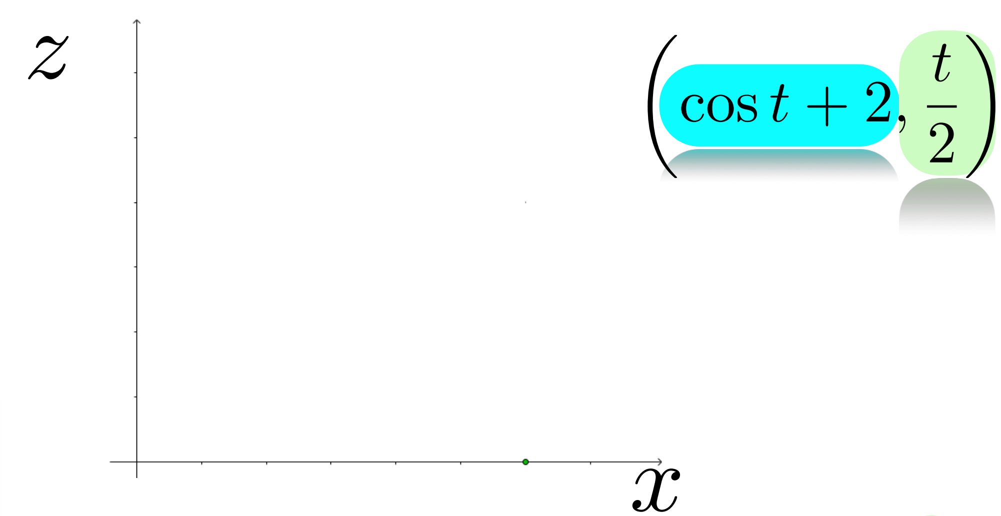
# Faites les exercices suivants

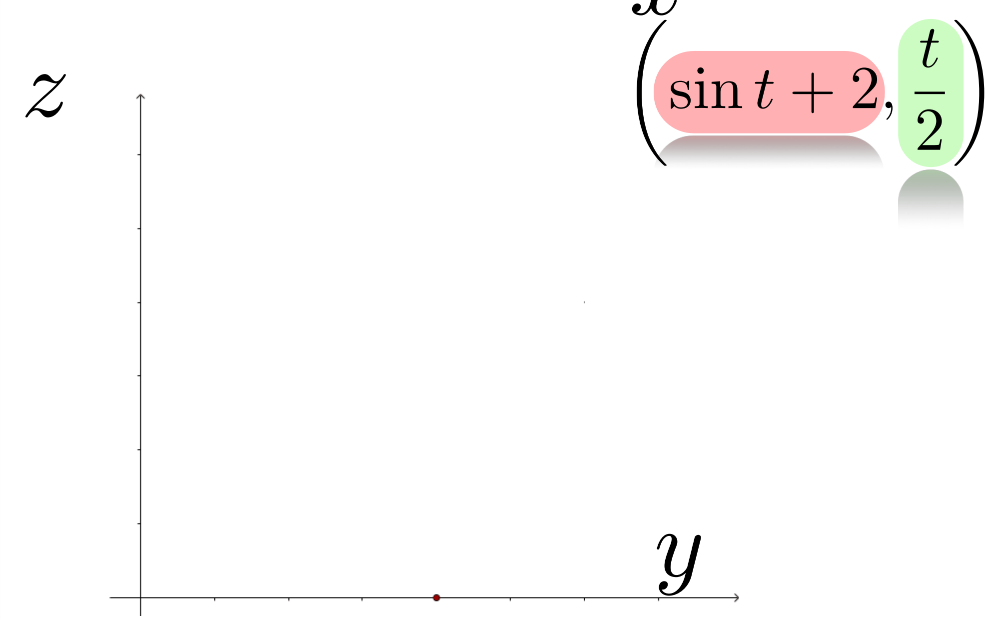
p.700 # 5, 7, 9, 10, 12 et 16

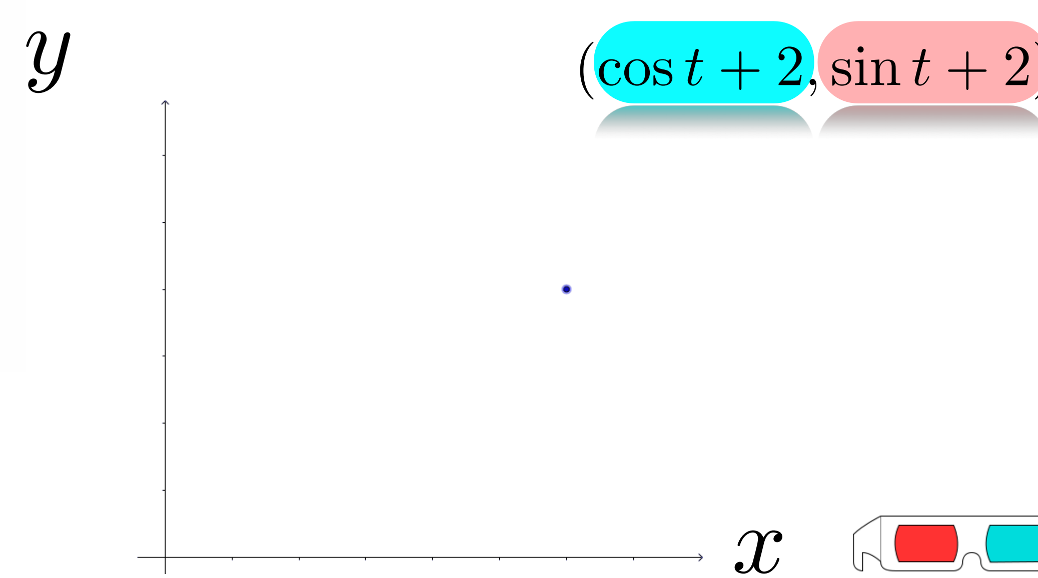
Une façon pour aider à visualiser une courbe dans l'espace est de regarder ses projections sur les plans de coordonnés.

$$\vec{r}(t) = \left(\cos t + 2, \sin t + 2, \frac{t}{2}\right)$$



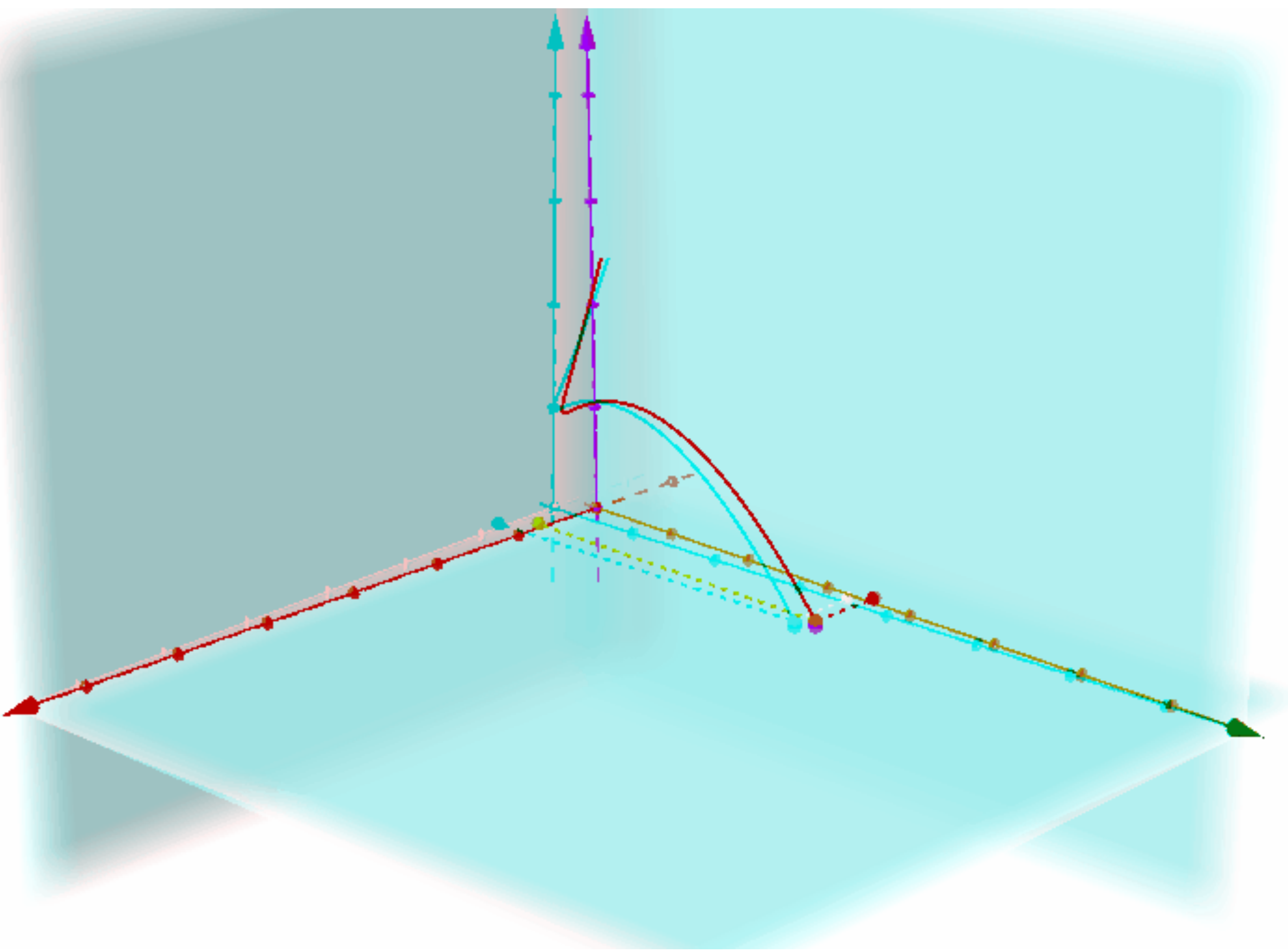


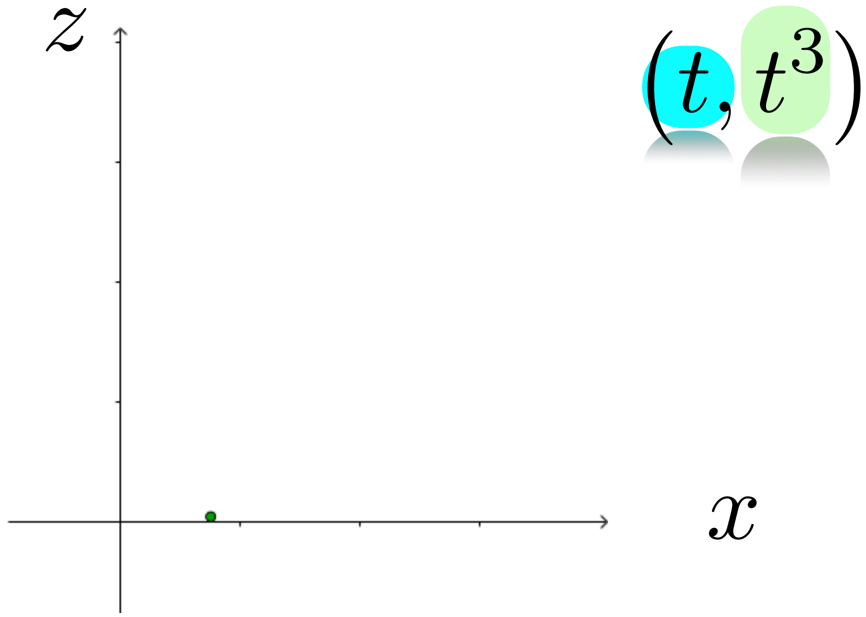



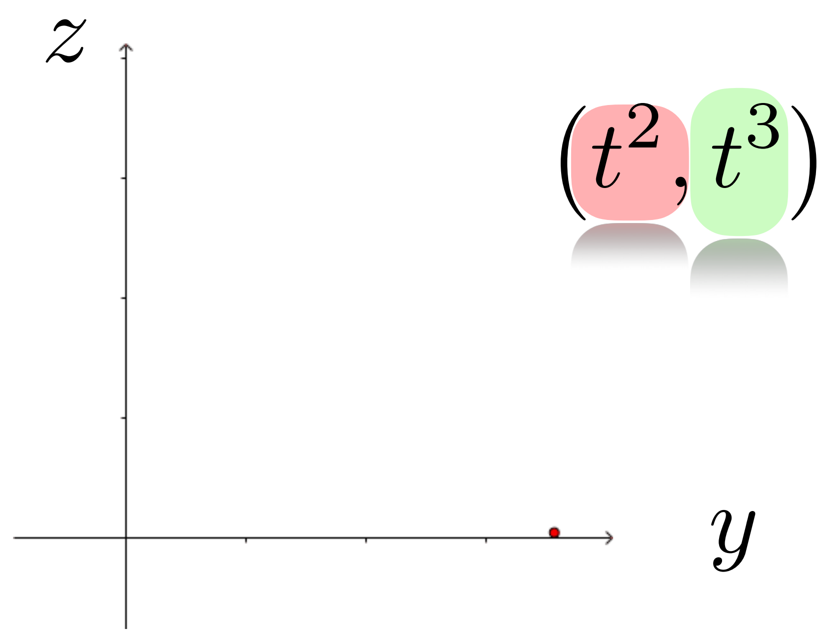



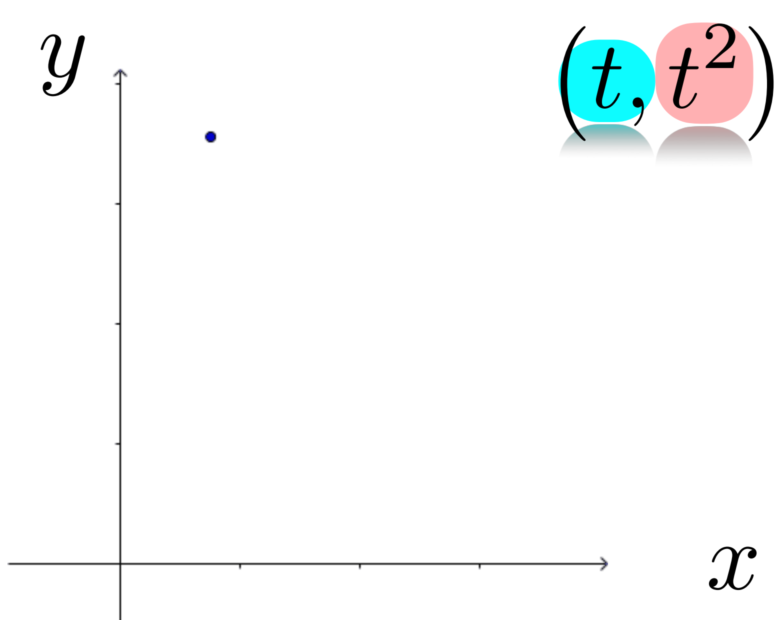
## Exemple

$$\vec{r}(t) = (t, t^2, t^3)$$

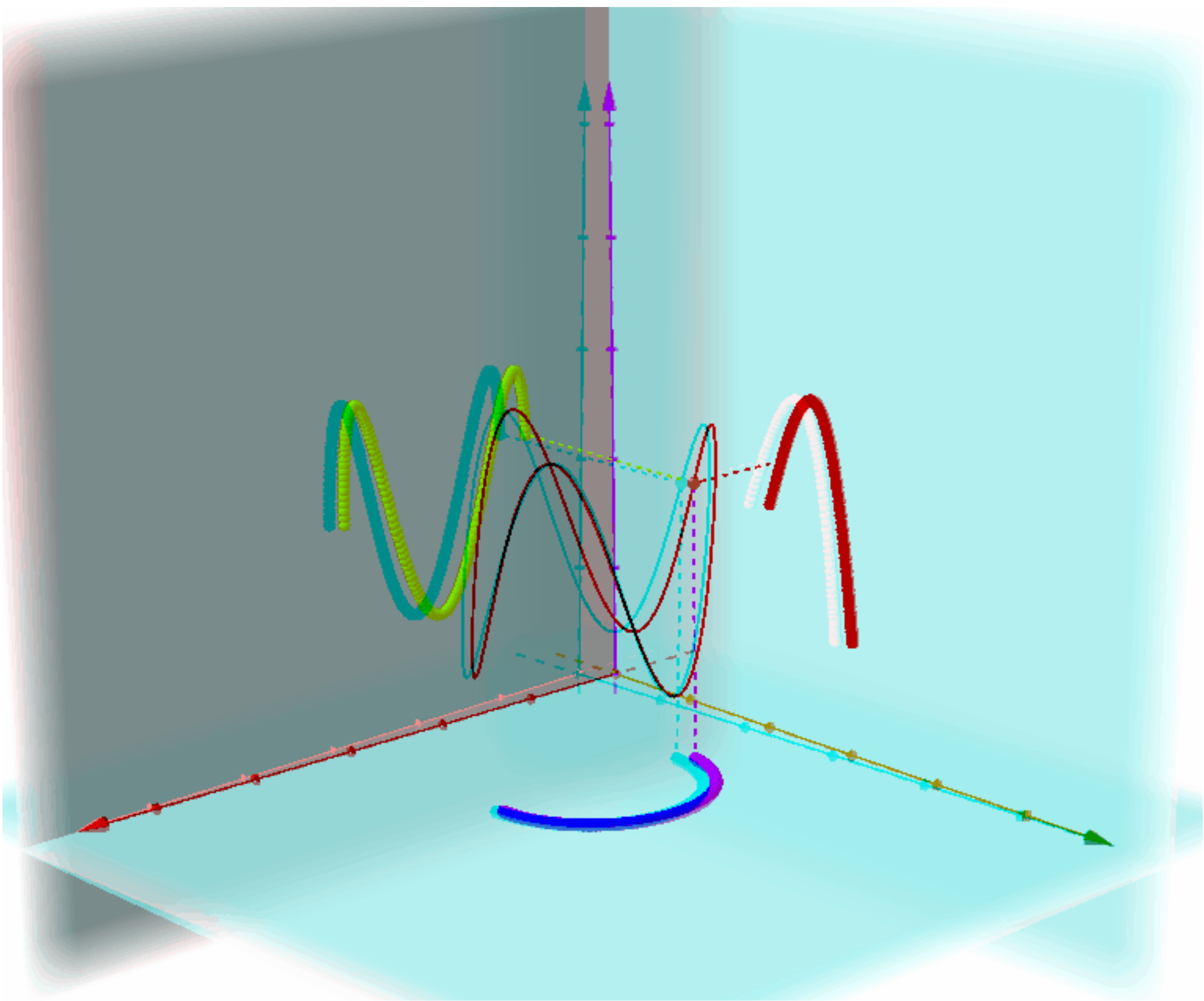


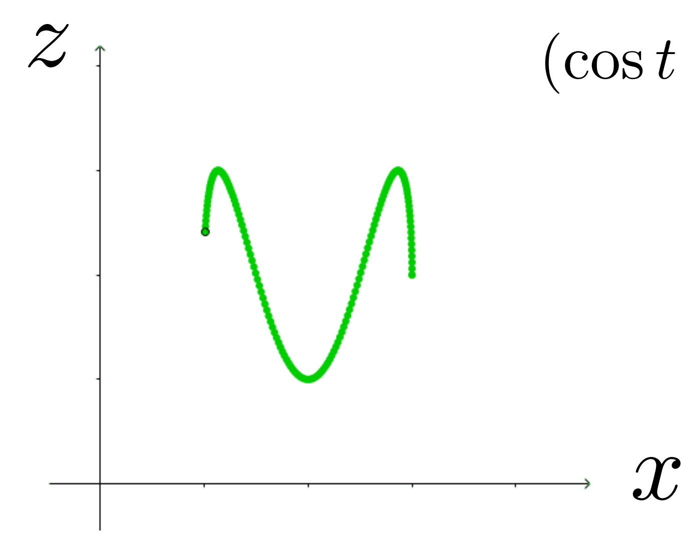


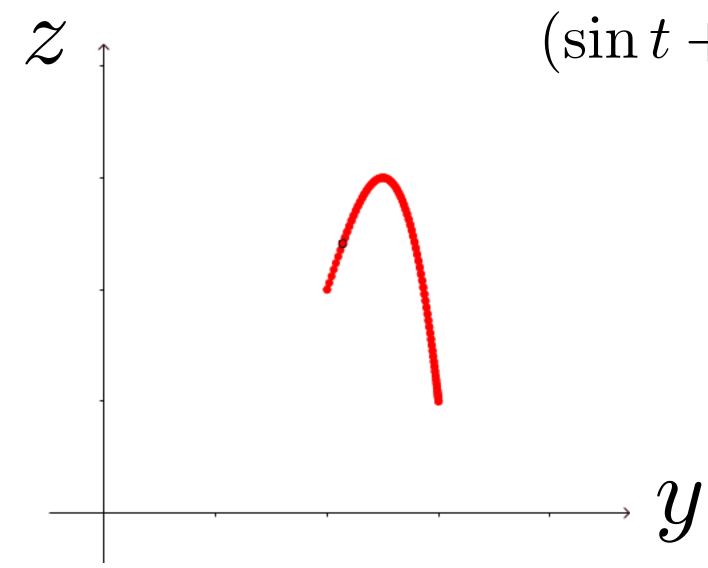



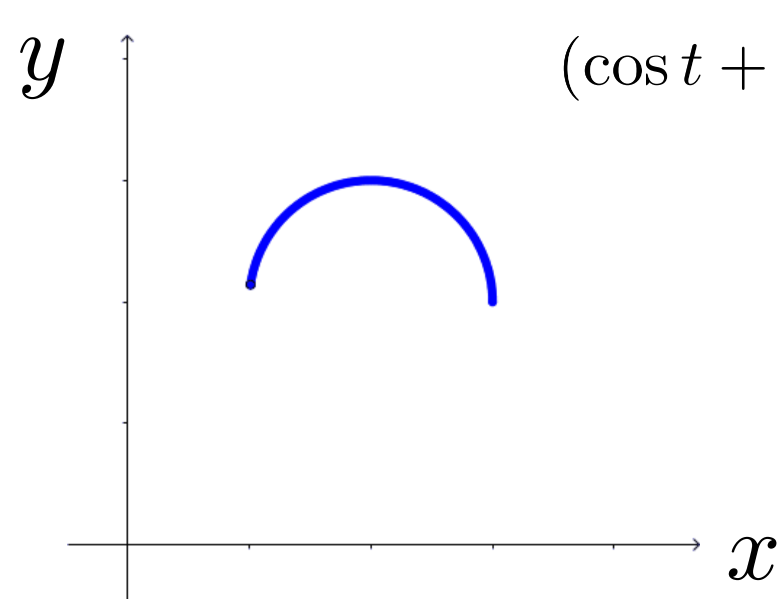



$$\vec{r}(t) = (\cos t + 2, \sin t + 2, \sin 3t + 2)$$

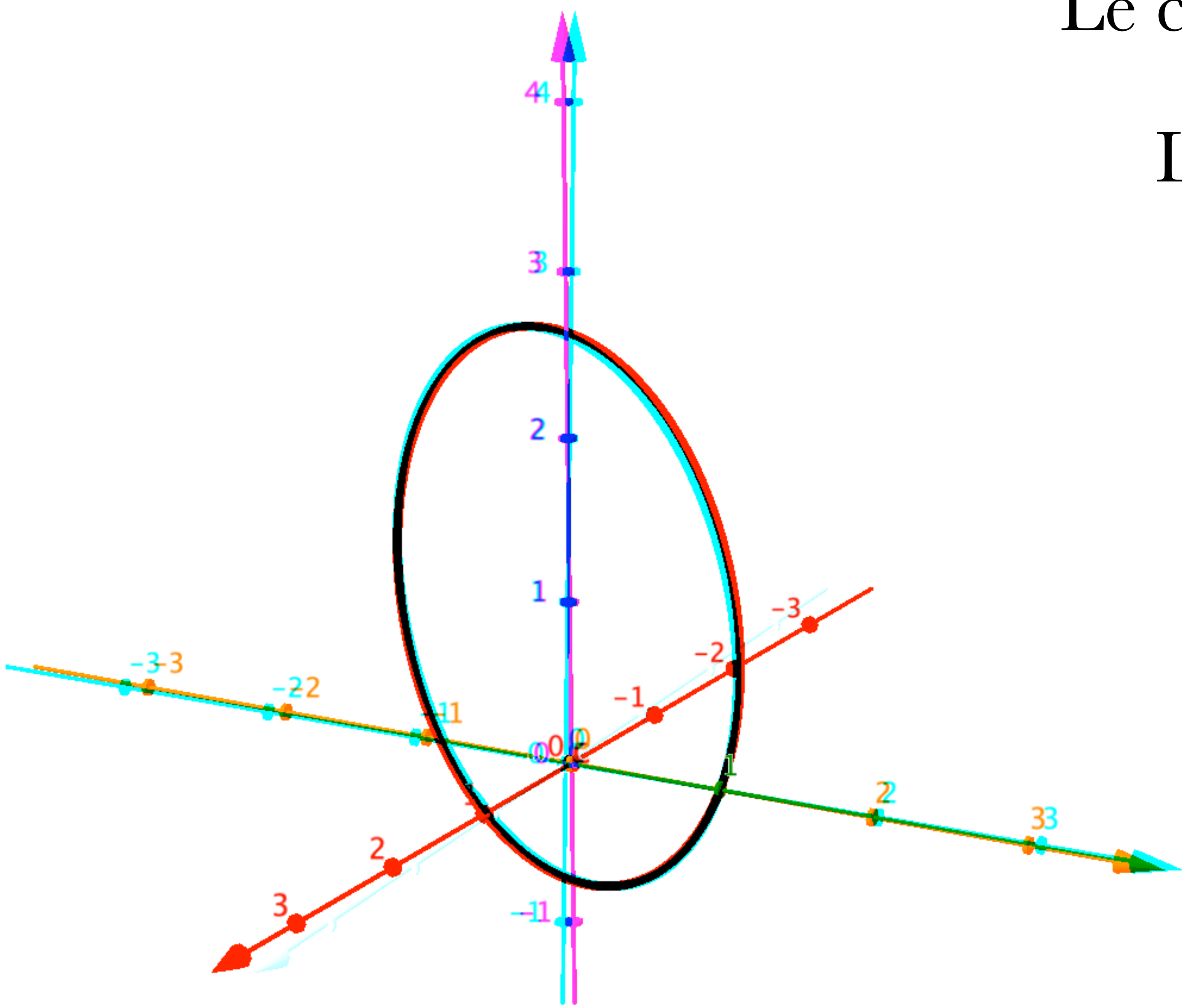




On peut construire certaines courbes en regardant l'intersection de deux surfaces.




Le cylindre $x^2 + y^2 = 1$

Le plan $x + y + z = 1$

$$x = \cos t$$

$$y = \sin t$$

$$z = 1 - x - y$$

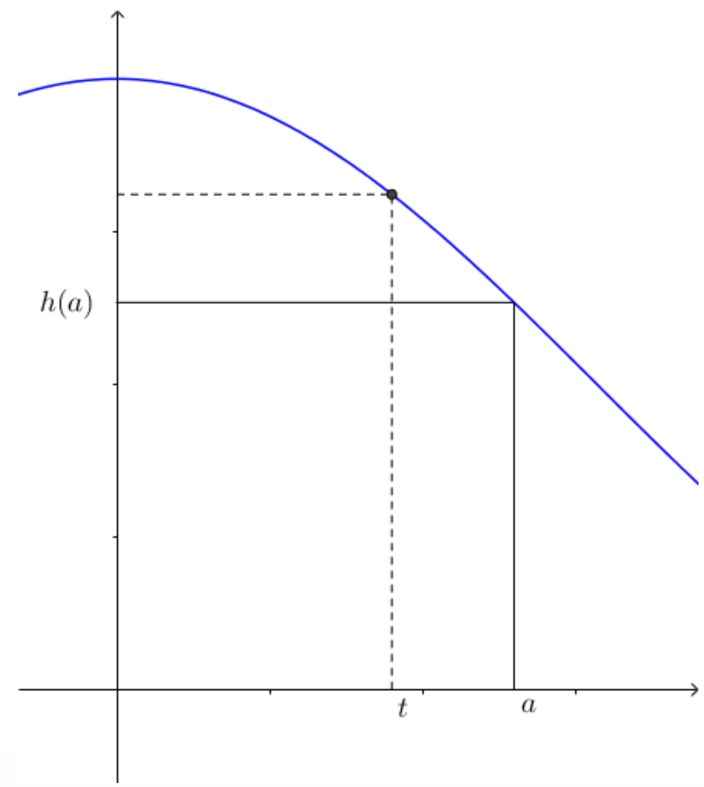
$$= 1 - \cos t - \sin t$$

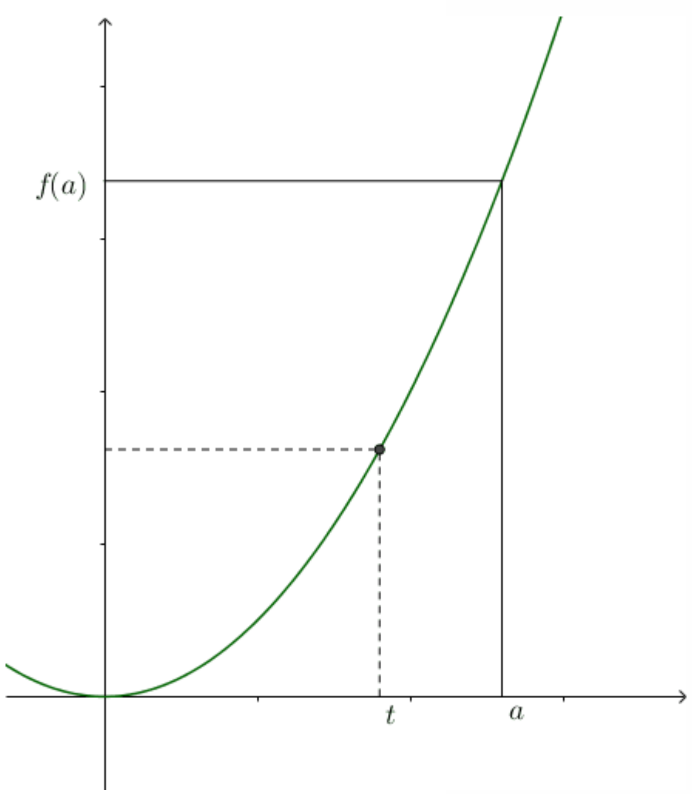
$$\vec{r}(t) = (\cos t, \sin t, 1 - \cos t - \sin t)$$

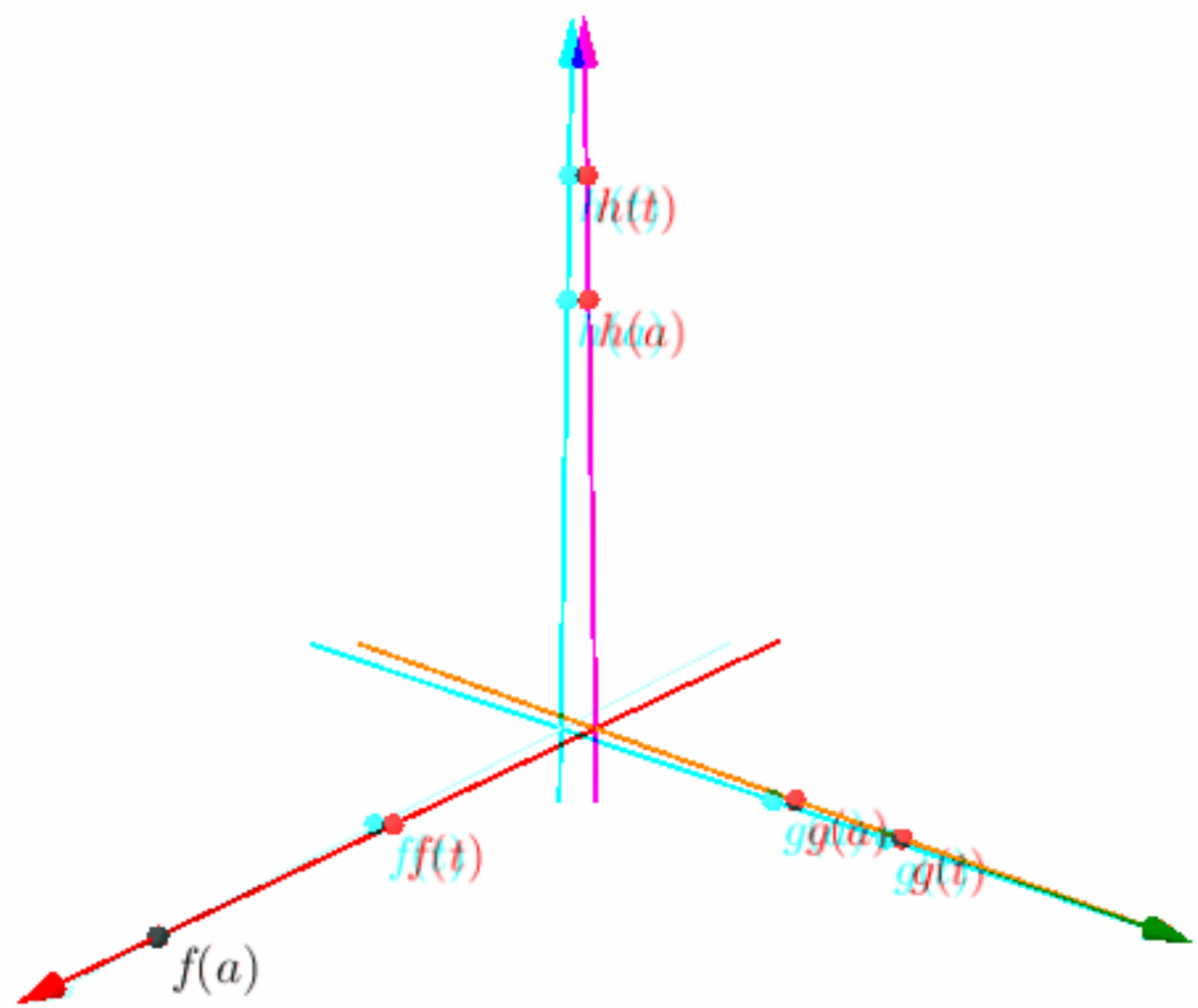
# Faites les exercices suivants

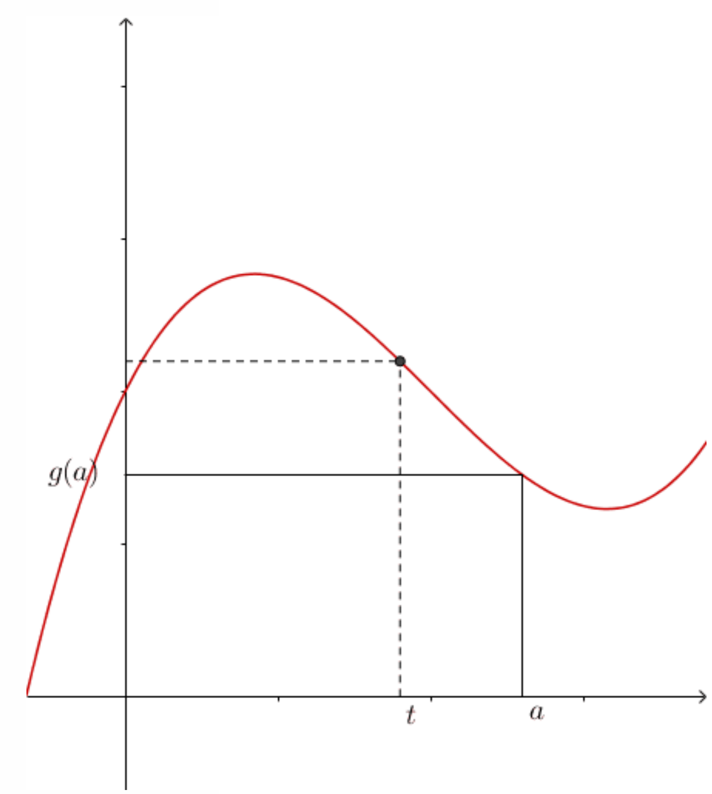
p.700 # 13, 14, 19 à 28

$$\vec{r}(t) = (f(t), g(t), h(t))$$

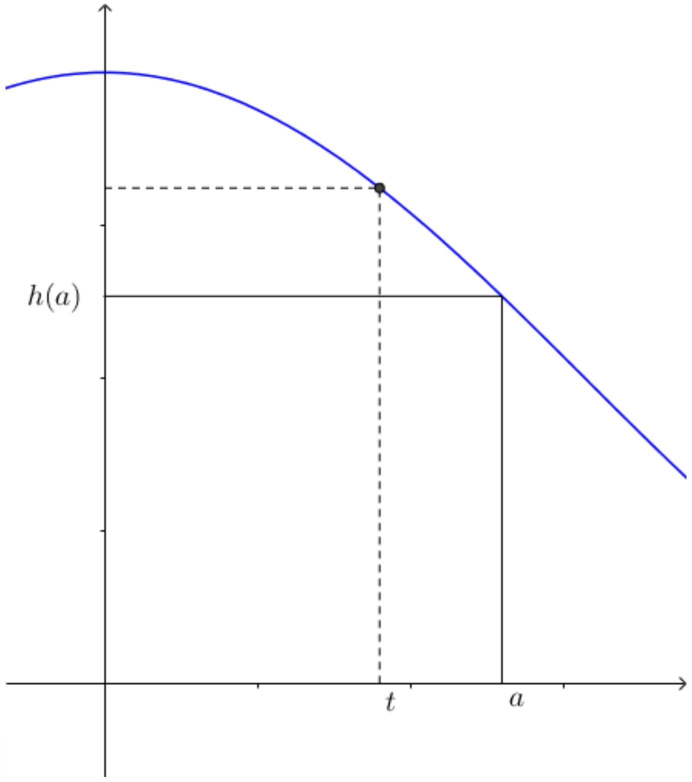


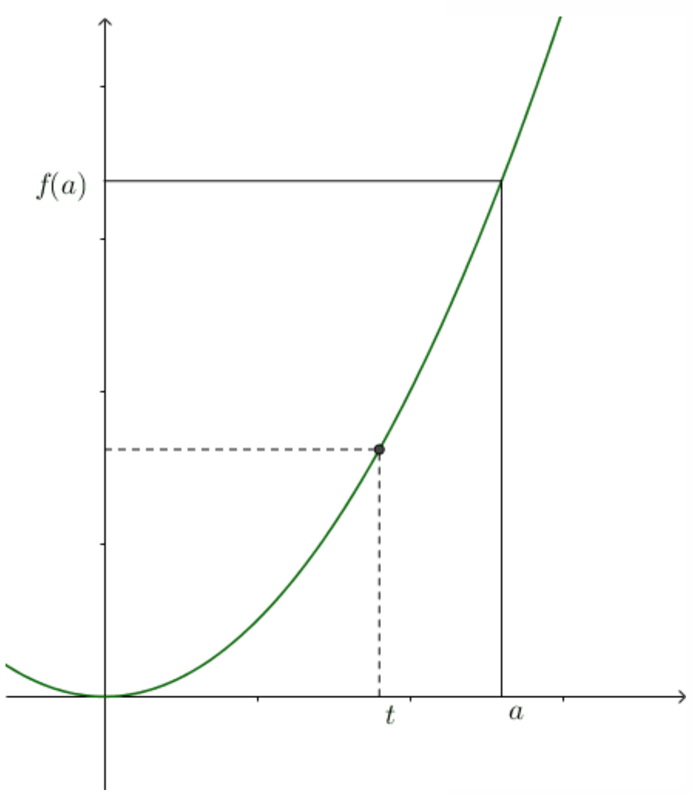



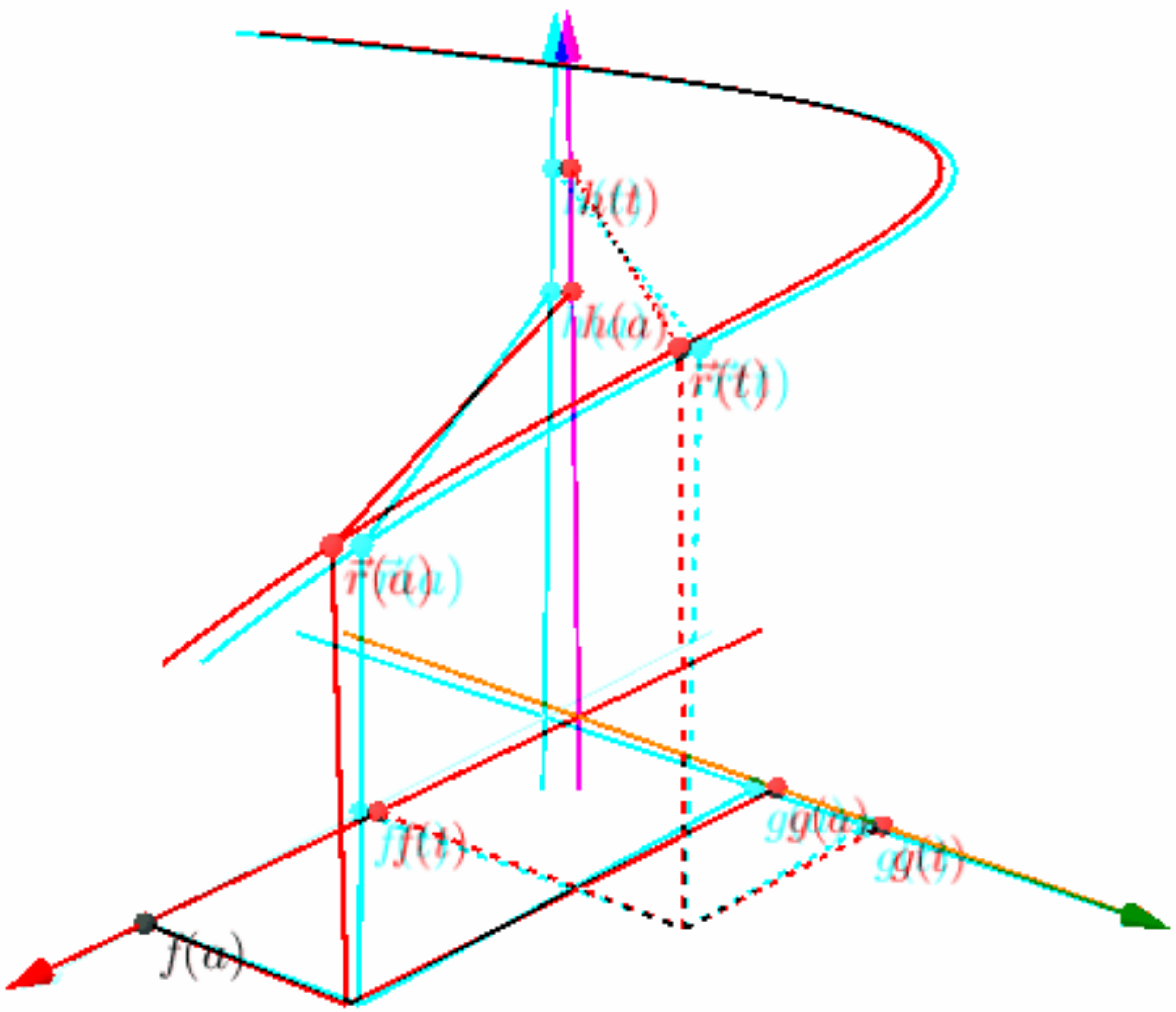



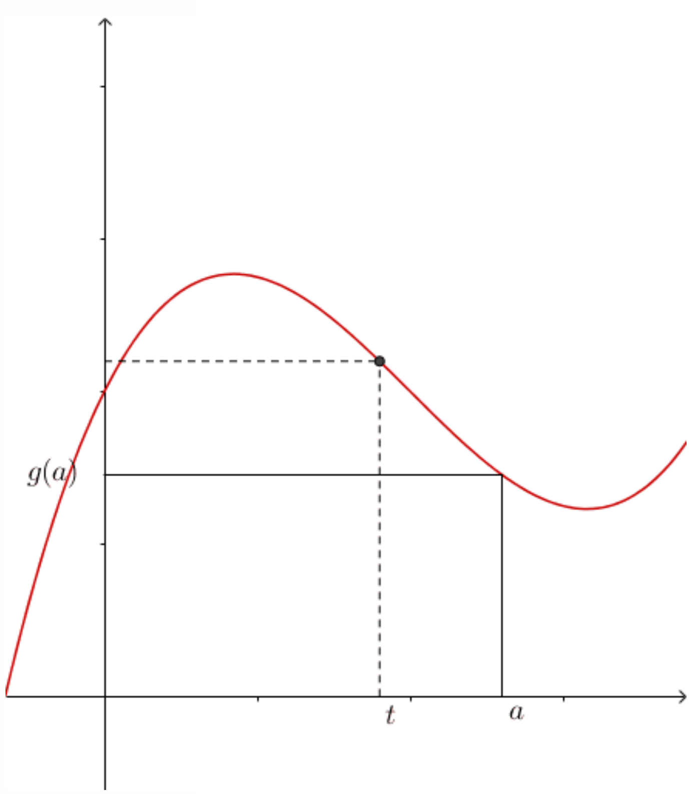



$$\vec{r}(t) = (f(t), g(t), h(t))$$



## Définition

$$\vec{r}(t) = (f(t), g(t), h(t))$$

La limite d'une fonction vectorielle est

$$\lim_{t \to a} \vec{r}(t) = \left(\lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t)\right)$$

## Définition

On dit qu'une fonction vectorielle est continue en $t = a$ si

$$\lim_{t \to a} \vec{r}(t) = \vec{r}(a)$$

**Exemple** $\vec{r}(t) = (t, \sin t, \cos t)$

$$\lim_{t \to \frac{\pi}{2}} \vec{r}(t) = \left( \lim_{t \to \frac{\pi}{2}} t, \lim_{t \to \frac{\pi}{2}} \sin t, \lim_{t \to \frac{\pi}{2}} \cos t \right) = \left( \frac{\pi}{2}, 1, 0 \right)$$

**Exemple** $\vec{r}(t) = \left( e^{-t}, \frac{4}{1-t}, \frac{4t^2 + t}{5t^2 - 6} \right)$

$$\lim_{t \to \infty} \vec{r}(t) = \left( \lim_{t \to \infty} e^{-t}, \lim_{t \to \infty} \frac{4}{1-t}, \lim_{t \to \infty} \frac{4t^2 + t}{5t^2 - 6} \right) = \left( 0, 0, \frac{4}{5} \right)$$

# Faites les exercices suivants

p.699 # 1 à 4

On vient de voir comment obtenir une courbe à l'aide d'une fonction vectorielle à un paramètre.

$$\vec{r}(t)$$

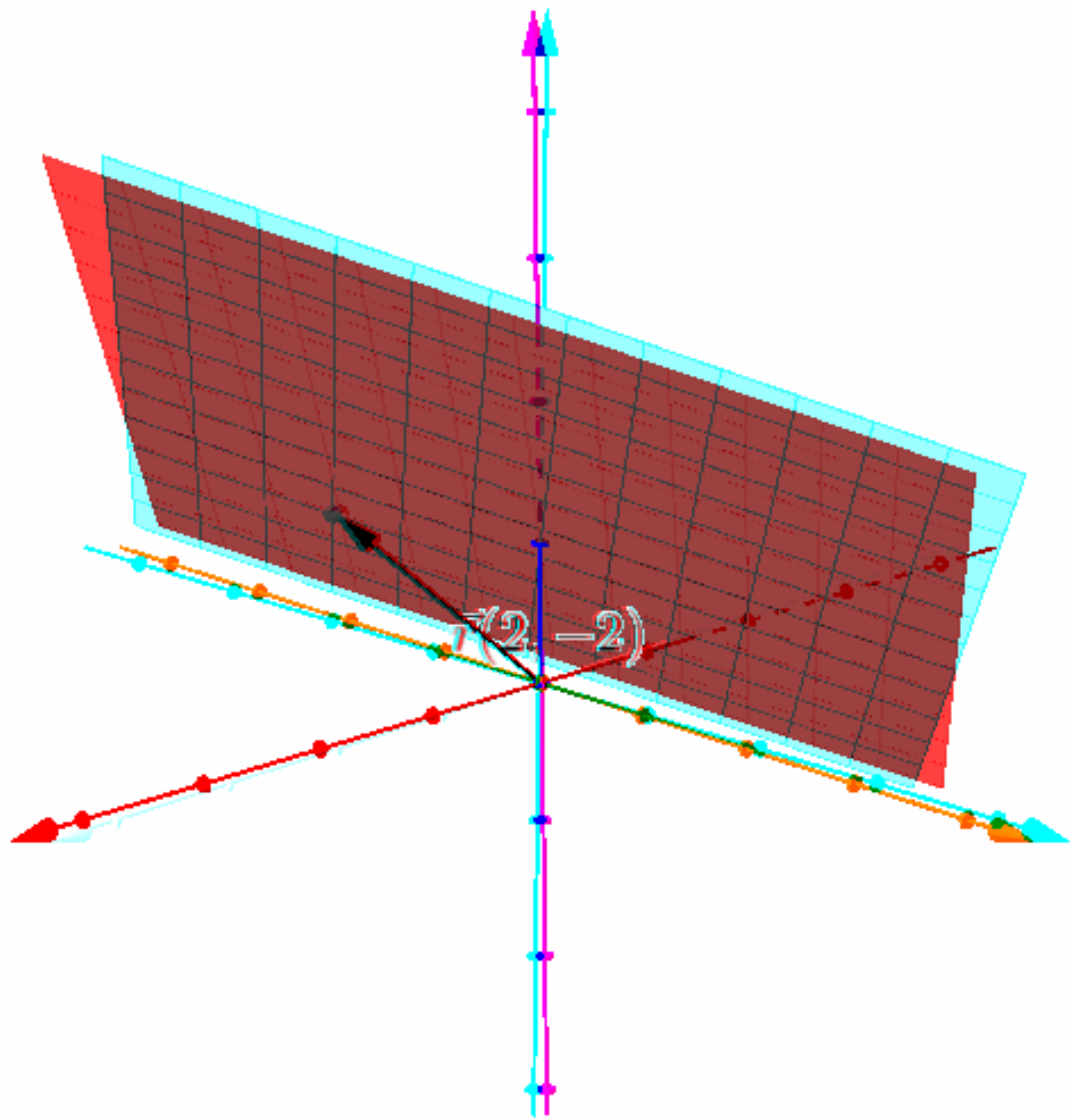
De manière analogue, on peut obtenir une surface à l'aide d'une fonction vectorielle à deux paramètres.

$$\vec{r}(u,v) = (f(u,v), g(u,v), h(u,v))$$

**Exemple**

**Plan**

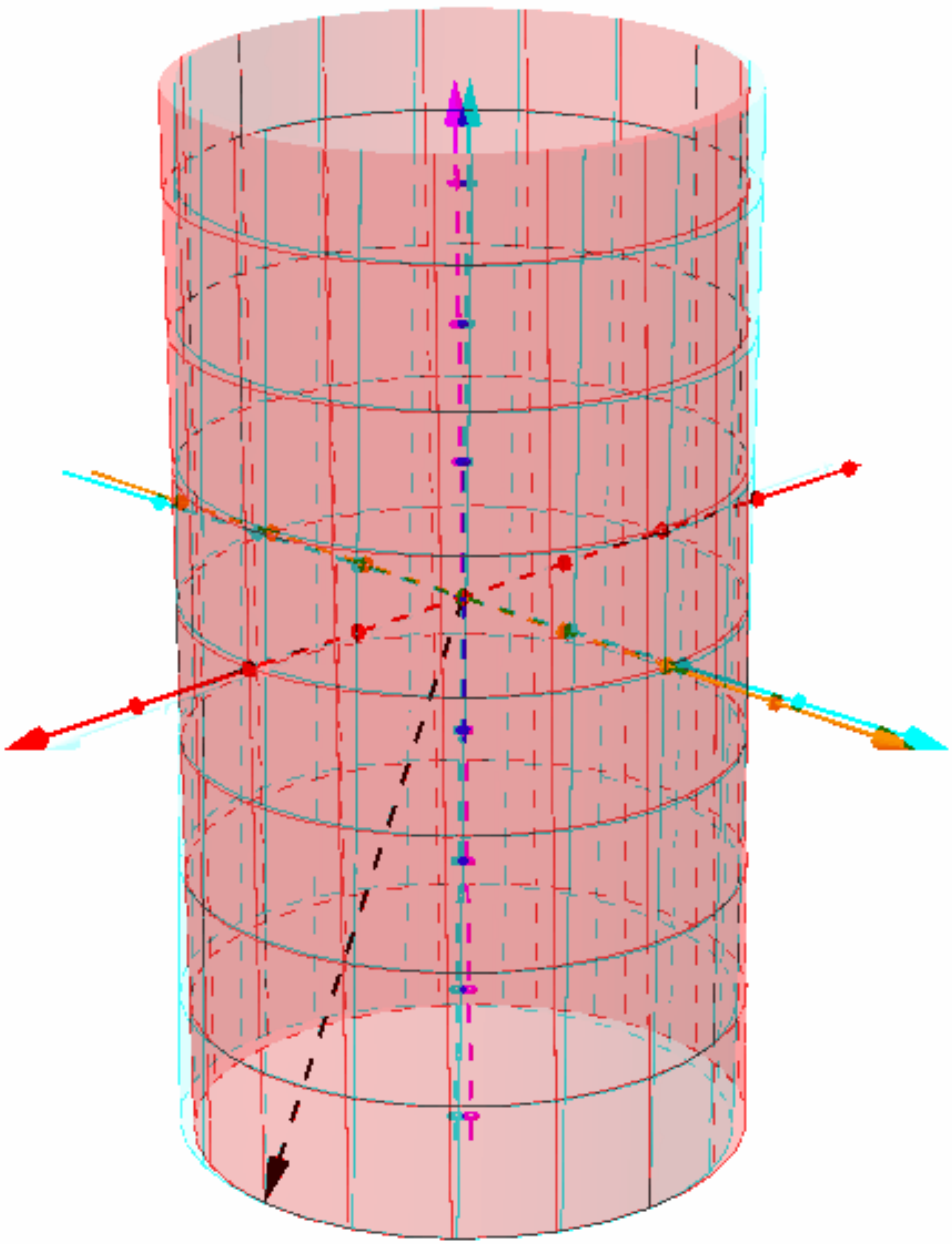
$$\vec{r}(u,v) = \left(1+u-v, 1-u+v, 3+\frac{u}{2}+v\right)$$
$$= (1,1,3) + u\left(1,-1,\frac{1}{2}\right) + v(-1,1,1)$$



Exemple

$$\vec{r}(\theta, h) = (\cos\theta, \sin\theta, h)$$

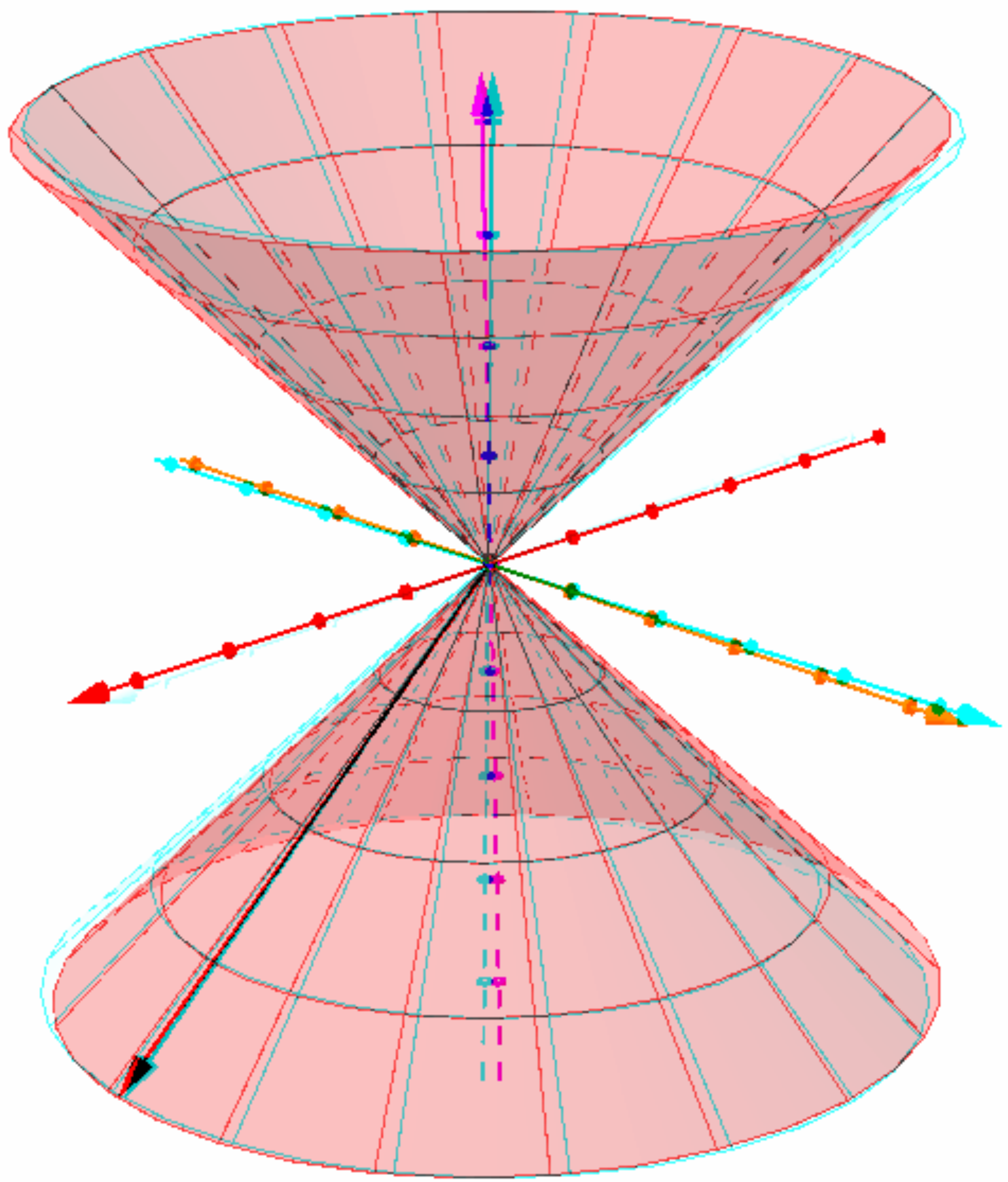
Cylindre

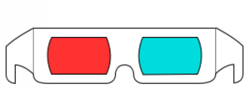


Exemple

$$\vec{r}(t,\theta) = (t\cos\theta, t\sin\theta, t)$$

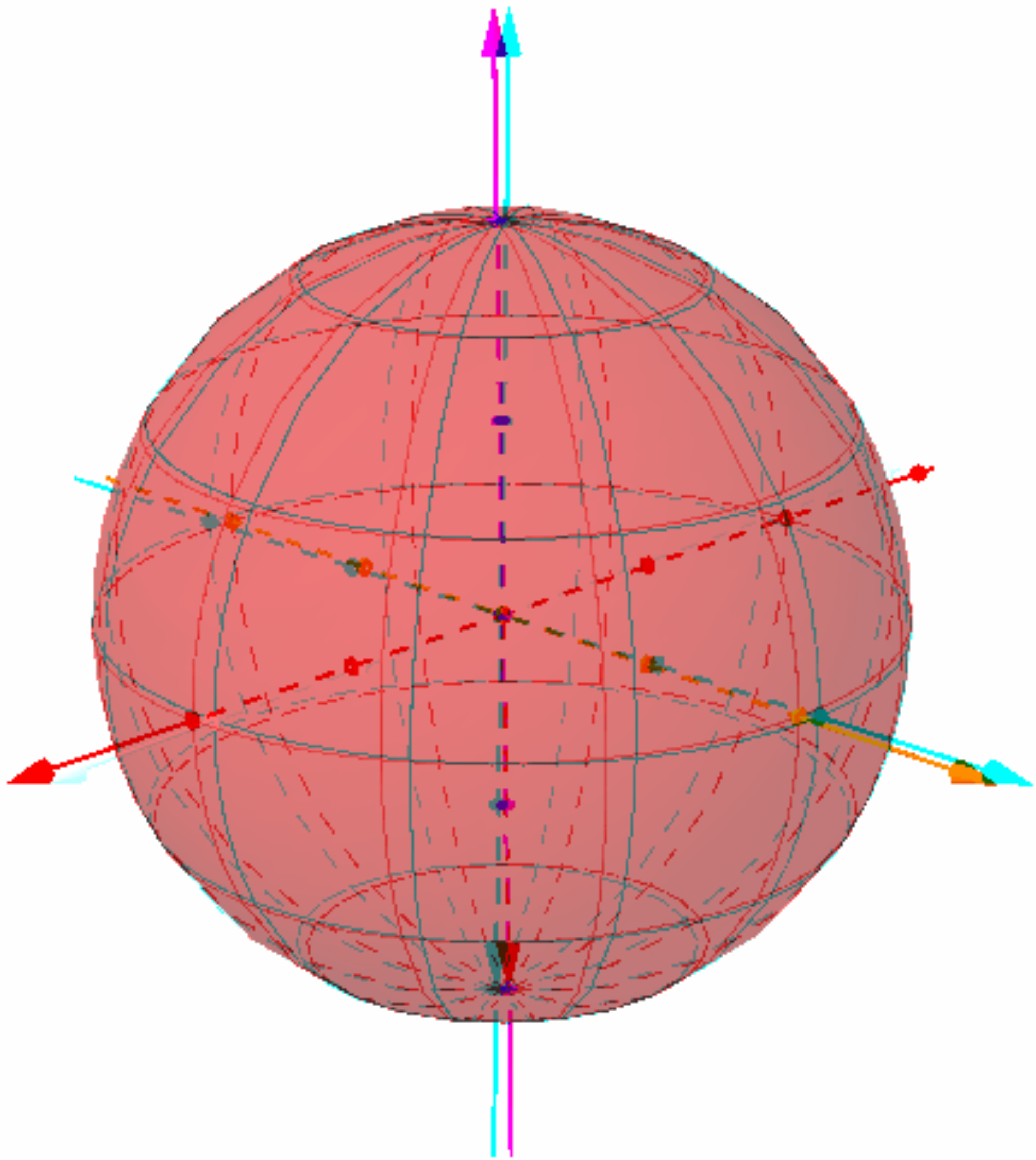
Cône



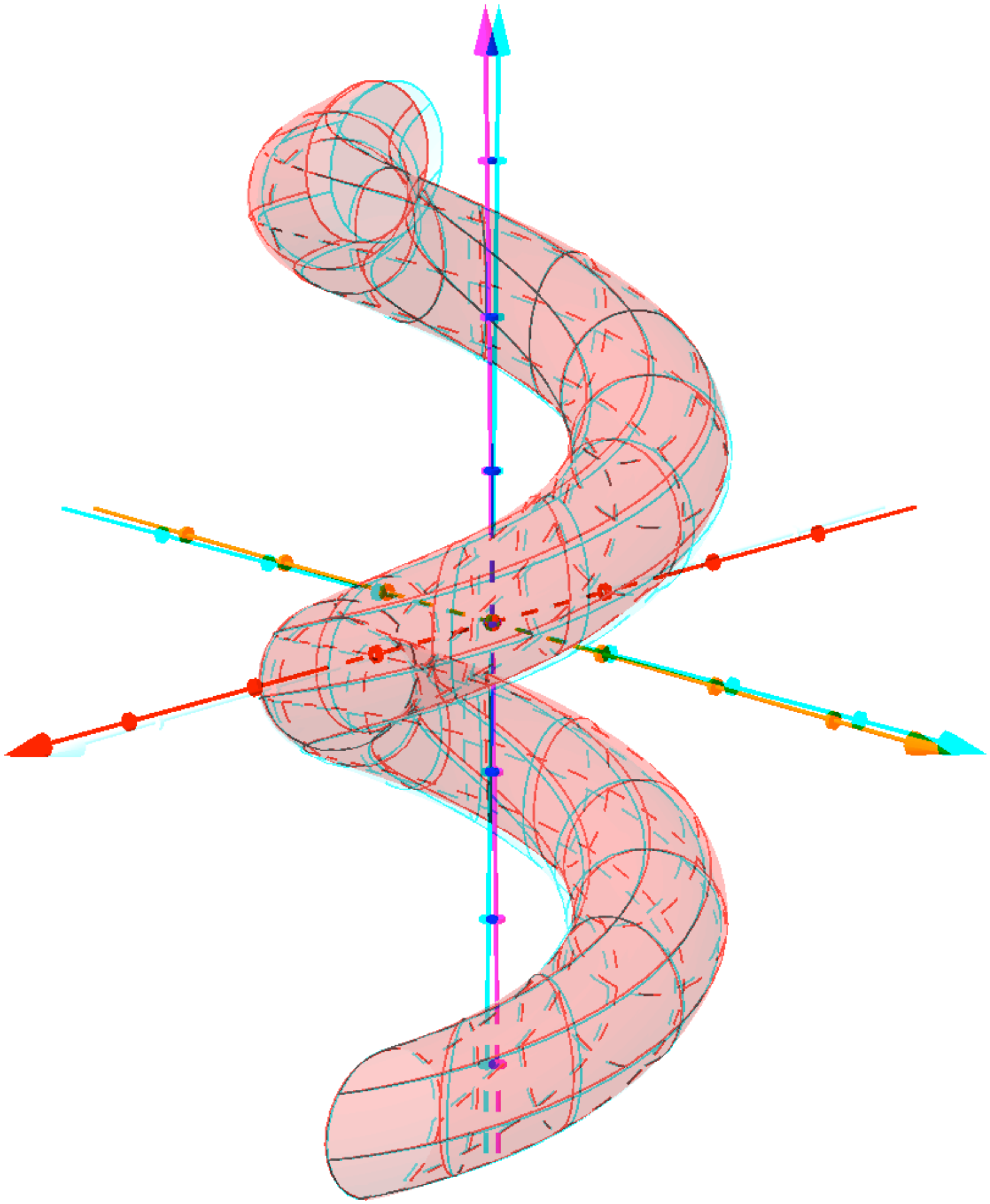


**Exemple**

$\vec{r}(\theta, \phi) = (\cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi)$ Sphère



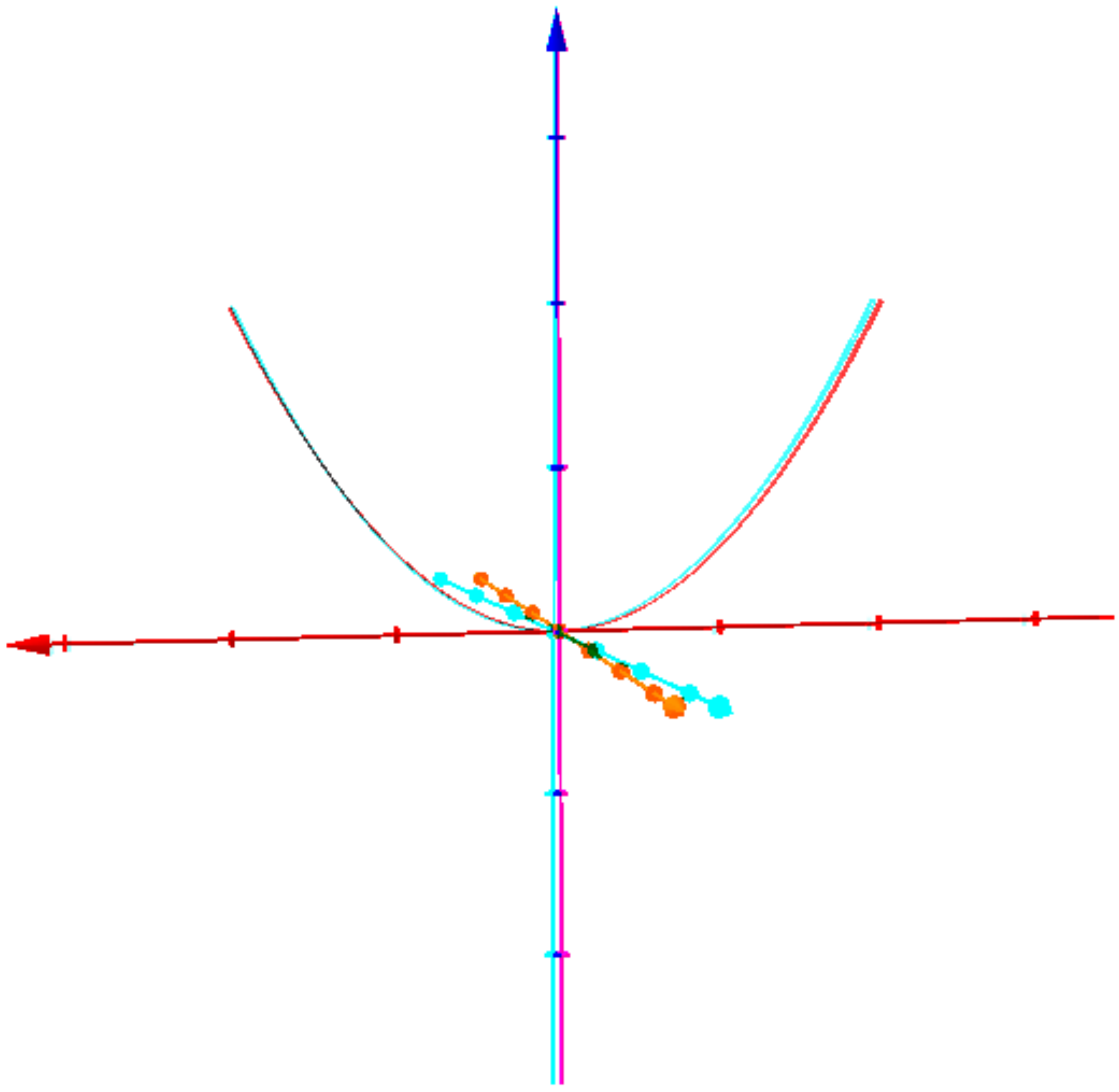
Exemple $\vec{r}(u,v) = ((2+\sin v)\cos u, (2+\sin v)\sin u, u+\cos v)$



Exemple

On peut aussi voir une surface de révolution d'une fonction $(x, f(x))$, comme vu en NYB.

$$\vec{r}(x, \theta) = (x, f(x)\cos\theta, f(x)\sin\theta)$$



# Faites les exercices suivants

p.731 # 1 à 6 et 13 à 18

# Aujourd’hui, nous avons vu

- Fonctions vectorielles
- Comment les visualiser
- Limite
- Surfaces paramétrées

Devoir:

p.699 # 1 à 28, 35 à 40 et 43 à 45

p.731 # 1 à 26