Examen 3 (solutions)
201-GNF Calcul 3
18 avril 2019
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Question 1. (24%)

a) $\displaystyle\int_{-1}^{2}\int_{0}^{1}\frac{1+x}{1+y}\;dy\;dx=\int_{-1}^{2}(1+x)\ln|1+y|\Big|_{0}^{1}\;dx=\ln 2\int_{-1}^{2}(1+x)\;dx=\ln 2\left(x+\frac{x^2}{2}\right)\Big|_{-1}^{2}=\frac{9\ln 2}{2}$

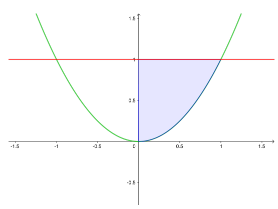
b) $\displaystyle\int_{0}^{2}\int_{x^2}^{2x}5x^2+3y^2\;dy\;dx=\int_{0}^{2}5x^2y+y^3\Big|_{x^2}^{2x}\;dx=\int_{0}^{2}18x^3-5x^4-x^6\;dx=\frac{18(2)^4}{4}-2^5-\frac{2^7}{7}=\frac{152}{7}$

Question 2. (12%)

$$\begin{aligned}\int_{0}^{2\pi}\int_{2\cos\theta}^{3}r^2\sin\theta\;dr\;d\theta&=\int_{0}^{2\pi}\frac{r^3}{3}\sin\theta\Big|_{2\cos\theta}^{3}\;d\theta=\int_{0}^{2\pi}9\sin\theta-\frac{8\cos^3\theta\sin\theta}{3}d\theta\\&=9\cos\theta+\frac{8}{12}\cos^4\theta\Big|_{0}^{2\pi}=0\end{aligned}$$

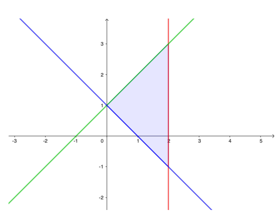
Question 3. (22%)

a)



$$\begin{aligned}\int_{0}^{1}\int_{0}^{\sqrt{y}}4x^3\sin(y^3)\;dx\;dy&=\int_{0}^{1}x^4\sin(y^3)\Big|_{0}^{\sqrt{y}}\;dy=\int_{0}^{1}y^2\sin(y^3)dy=-\frac{1}{3}\cos(y^3)\Big|_{0}^{1}\\&=-\frac{1}{3}\cos(1)+\frac{1}{3}\end{aligned}$$

b)



$$\int_0^2 \int_{-x+1}^{x+1} e^{x^2}\ dy\ dx = \int_0^2 e^{x^2}(x+1-(-x+1))\ dx = 2\int_0^2 xe^{x^2}\ dx$$
$$= e^{x^2}\Big|_0^2 = e^4 - 1$$

Question 4. (22%)

Utiliser les coordonnée cylindrique ou sphérique pour calculer les intégrales suivantes sur les régions spécifiées.

a)

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_1^2 \rho^3 e^{\rho^4} \sin^2\phi\cos\theta\ d\rho\ d\phi\ d\theta = \frac{1}{4}\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} e^{\rho^4}\sin^2\phi\cos\theta\Big|_1^2\ d\phi\ d\theta$$
$$= \frac{e^{16}-e}{4}\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin^2\phi\cos\theta\ d\phi\ d\theta$$
$$= \frac{e^{16}-e}{8}\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} (1-\cos 2\phi)\cos\theta\ d\phi\ d\theta$$
$$= \frac{e^{16}-e}{8}\int_0^{\frac{\pi}{2}} \left(\phi - \frac{\sin 2\phi}{2}\right)\cos\theta\Big|_0^{\frac{\pi}{2}}\ d\theta$$
$$= \frac{\pi(e^{16}-e)}{16}\int_0^{\frac{\pi}{2}} \cos\theta\ d\theta = = \frac{\pi(e^{16}-e)}{16}\sin\theta\Big|_0^{\frac{\pi}{2}} = \frac{\pi(e^{16}-e)}{16}$$

b)

$$\int_0^{2\pi}\int_1^2\int_0^{r\cos\theta} r^2\sin\theta\ dz\ dr\ d\theta = \int_0^{2\pi}\int_1^2 r^3\cos\theta\sin\theta\ dr\ d\theta = \int_0^{2\pi}\frac{r^4}{4}\cos\theta\sin\theta\Big|_1^2\ d\theta$$
$$= \frac{15}{4}\int_0^{2\pi}\cos\theta\sin\theta\ d\theta = \frac{15}{4}\ \frac{\sin^2\theta}{2}\Big|_0^{2\pi} = 0$$

Question 5. (10%)

$$\iint_R \sqrt{1+(-2x)^2+(2y)^2}dxdy = \iint_R \sqrt{1+4x^2+4y^2}dxdy$$
$$= \int_0^{2\pi}\int_1^2 r\sqrt{1+4r^2}\ dr\ d\theta = \frac{1}{8}\int_0^{2\pi}\frac{2(1+4r^2)^{\frac{3}{2}}}{3}\Big|_1^2\ d\theta$$
$$= \frac{\sqrt{17^3}-\sqrt{5^3}}{12}\int_0^{2\pi} d\theta = \frac{\pi\left(\sqrt{17^3}-\sqrt{5^3}\right)}{6}$$

Question 6. (10%)

$$\begin{aligned}
\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\[2ex] \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} &= \begin{vmatrix} \dfrac{v\cos(uv)}{\ln(v)} & \dfrac{u\cos(uv)\ln v - \frac{\sin(uv)}{v}}{\ln^2(v)} \\[2ex] -\sin(u)ve^{\cos(u)} & e^{\cos(u)} \end{vmatrix} \\
&= \begin{vmatrix} \dfrac{v\cos(uv)}{\ln(v)} & \dfrac{uv\cos(uv)\ln v - \sin(uv)}{v\ln^2(v)} \\[2ex] -\sin(u)ve^{\cos(u)} & e^{\cos(u)} \end{vmatrix} \\
&= e^{\cos(u)}\frac{v\cos(uv)}{\ln(v)} + \sin(u)ve^{\cos(u)}\frac{uv\cos(uv)\ln v - \sin(uv)}{v\ln^2(v)} \\
&= e^{\cos(u)}\left(\frac{v\cos(uv)\ln(v) + uv\sin u\cos(uv)\ln v - \sin u\sin(uv)}{\ln^2(v)}\right)
\end{aligned}$$