Examen 2 (solutions)
201-NYA Calcul Différentiel
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Question 1. (5%)

C'est le taux de variation instantané ; $f'(a) = \lim_{\Delta x \to 0} \dfrac{f(a+\Delta x) - f(a)}{\Delta x}$. On peut interpréter géométriquement cette valeur comme étant la pente de la droite tangente à la fonction au point $(a, f(a))$.

Question 2. (10%)

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\sqrt{2(2+h)+3} - \sqrt{2(2)+3}}{h} = \lim_{h\to 0} \frac{\sqrt{7+2h}-\sqrt{7}}{h} \\ &= \lim_{h\to 0} \frac{(\sqrt{7+2h}-\sqrt{7})(\sqrt{7+2h}+\sqrt{7})}{h(\sqrt{7+2h}+\sqrt{7})} = \lim_{h\to 0} \frac{7+2h-7}{h(\sqrt{7+2h}+\sqrt{7})} = \lim_{h\to 0} \frac{2h}{h(\sqrt{7+2h}+\sqrt{7})} \\ &= \lim_{h\to 0} \frac{2}{\sqrt{7+2h}+\sqrt{7}} = \frac{2}{2\sqrt{7}} = \frac{\sqrt{7}}{7} \end{aligned}$$

Question 3. (10%)

$$f'(x) = \left(\frac{1}{x^2-1}\right)' = \left((x^2-1)^{-1}\right)' = -(x^2-1)^{-2}(2x) = \frac{-2x}{(x^2-1)^2}$$

et donc $f'(2) = \dfrac{-4}{(3)^2} = \dfrac{-4}{9}$. L'équation de la droite a donc la forme $y = -\dfrac{4}{9}x + b$ et passe par le point $\left(2, \dfrac{1}{3}\right)$. D'où $\dfrac{1}{3} = -\dfrac{4}{9}(2) + b \Longrightarrow b = \dfrac{1}{3} + \dfrac{8}{9} = \dfrac{11}{9}$. L'équation de la droite est

$$y = -\frac{4}{9}x + \frac{11}{9}$$

Question 4. (35%)

a) $f'(x) = 21x^6 - \dfrac{28}{3}x^{\frac{1}{3}} + 21x^{-4} + 0$

b) $f'(x) = 10x^4\sqrt{3x^5+x} + \dfrac{(2x^5-2)(15x^4+1)}{2\sqrt{3x^5+x}}$

c) $f'(x) = \dfrac{(2x-3)(2x-5)-(x^2-3x)(2)}{(2x-5)^2}$

d) $f'(x) = 50(8x^3-6x)^{49}(24x^2-6)$

e) $f'(x) = \dfrac{(12x^2+5)(x^2-5)\sqrt[3]{x^7+2} - (4x^3+5x)\left(2x\sqrt[3]{x^7+2} + (x^2-5)\frac{7x^6}{3\sqrt[3]{(x^7+2)^2}}\right)}{(x^2-5)^2\sqrt[3]{(x^7+2)^2}}$

Question 5. (20%)

a)

$$f'(x) = 7(x^2-2)^6(2x)(5x-7)^9 + (x^2-2)^7 9(5x-7)^8(5) = 14x(x^2-2)^6(5x-7)^9 + 45(x^2-2)^7(5x-7)^8$$
$$= (x^2-2)^6(5x-7)^8\left(14x(5x-7) + 45(x^2-2)\right) = (x^2-2)^6(5x-7)^8\left(70x^2 - 98x + 45x^2 - 90\right)$$
$$= (x^2-2)^6(5x-7)^8\left(115x^2 - 98x - 90\right)$$

b)

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}+1) - \frac{1}{2\sqrt{x}}(\sqrt{x}-1)}{(\sqrt{x}+1)^2} = \frac{(\sqrt{x}+1)-(\sqrt{x}-1)}{2\sqrt{x}(\sqrt{x}+1)^2} = \frac{2}{2\sqrt{x}(\sqrt{x}+1)^2} = \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$$

Question 6. (10%)

$$2(x^3y^2)' - \left(\frac{3x}{y}\right)' = (5)' - (y^3)'$$
$$2(3x^2y^2 + x^3 2yy') - \frac{3y - 3xy'}{y^2} = 0 - 3y^2y'$$
$$2y^2(3x^2y^2 + x^3 2yy') - 3y + 3xy' = -3y^4y'$$
$$6x^2y^4 + 4x^3y^3y' - 3y + 3xy' = -3y^4y'$$
$$(4x^3y^3 + 3x + 3y^4)y' = 3y - 6x^2y^4$$
$$y' = \frac{3y - 6x^2y^4}{4x^3y^3 + 3x + 3y^4}$$

Question 7. (10%)

$$f'(x) = \frac{1}{2}(x^2-3)^{-\frac{1}{2}}(2x) = x(x^2-3)^{-\frac{1}{2}}$$
$$f''(x) = (x^2-3)^{-\frac{1}{2}} + x\left(-\frac{1}{2}(x^2-3)^{-\frac{3}{2}}(2x)\right) = (x^2-3)^{-\frac{1}{2}} - x^2(x^2-3)^{-\frac{3}{2}}$$
$$f'''(x) = \left(-\frac{1}{2}(x^2-3)^{-\frac{3}{2}}(2x)\right) - 2x(x^2-3)^{-\frac{3}{2}} - x^2\left(-\frac{3}{2}(x^2-3)^{-\frac{5}{2}}(2x)\right)$$
$$= -x(x^2-3)^{-\frac{3}{2}} - 2x(x^2-3)^{-\frac{3}{2}} + 3x^3(x^2-3)^{-\frac{5}{2}}$$
$$= -3x(x^2-3)^{-\frac{3}{2}} + 3x^3(x^2-3)^{-\frac{5}{2}}$$