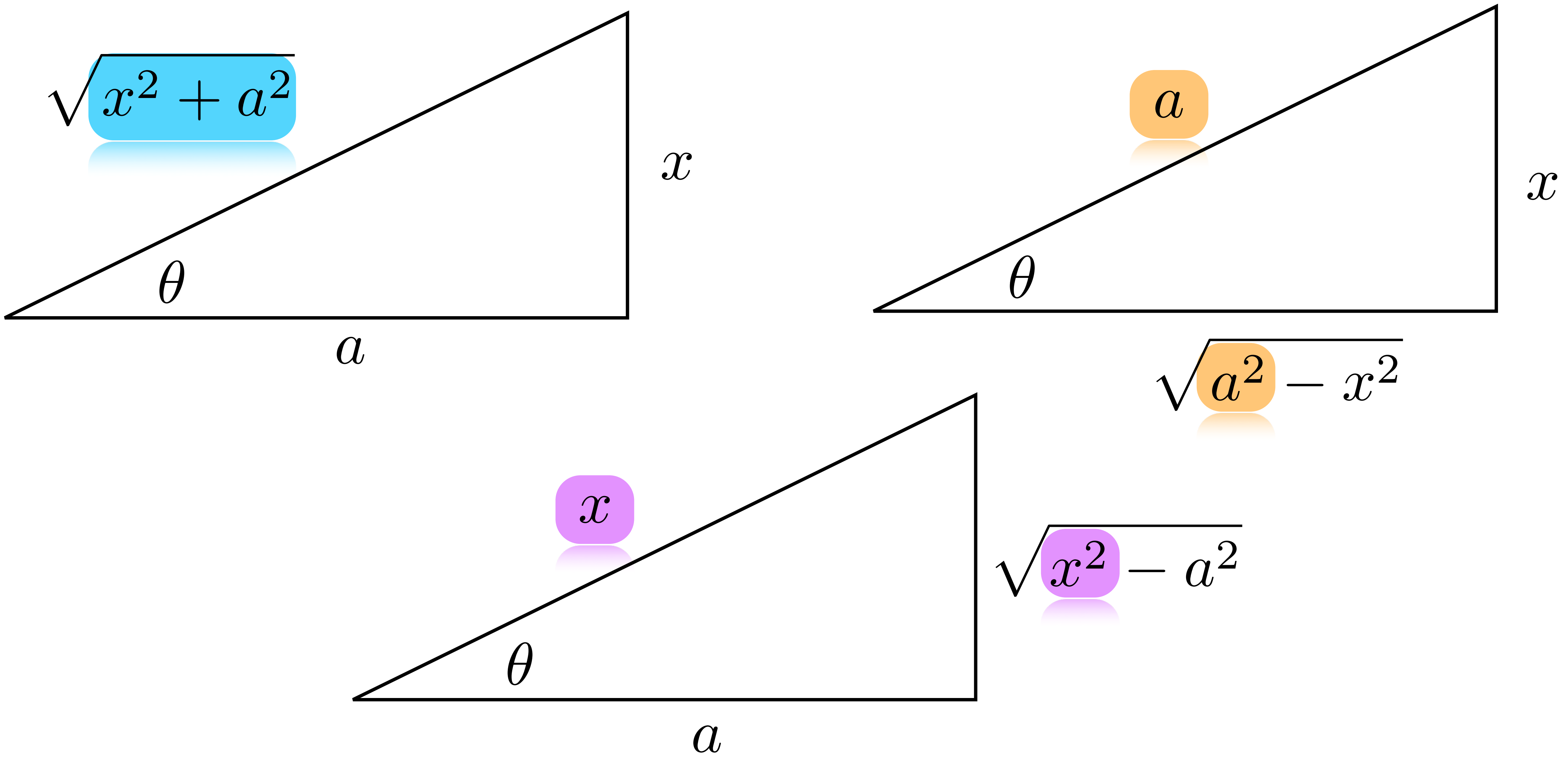
2.4 SUBSTITUTION TRIGONOMÉTRIQUE 2

cours 12

Au dernier cours, nous avons vu

✓ Substitution trigonométrique



Aujourd'hui, nous allons voir

- ✓ Forme plus général des substitutions trigonométriques.

La substitution trigonométrique s'applique lorsqu'on a une expression de la forme.

$$x^2 + a^2 \qquad x^2 - a^2 \qquad a^2 - x^2$$

On pourrait aussi l'appliquer après un changement de variable à des expressions de la forme

$$(bx+c)^2 + a^2 \qquad (bx+c)^2 - a^2 \qquad a^2 - (bx+c)^2$$

Or les expressions sont rarement sous cette forme, il va falloir faire une petite...

Complétion de carré

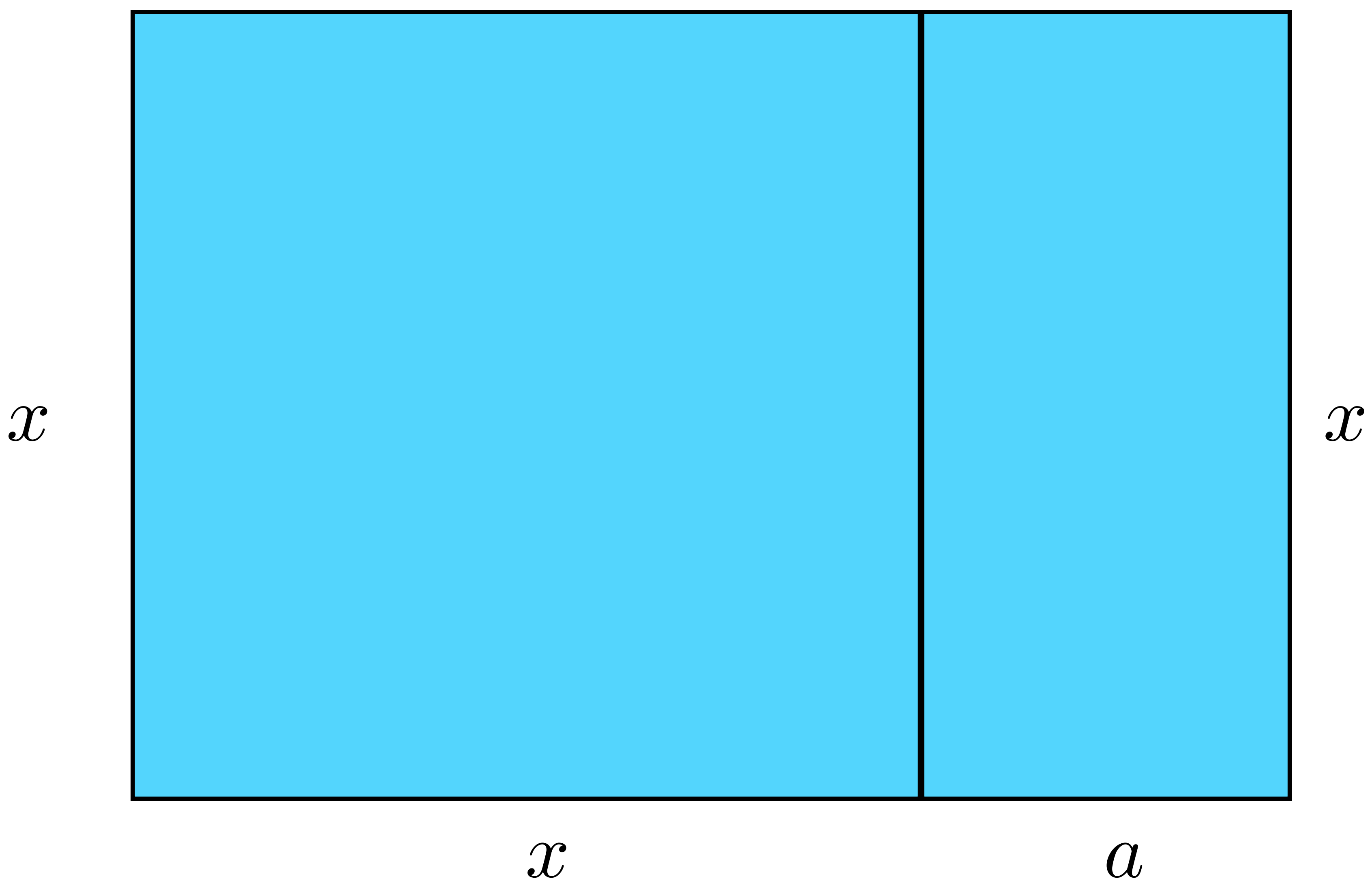
$$ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

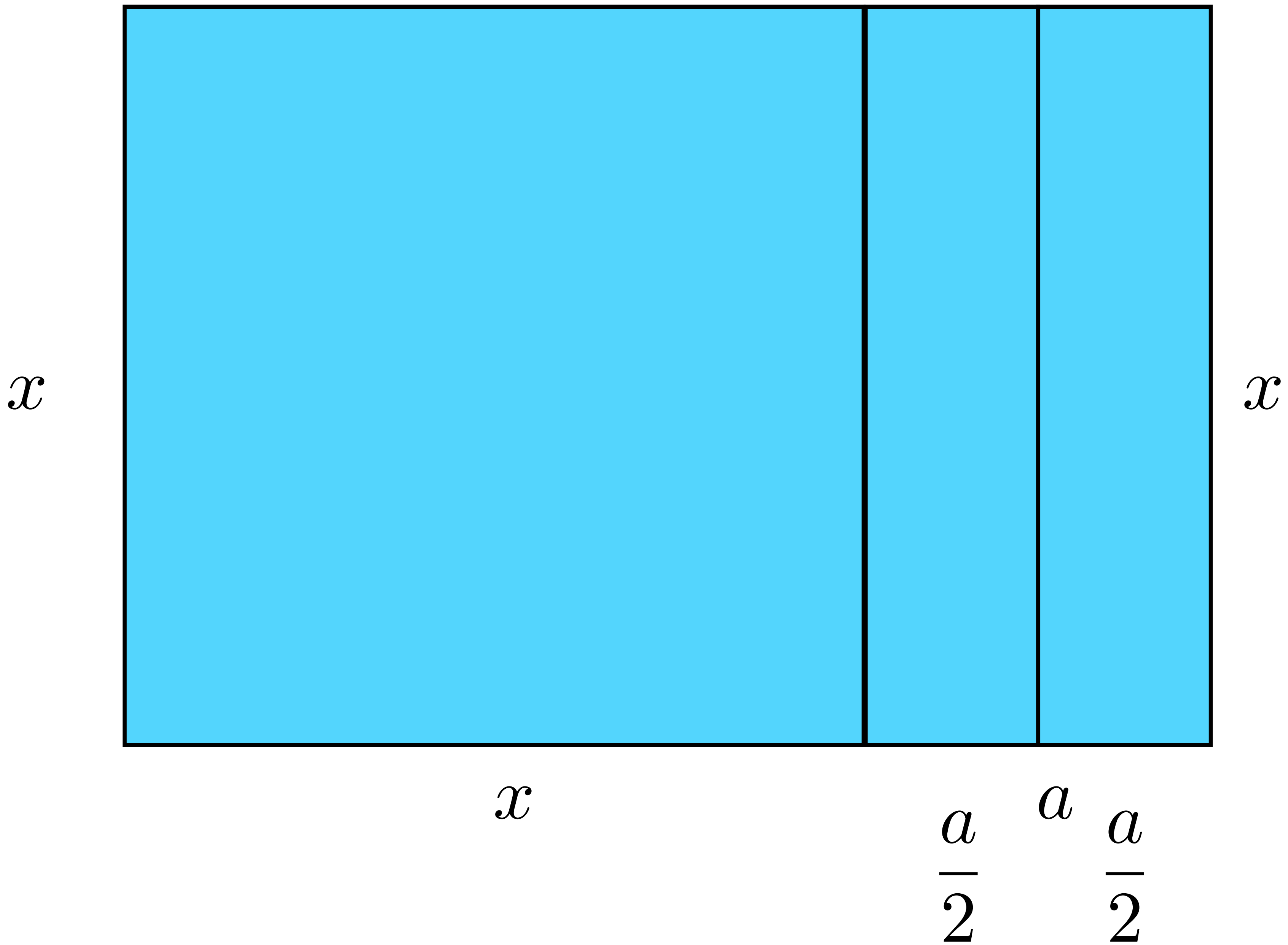
$$x^2 + ax + b$$

$$= (x^2 + ax) + b$$

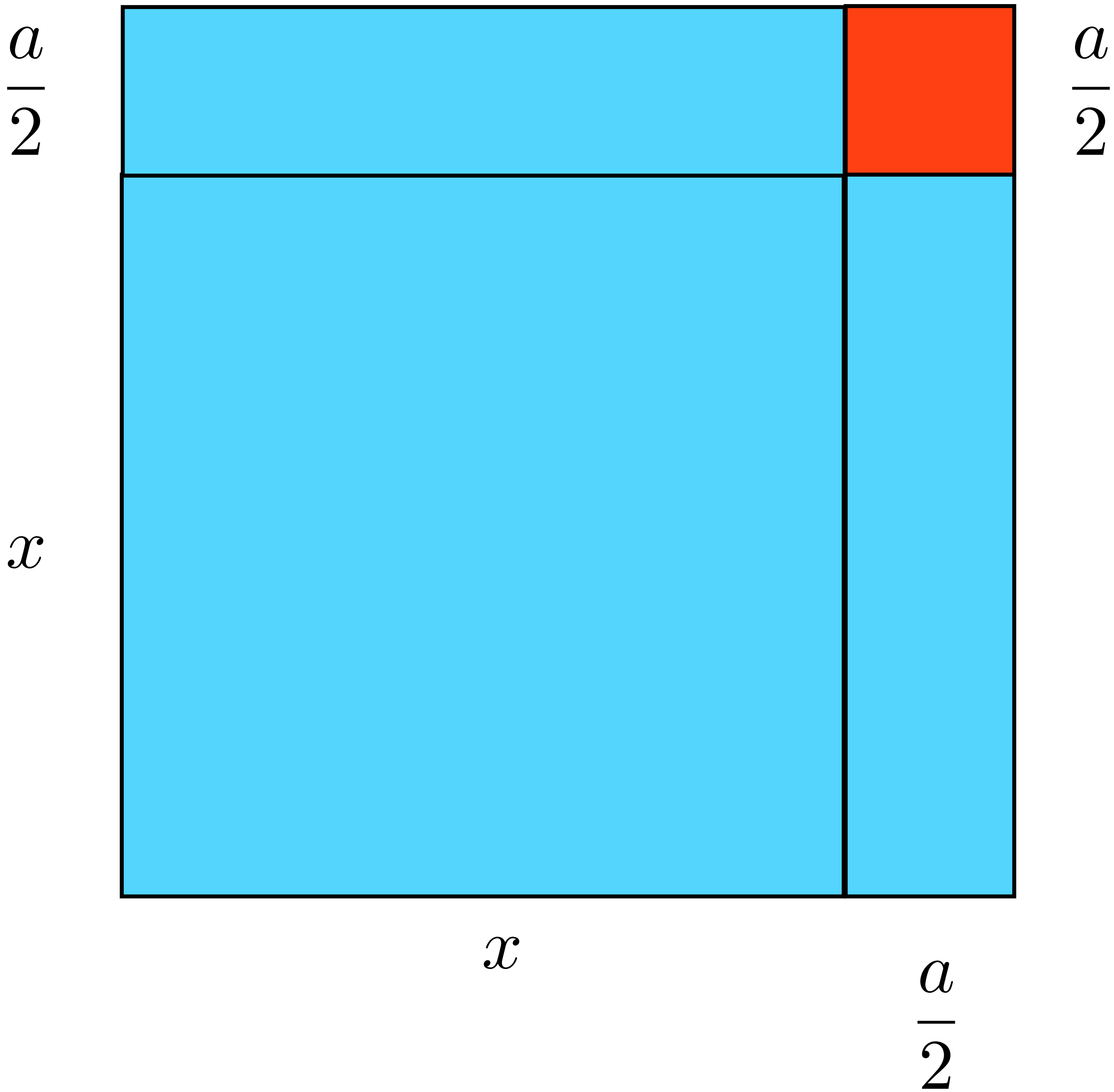
$x^2 + ax$



$$x^2 + ax = x^2 + \frac{a}{2}x + \frac{a}{2}x$$

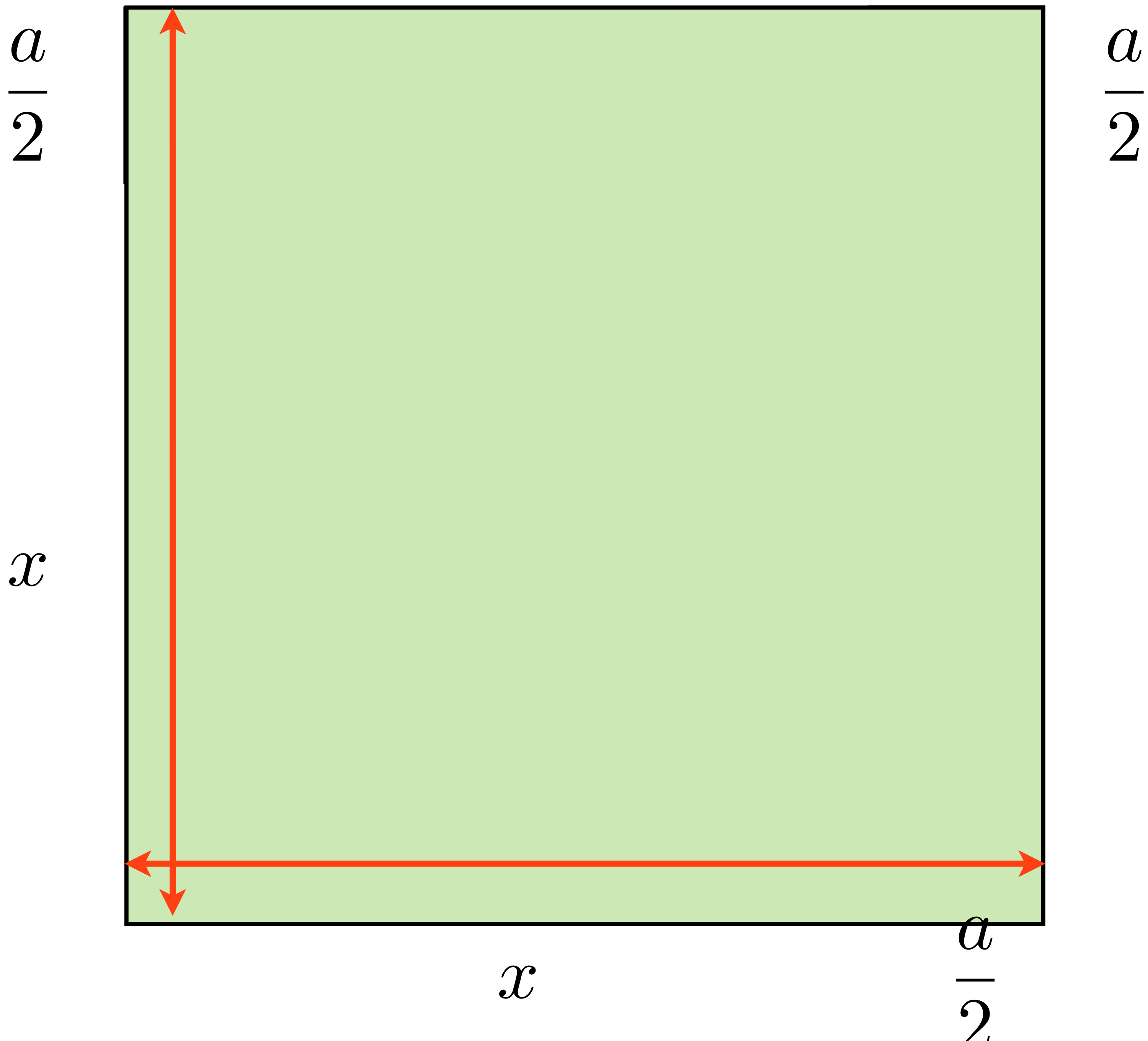


$$x^2 + ax = x^2 + \frac{a}{2}x + \frac{a}{2}x + \left(\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2$$



$$x^2 + ax = x^2 + \frac{a}{2}x + \frac{a}{2}x + \left(\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2$$

$$= \left(x^2 + ax + \frac{a^2}{2^2}\right) - \frac{a^2}{2^2} \qquad = \left(x + \frac{a}{2}\right)^2 - \frac{a^2}{2^2}$$



Exemple

$$
\begin{aligned}
2x^2 + 8x - 6 &= 2(x^2 + 4x - 3) \\
&= 2(x^2 + 2x + 2x - 3) \\
&= 2\left(x^2 + 2x + 2x + 2^2 - 2^2 - 3\right) \\
&= 2\left(x^2 + 4x + 4 - 7\right) \\
&= 2\left((x + 2)^2 - 7\right)
\end{aligned}
$$

Faites les exercices suivants

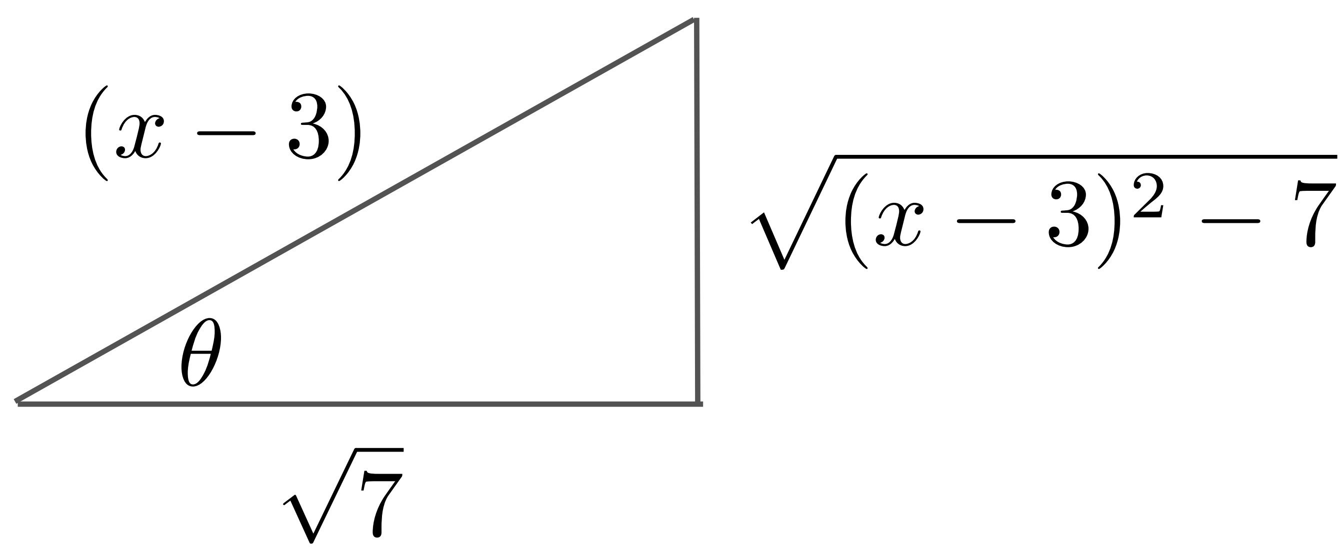
Section 2 # 16

Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx = \int \frac{1}{(x-3)^2 - 7}\, dx$$

$$x^2 - 6x + 2 = x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 2$$

$$= x^2 - 6x + 9 - 7 = (x-3)^2 - 7$$



$$\frac{x-3}{\sqrt{7}} = \sec\theta$$

$$x - 3 = \sqrt{7}\sec\theta$$

$$x = \sqrt{7}\sec\theta + 3$$

$$dx = \sqrt{7}\sec\theta\tan\theta\ d\theta$$

$$\frac{\sqrt{(x-3)^2 - 7}}{\sqrt{7}} = \tan\theta$$

$$\sqrt{(x-3)^2 - 7} = \sqrt{7}\tan\theta$$

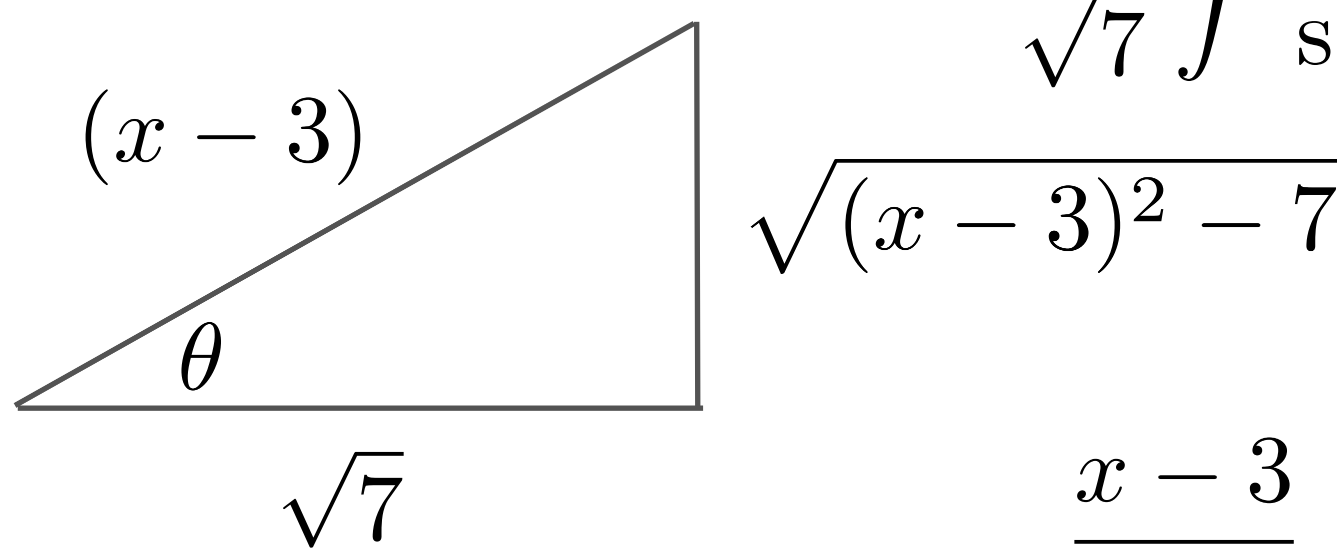
$$(x-3)^2 - 7 = 7\tan^2\theta$$

Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx = \int \frac{1}{(x-3)^2 - 7}\, dx$$

$$= \int \frac{\sqrt{7}\sec\theta\tan\theta}{7\tan^2\theta}\, d\theta = \frac{1}{\sqrt{7}} \int \frac{\sec\theta}{\tan\theta}\, d\theta = \frac{1}{\sqrt{7}} \int \frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta}\, d\theta$$

$$= \frac{1}{\sqrt{7}} \int \frac{1}{\sin\theta}\, d\theta = \frac{1}{\sqrt{7}} \int \csc\theta \; d\theta$$



$$x - 3 = \sqrt{7}\sec\theta$$

$$\frac{x-3}{\sqrt{7}} = \sec\theta$$

$$x = \sqrt{7}\sec\theta + 3$$

$$\frac{\sqrt{(x-3)^2 - 7}}{\sqrt{7}} = \tan\theta$$

$$dx = \sqrt{7}\sec\theta\tan\theta \; d\theta$$

$$\sqrt{(x-3)^2 - 7} = \sqrt{7}\tan\theta$$

$$(x-3)^2 - 7 = 7\tan^2\theta$$

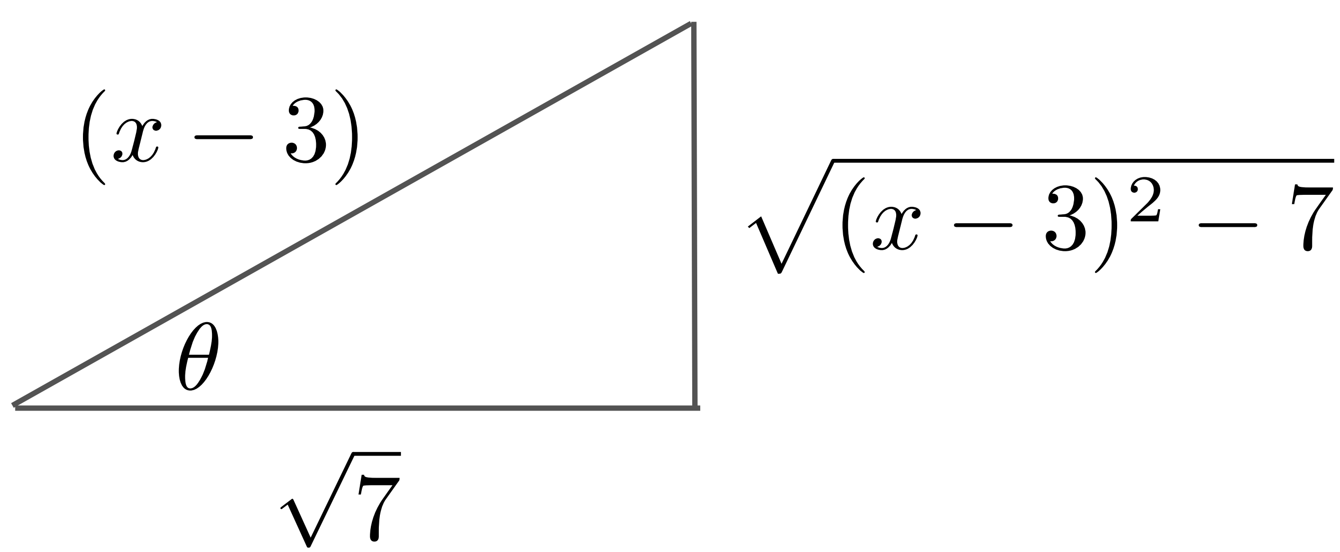
Exemple

$$\int \frac{1}{x^2-6x+2}\,dx = \int \frac{1}{(x-3)^2-7}\,dx$$

$$= \frac{1}{\sqrt{7}}\int \csc\theta\;d\theta = -\frac{1}{\sqrt{7}}\ln|\csc\theta+\cot\theta| + C$$

$$= -\frac{1}{\sqrt{7}}\ln\left|\frac{x-3}{\sqrt{(x-3)^2-7}} + \frac{\sqrt{7}}{\sqrt{(x-3)^2-7}}\right| + C$$

$$= -\frac{1}{\sqrt{7}}\ln\left|\frac{x-3+\sqrt{7}}{\sqrt{(x-3)^2-7}}\right| + C = -\frac{1}{\sqrt{7}}\ln\left|\frac{x-3+\sqrt{7}}{\sqrt{x^2-6x+2}}\right| + C$$



Faites les exercices suivants

Section 2 # 17, 18

Exemple $\displaystyle\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$

$$u = \cos x$$

$$du = -\sin x \; dx$$

$$= \int \frac{\sin x}{1 - \cos^2 x + \cos x}\, dx$$

$$= -\int \frac{1}{1 - u^2 + u}\, du$$

$$= \int \frac{1}{u^2 - u - 1}\, du$$

$$= \int \frac{1}{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}\, du$$

$$u^2 - u - 1$$

$$= u^2 - u + \frac{1}{4} - \frac{1}{4} - 1$$

$$= \left(u - \frac{1}{2}\right)^2 - \frac{5}{4}$$

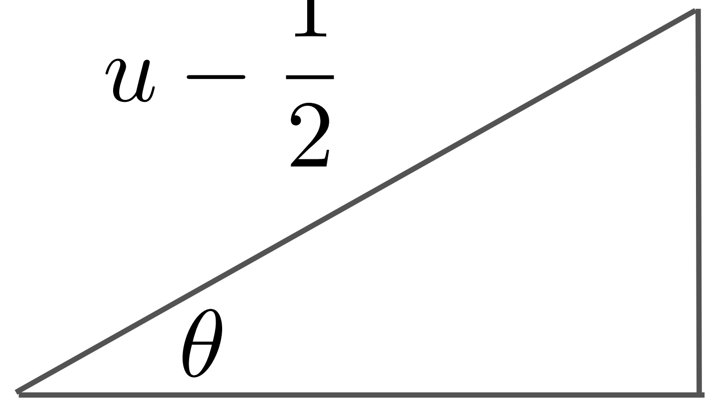
Exemple

$$\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$$

$$u = \cos x$$

$$u^2 - u - 1$$

$$= \int \frac{1}{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}\, du$$



$$\sqrt{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}$$

$$\frac{u - \frac{1}{2}}{\frac{\sqrt{5}}{2}} = \sec\theta \qquad u - \frac{1}{2} = \frac{\sqrt{5}}{2}\sec\theta \qquad u = \frac{\sqrt{5}}{2}\sec\theta + \frac{1}{2}$$

$$\frac{\sqrt{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}}{\frac{\sqrt{5}}{2}} = \tan\theta \qquad du = \frac{\sqrt{5}}{2}\sec\theta\tan\theta\ d\theta$$

$$\sqrt{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}} = \frac{\sqrt{5}}{2}\tan\theta \qquad \left(u - \frac{1}{2}\right)^2 - \frac{5}{4} = \frac{5}{4}\tan^2\theta$$

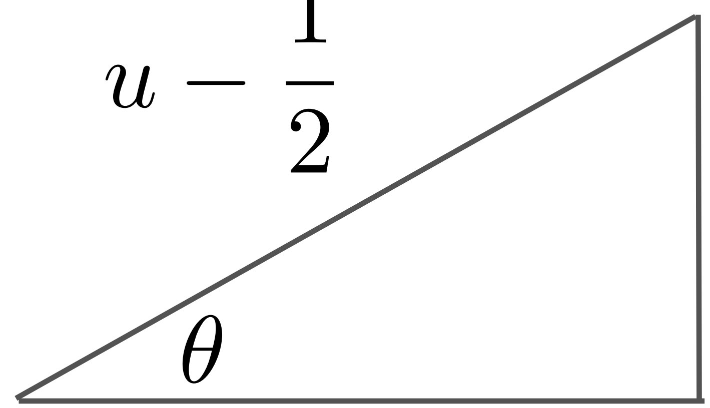
Exemple

$$\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$$

$$u = \cos x$$

$$u^2 - u - 1$$

$$= \int \frac{1}{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}\, du$$



$$= \int \frac{\frac{\sqrt{5}}{2} \sec\theta \tan\theta}{\frac{5}{4}\tan^2\theta}\, d\theta$$

$$u = \frac{\sqrt{5}}{2}\sec\theta + \frac{1}{2}$$

$$= \frac{2}{\sqrt{5}} \int \frac{\sec\theta}{\tan\theta}\, d\theta$$

$$du = \frac{\sqrt{5}}{2}\sec\theta \tan\theta \; d\theta$$

$$= \frac{2}{\sqrt{5}} \int \frac{1}{\cos\theta} \frac{\cos\theta}{\sin\theta}\, d\theta$$

$$\left(u - \frac{1}{2}\right)^2 - \frac{5}{4} = \frac{5}{4}\tan^2\theta$$

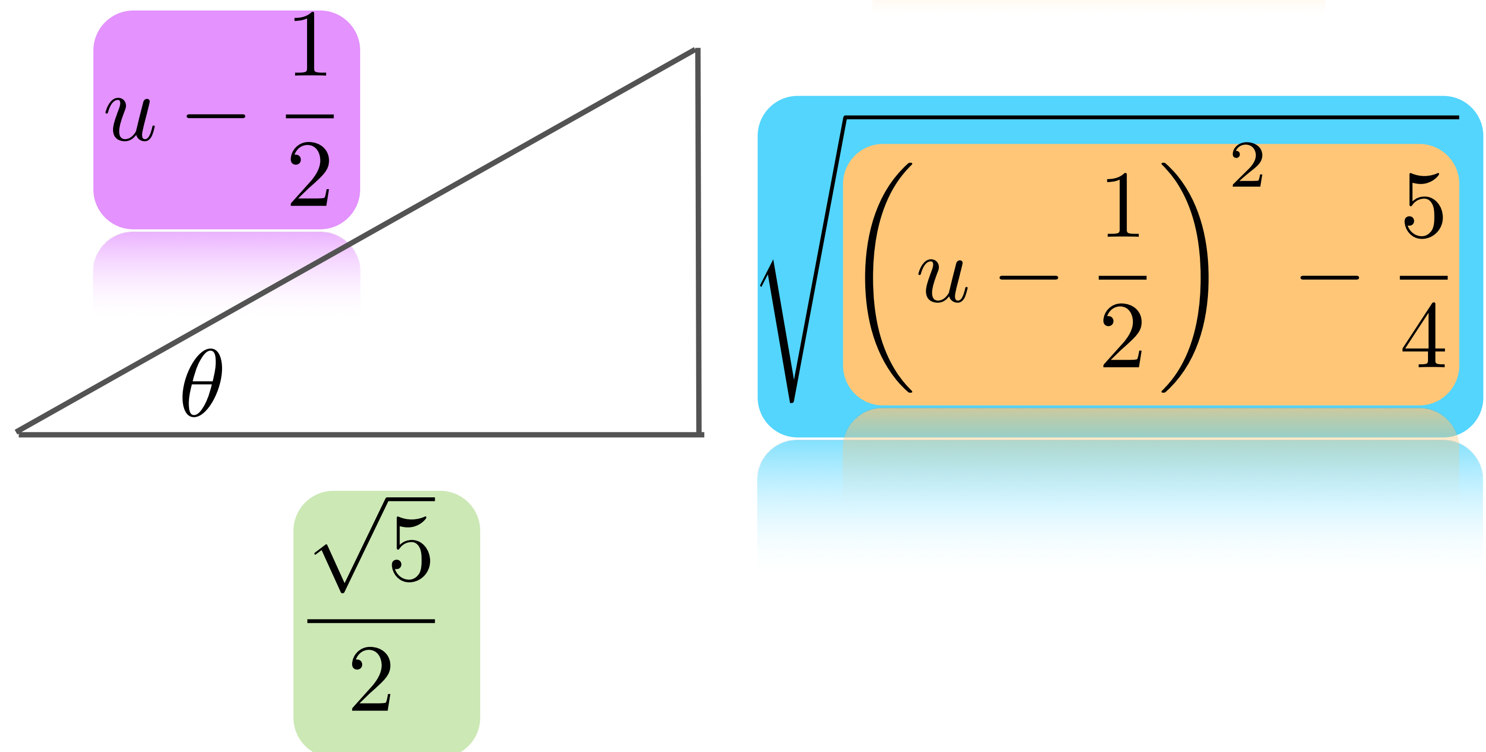
$$= \frac{2}{\sqrt{5}} \int \frac{1}{\sin\theta}\, d\theta$$

Exemple

$$\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$$

$$u = \cos x$$

$$u^2 - u - 1$$



$$= \frac{2}{\sqrt{5}} \int \frac{1}{\sin\theta}\, d\theta$$

$$= \frac{2}{\sqrt{5}} \int \csc\theta\ d\theta$$

$$= -\frac{2}{\sqrt{5}} \ln\left|\csc\theta + \cot\theta\right| + C$$

$$= -\frac{2}{\sqrt{5}} \ln\left|\frac{u - \frac{1}{2}}{\sqrt{u^2 - u - 1}} + \frac{\frac{\sqrt{5}}{2}}{\sqrt{u^2 - u - 1}}\right| + C$$

$$= -\frac{2}{\sqrt{5}} \ln\left|\frac{2u - 1 + \sqrt{5}}{2\sqrt{u^2 - u - 1}}\right| + C$$

Exemple $\displaystyle\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$

$u = \cos x$

$$= -\frac{2}{\sqrt{5}} \ln \left| \frac{2u - 1 + \sqrt{5}}{2\sqrt{u^2 - u - 1}} \right| + C$$

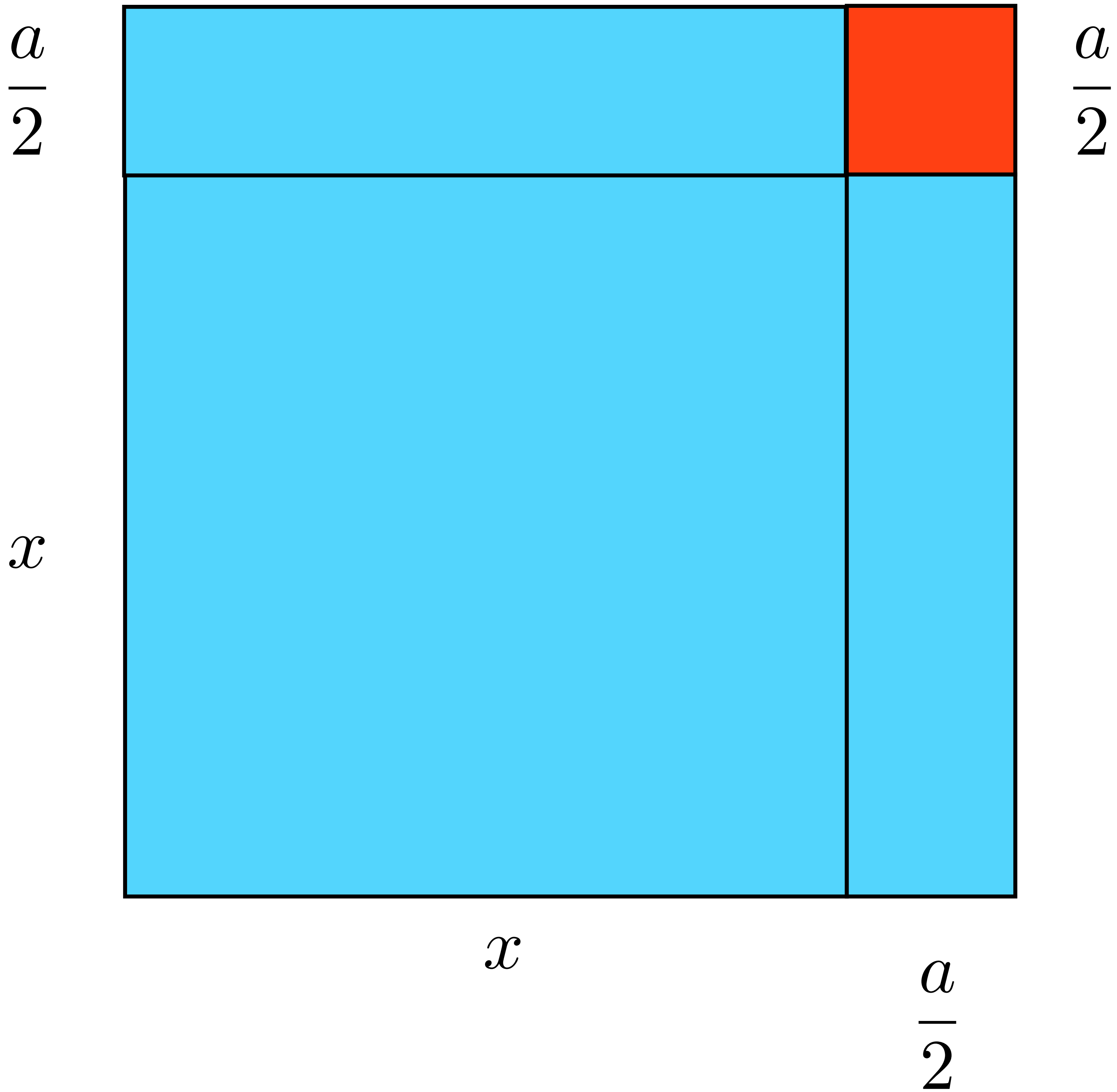
$$= -\frac{2}{\sqrt{5}} \ln \left| \frac{2\cos x - 1 + \sqrt{5}}{2\sqrt{\cos^2 x - \cos x - 1}} \right| + C$$

Faites les exercices suivants

Section 2 # 17, 18

Aujourd'hui, nous avons vu

✓ Complétion de carré



Devoir: Section 2.4