2.4 SUBSTITUTION TRIGONOMÉTRIQUE 2

cours 12

Au dernier cours, nous avons vu

- ✓ Substitution trigonométrique

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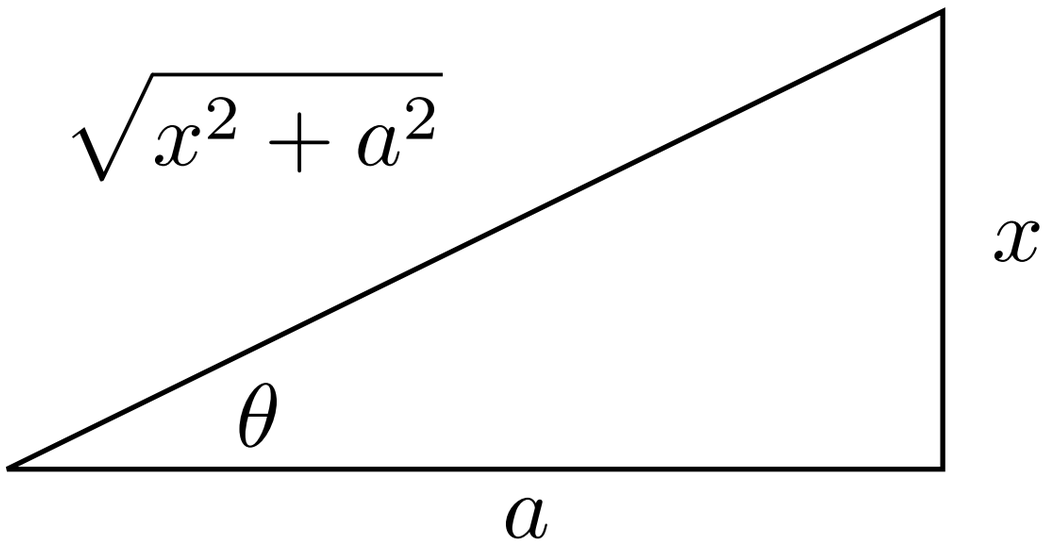
- ✓ Substitution trigonométrique



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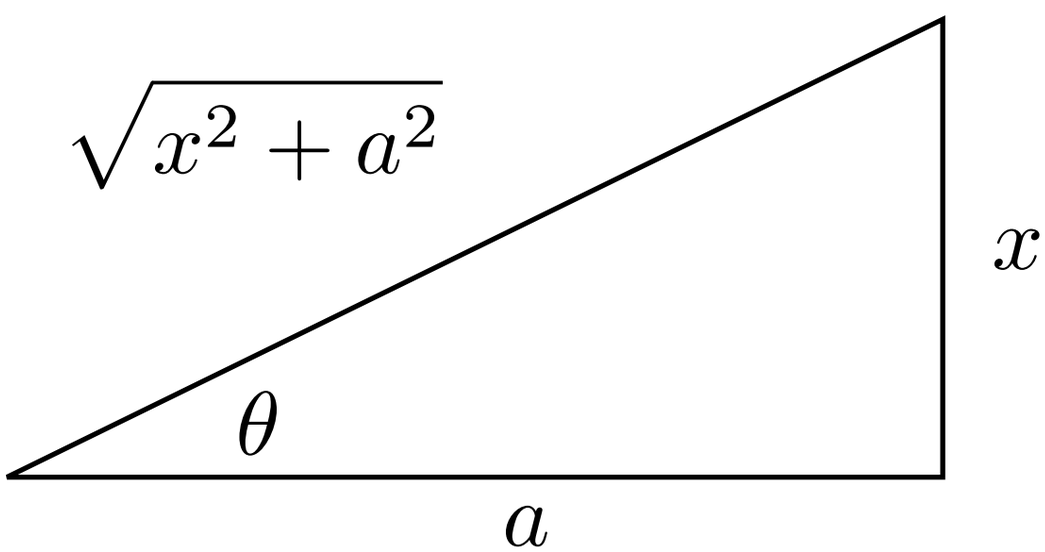

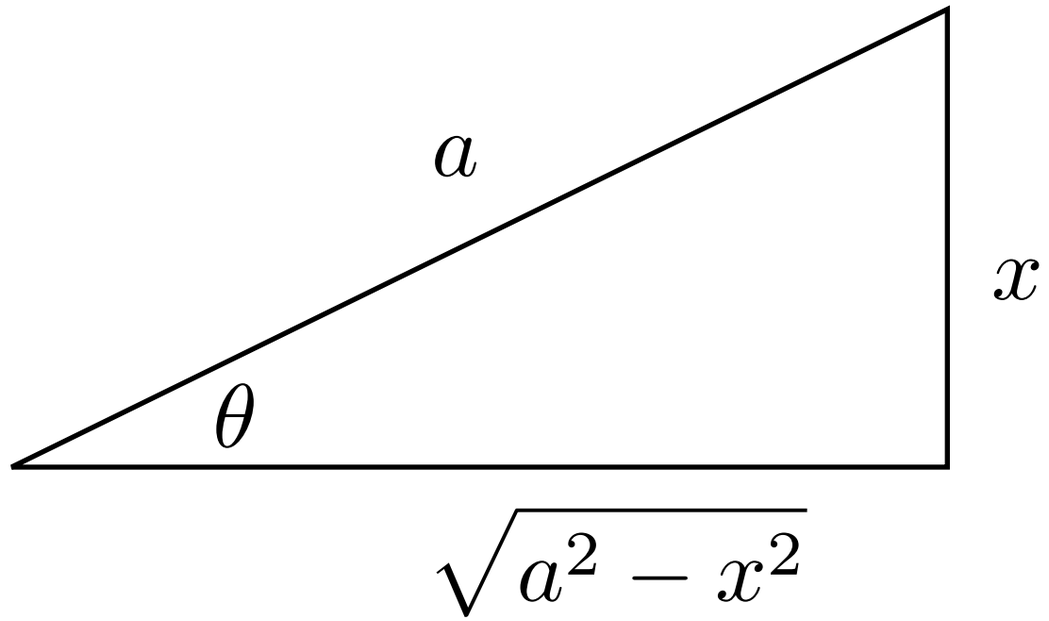

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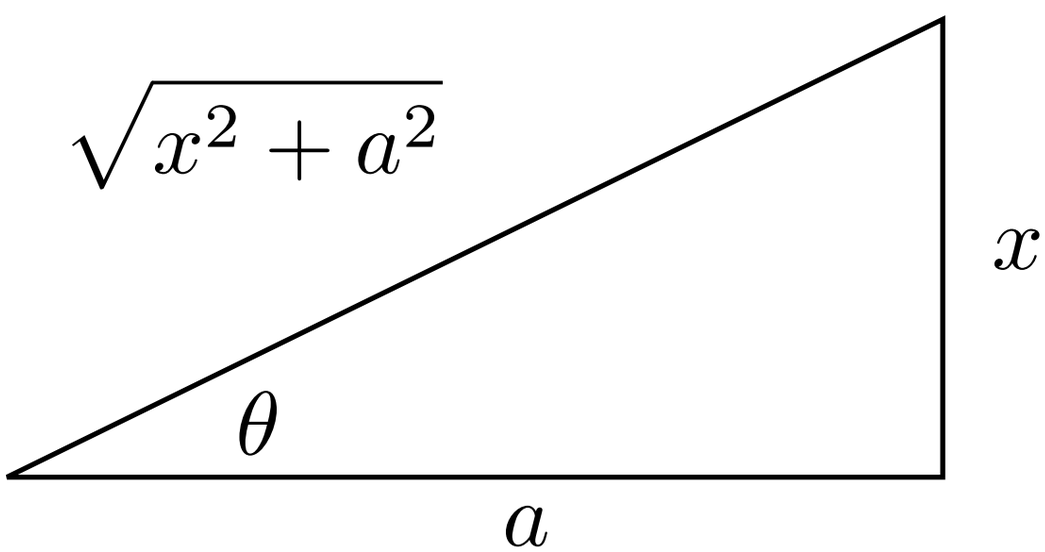

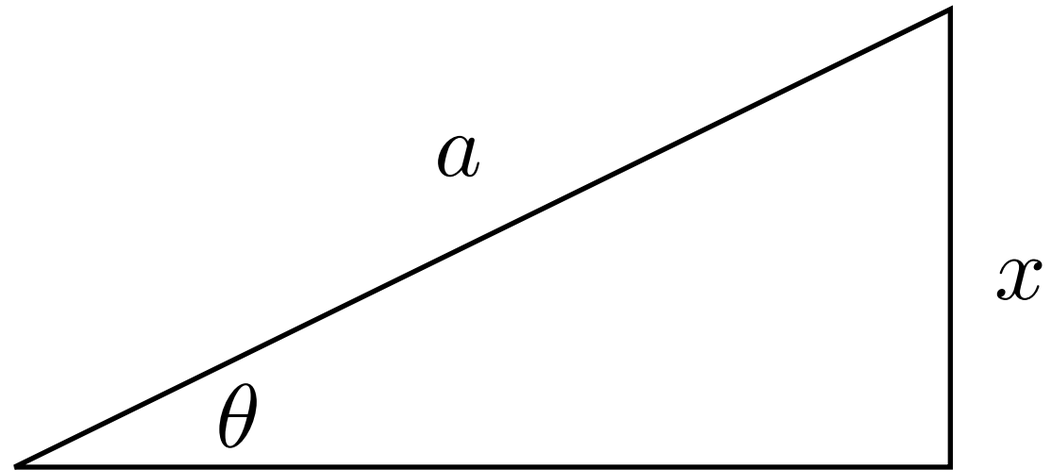

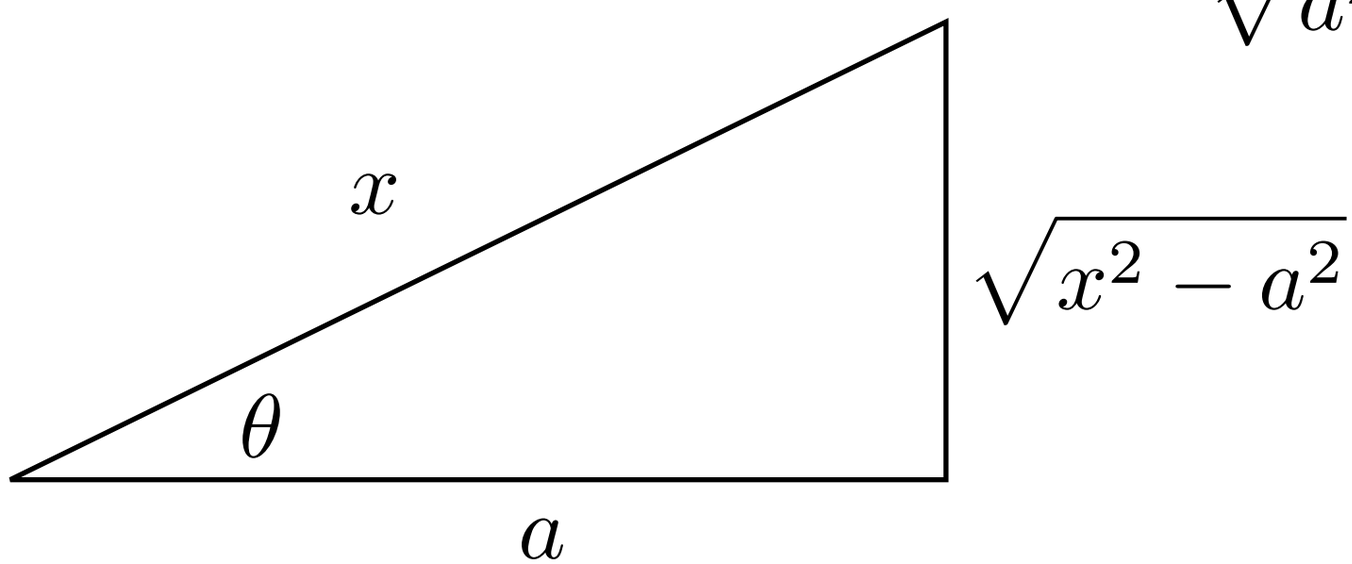

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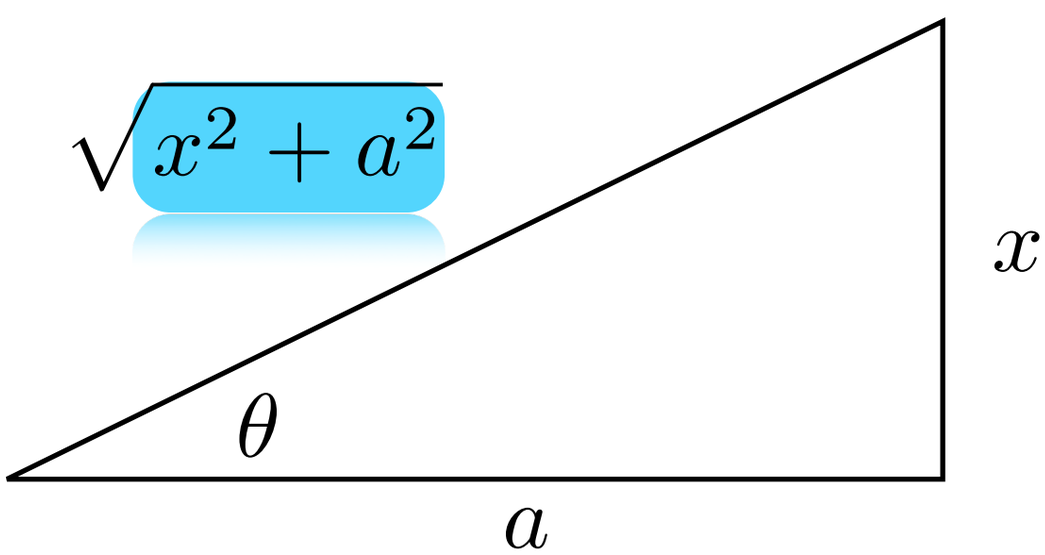

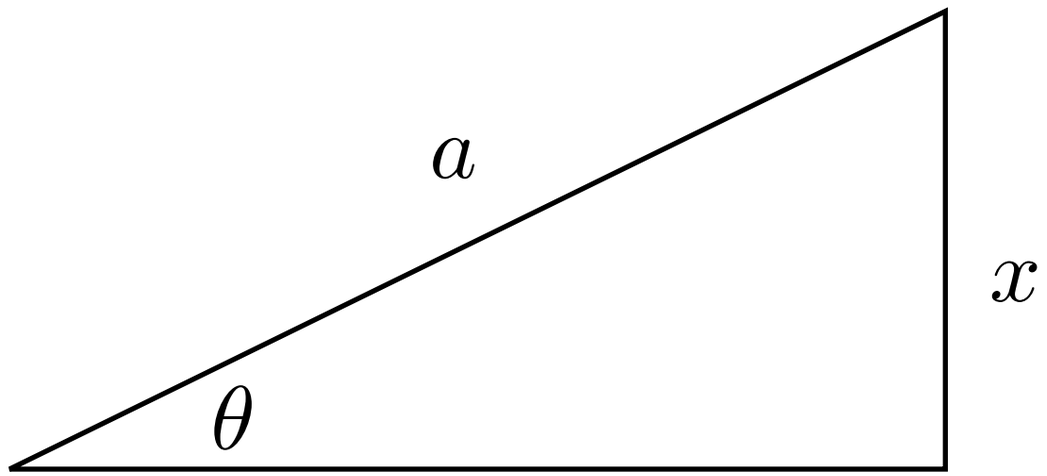

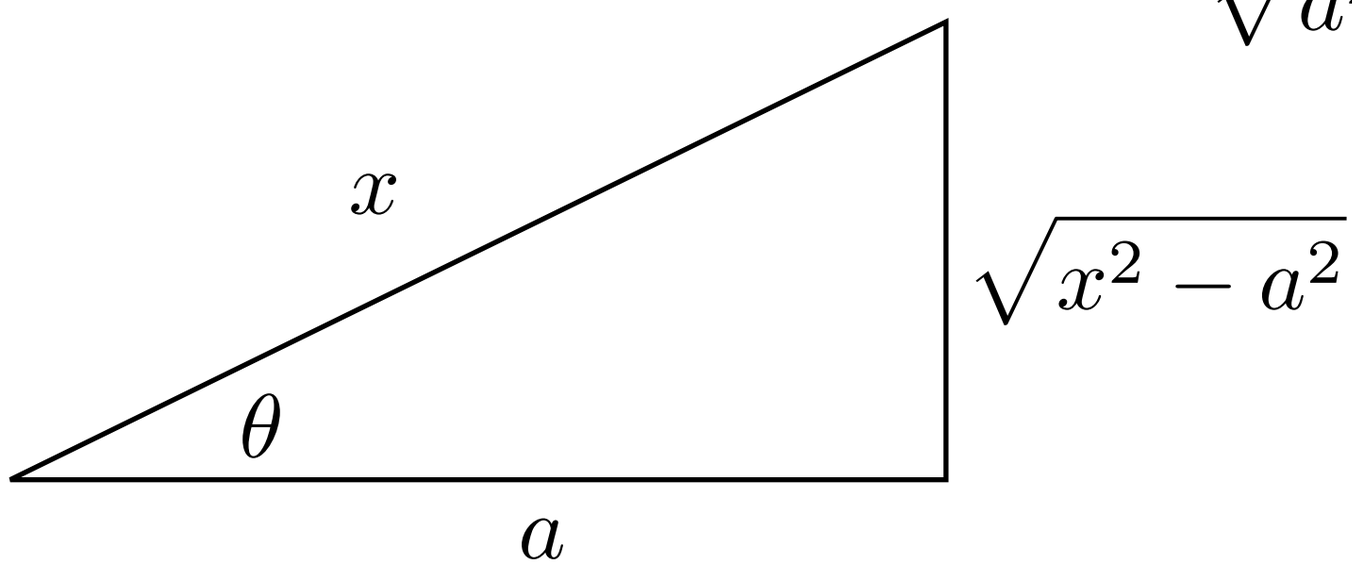

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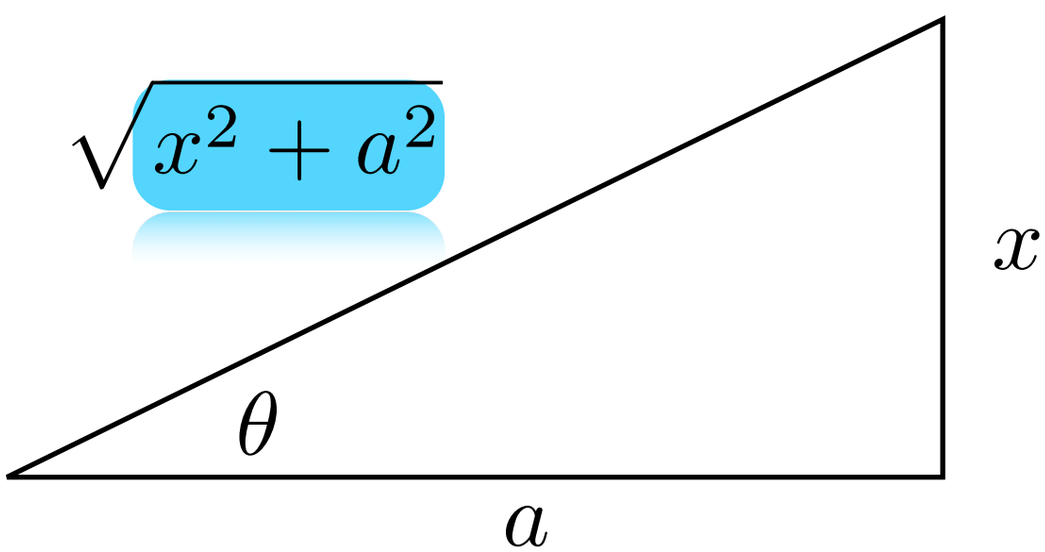

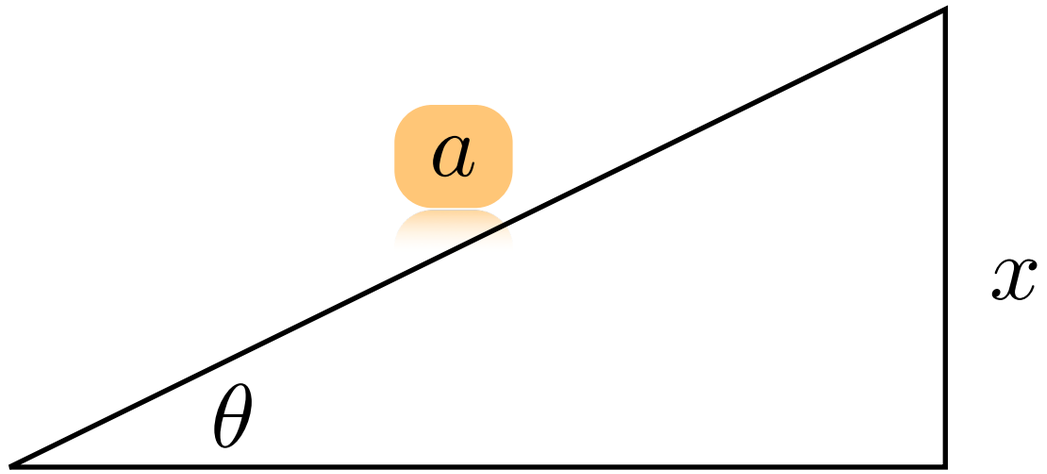

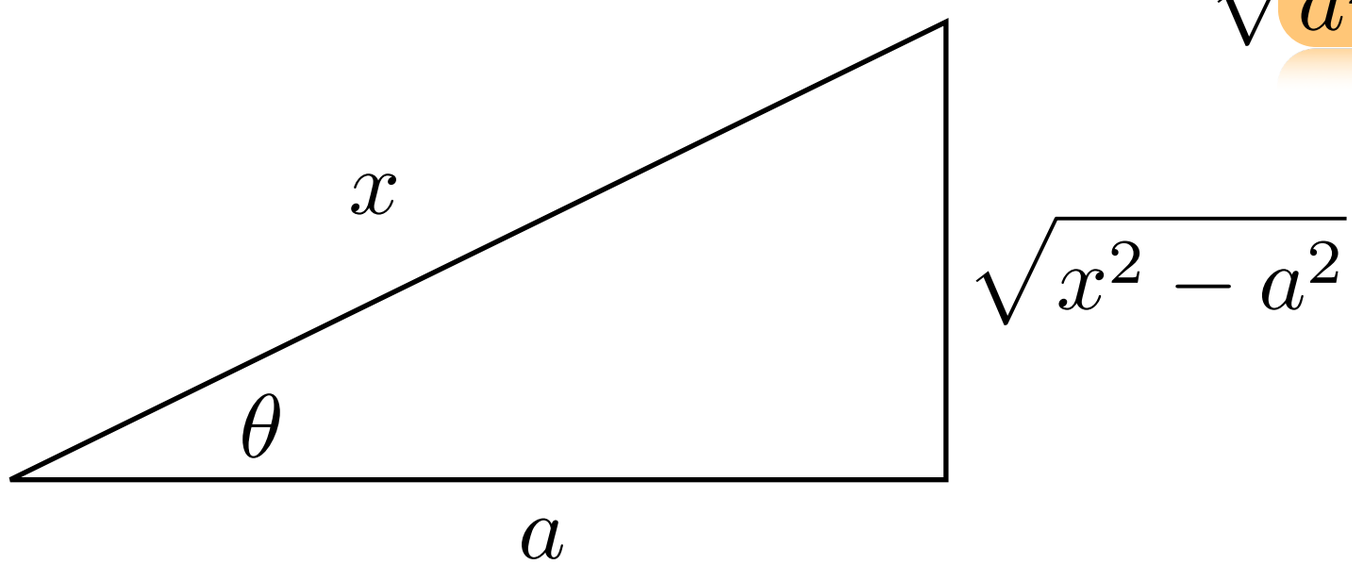

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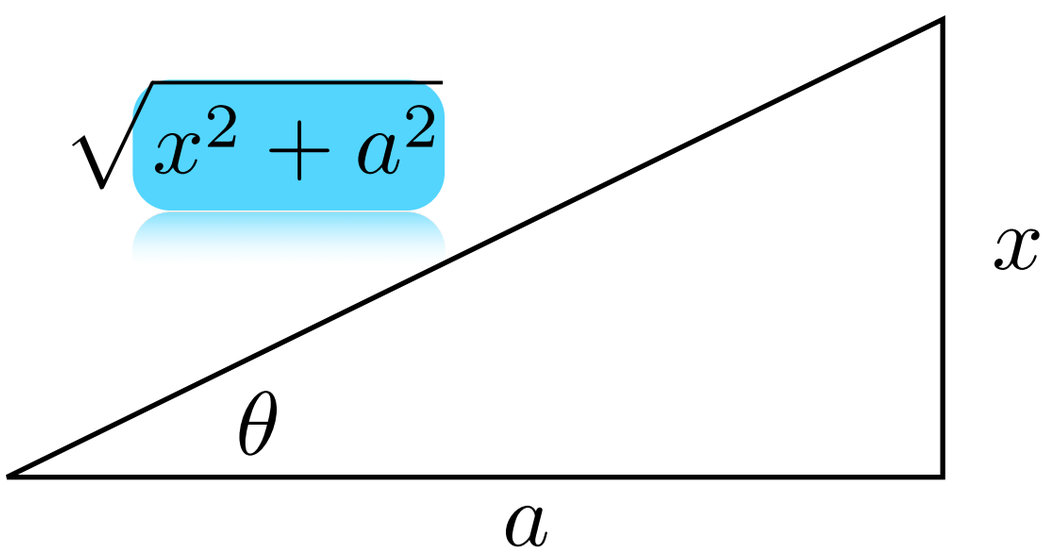

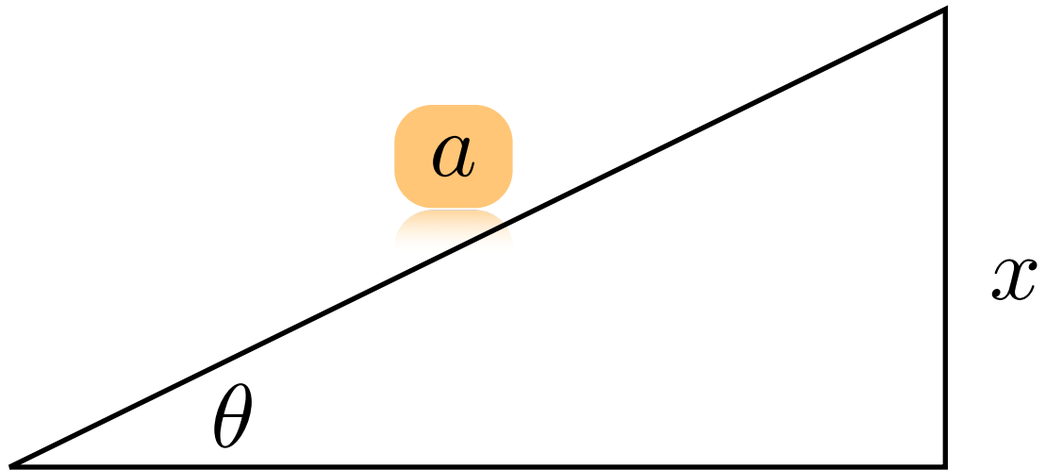

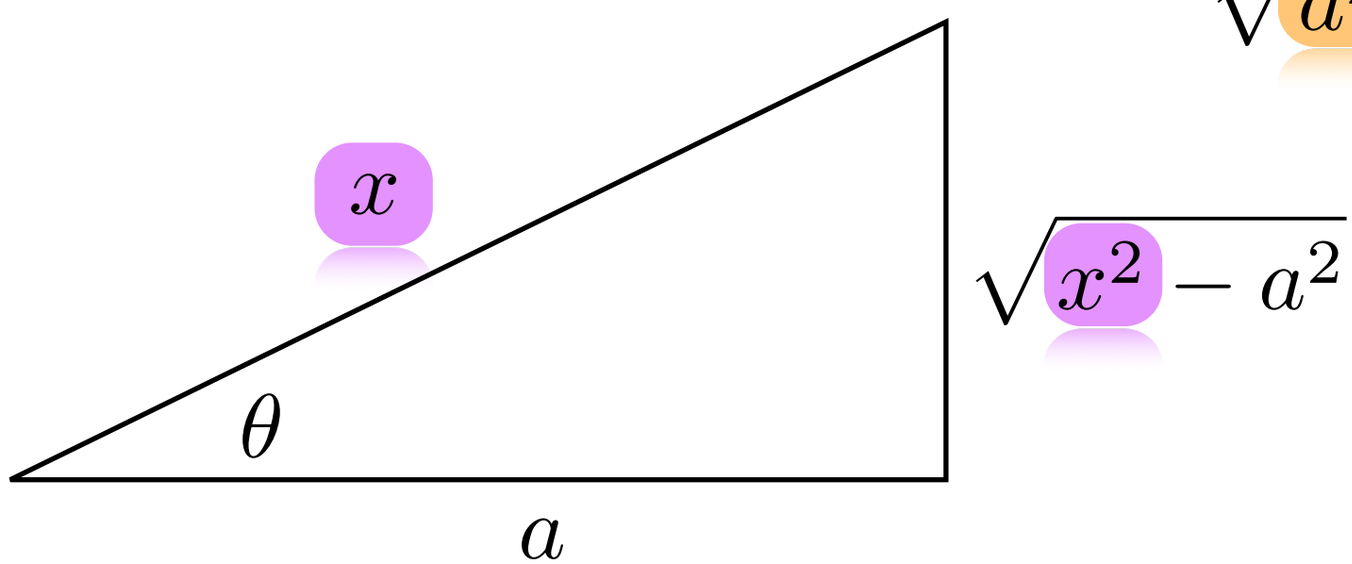

Aujourd'hui, nous allons voir

- ✓ Forme plus général des substitutions trigonométriques.

La substitution trigonométrique s'applique lorsqu'on a une expression de la forme.

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$$x^2 + a^2$$

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On pourrait aussi l'appliquer après un changement de variable à des expressions de la forme

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$$(bx + c)^2 + a^2 \qquad (bx + c)^2 - a^2$$

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$$(bx + c)^2 + a^2 \qquad (bx + c)^2 - a^2 \qquad a^2 - (bx + c)^2$$

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$$(bx + c)^2 + a^2 \qquad (bx + c)^2 - a^2 \qquad a^2 - (bx + c)^2$$

Or les expressions sont rarement sous cette forme, il va falloir faire une petite...

Complétion de carré

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$$ax^2 + bx + c$$

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$$ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

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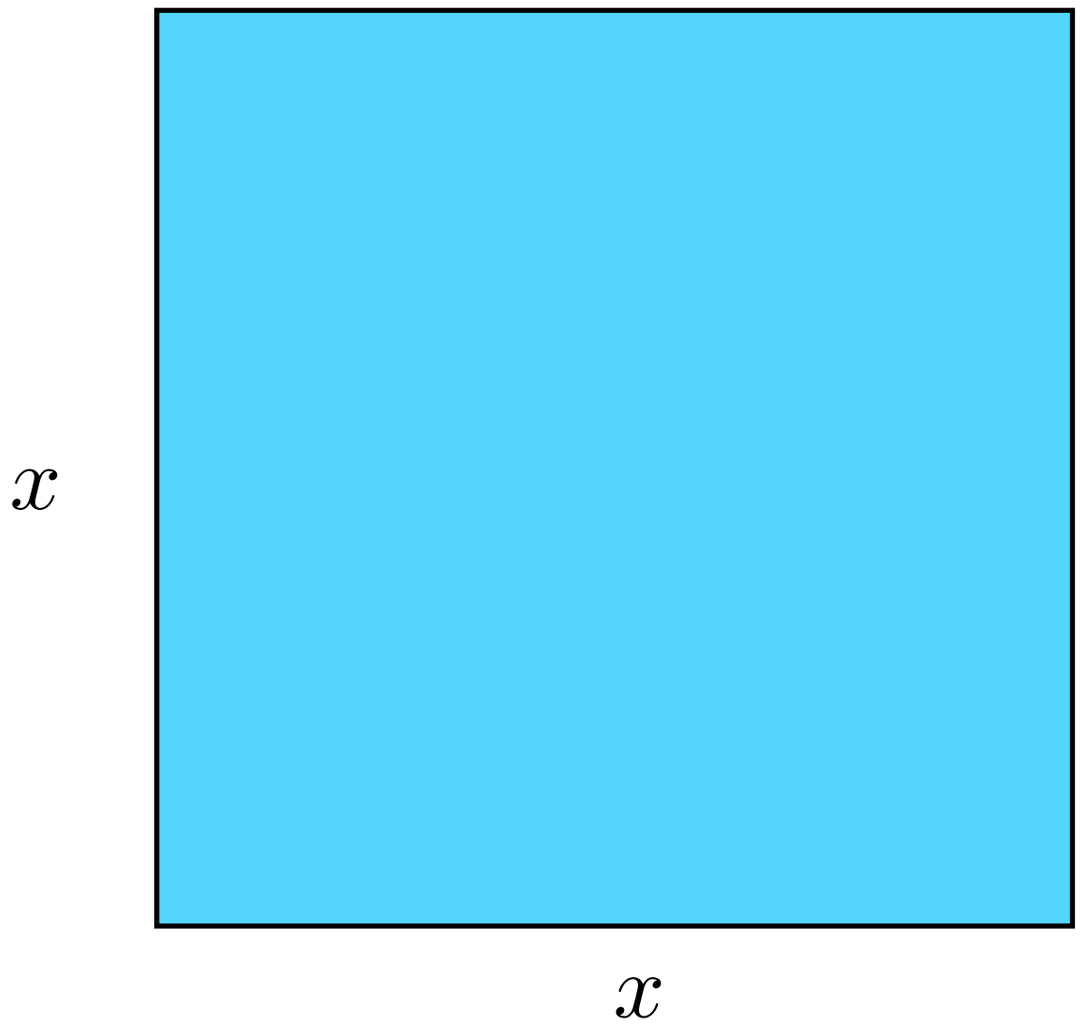
$$x^2 + ax + b$$

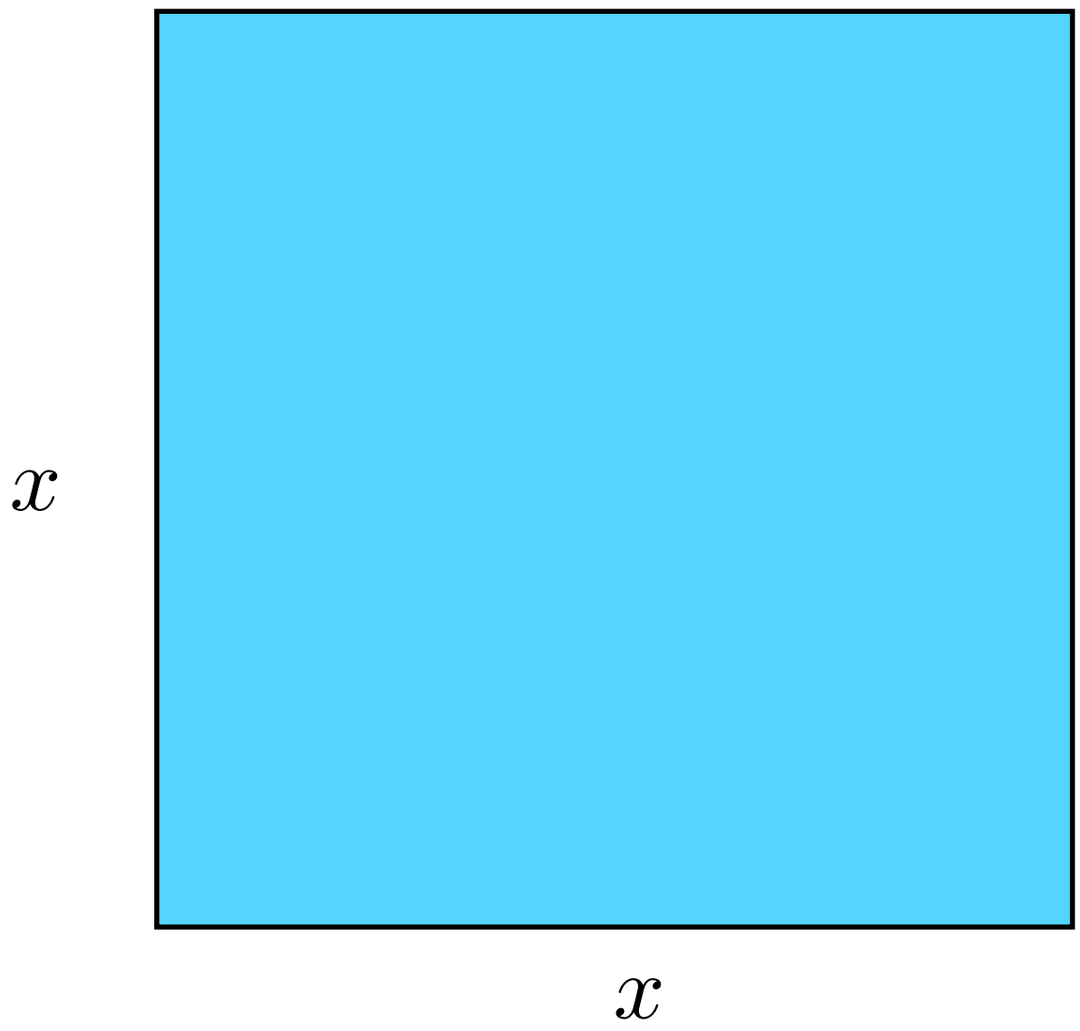
$$= (x^2 + ax) + b$$

$$x^2 + ax$$

$$x^2 + ax$$



$$x^2 + ax$$



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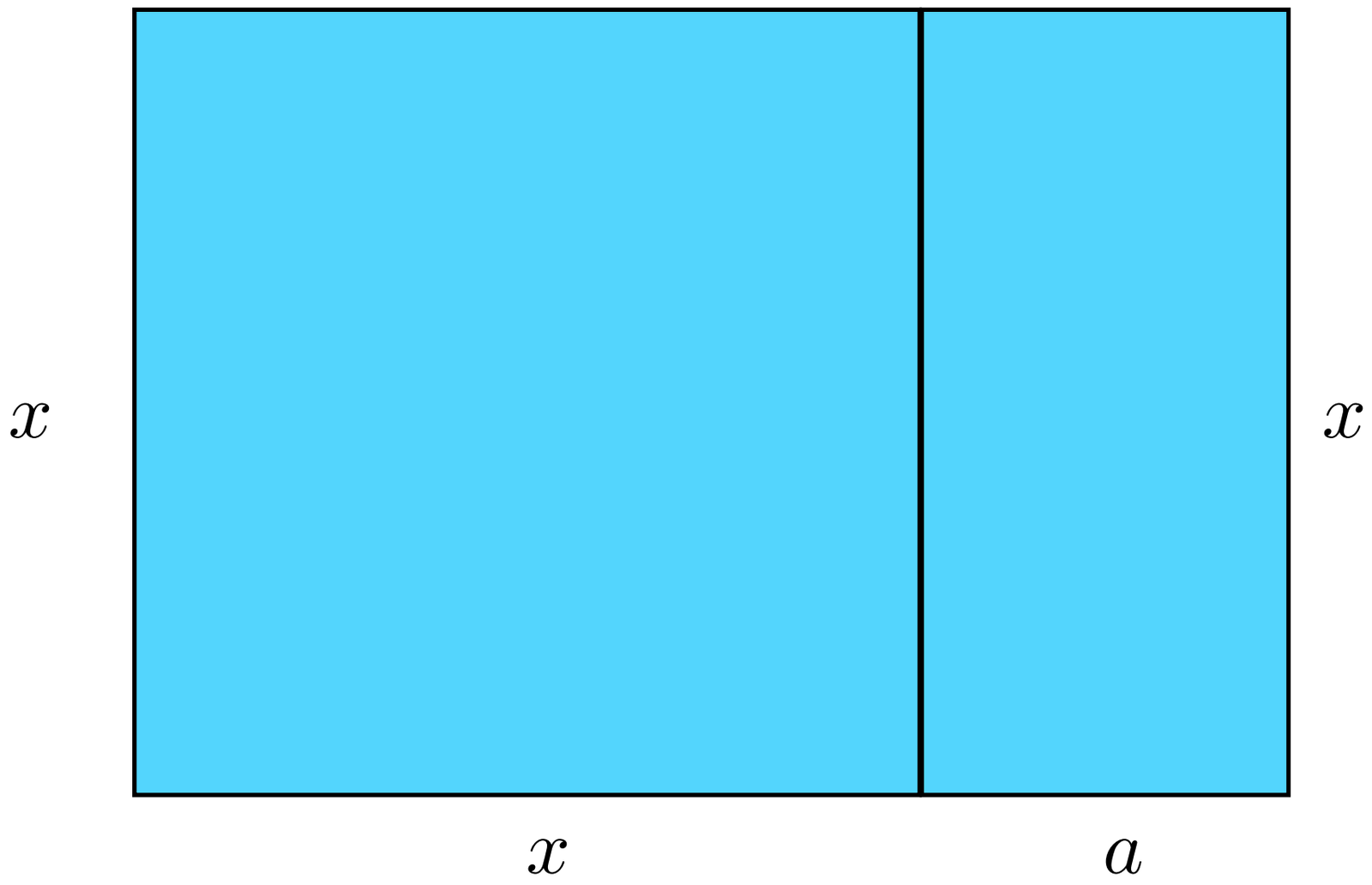

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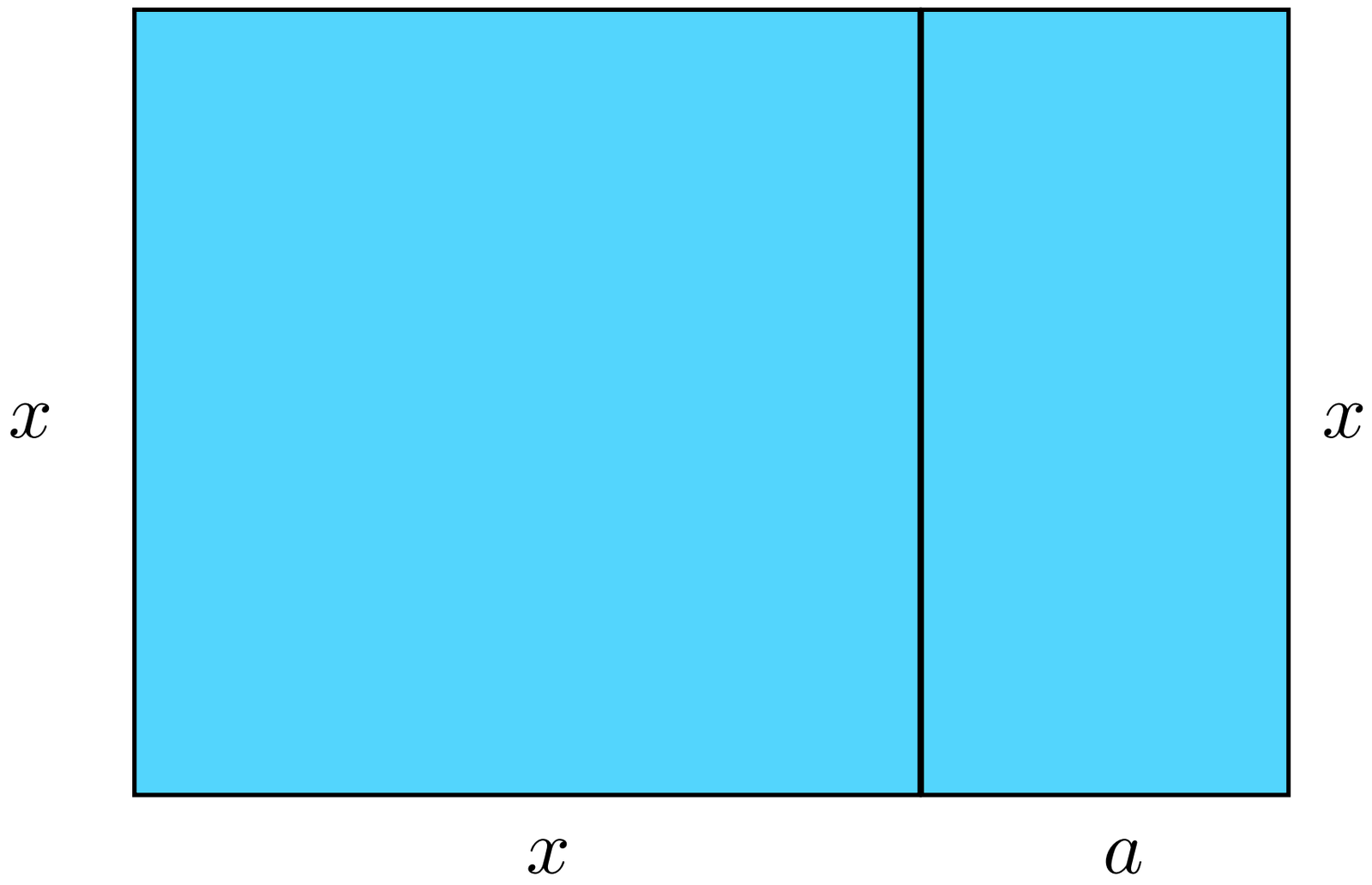

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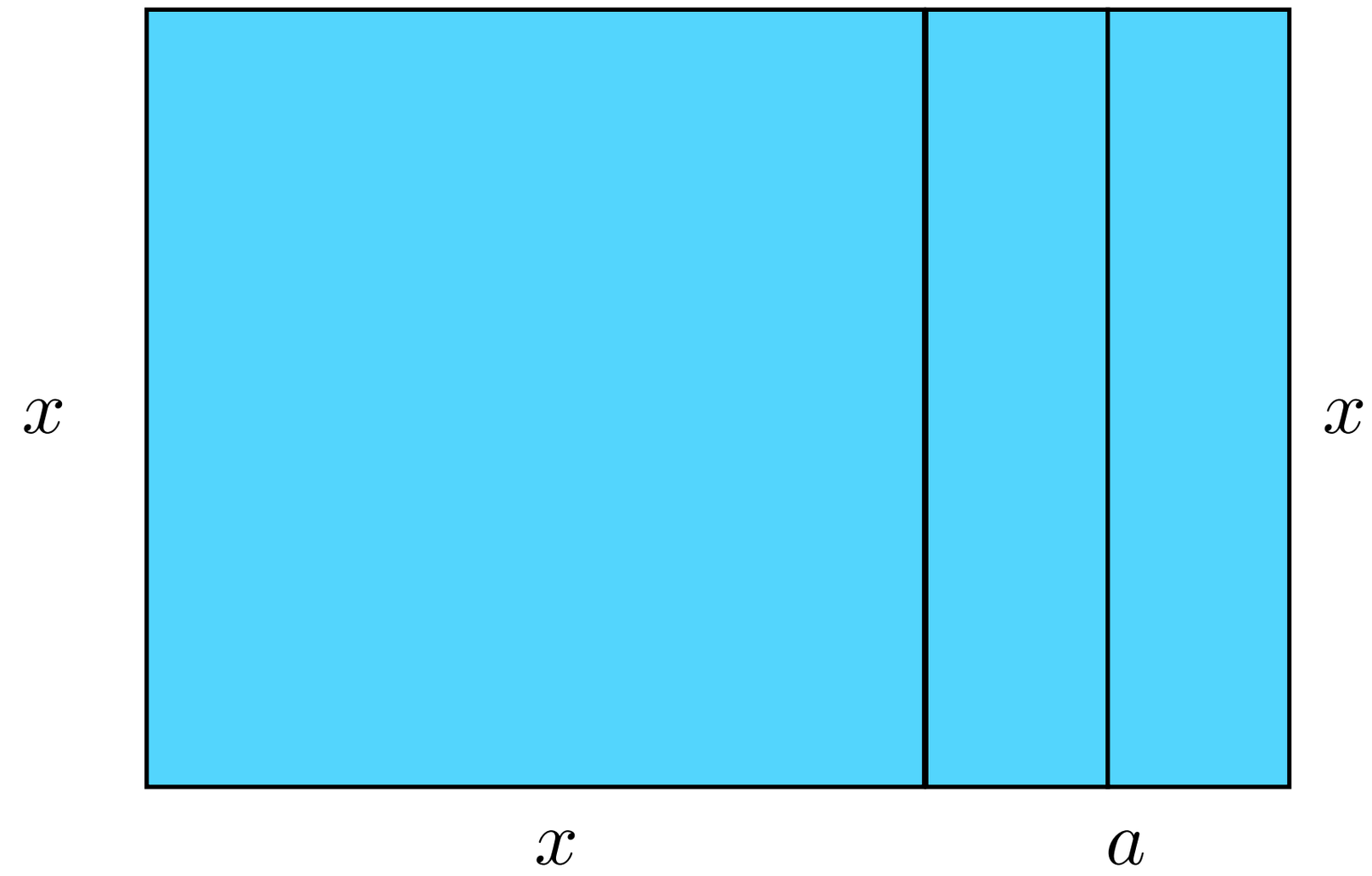

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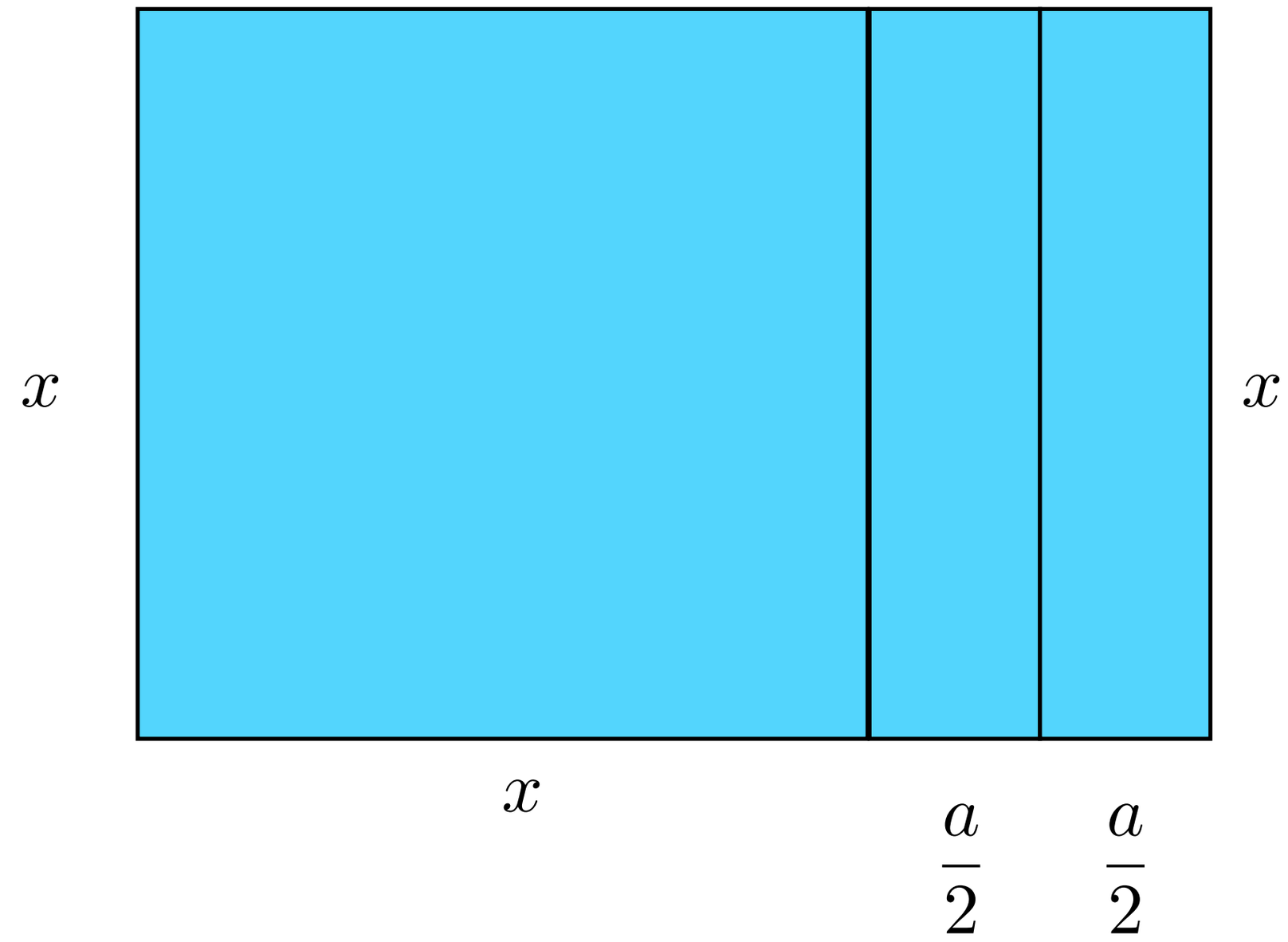
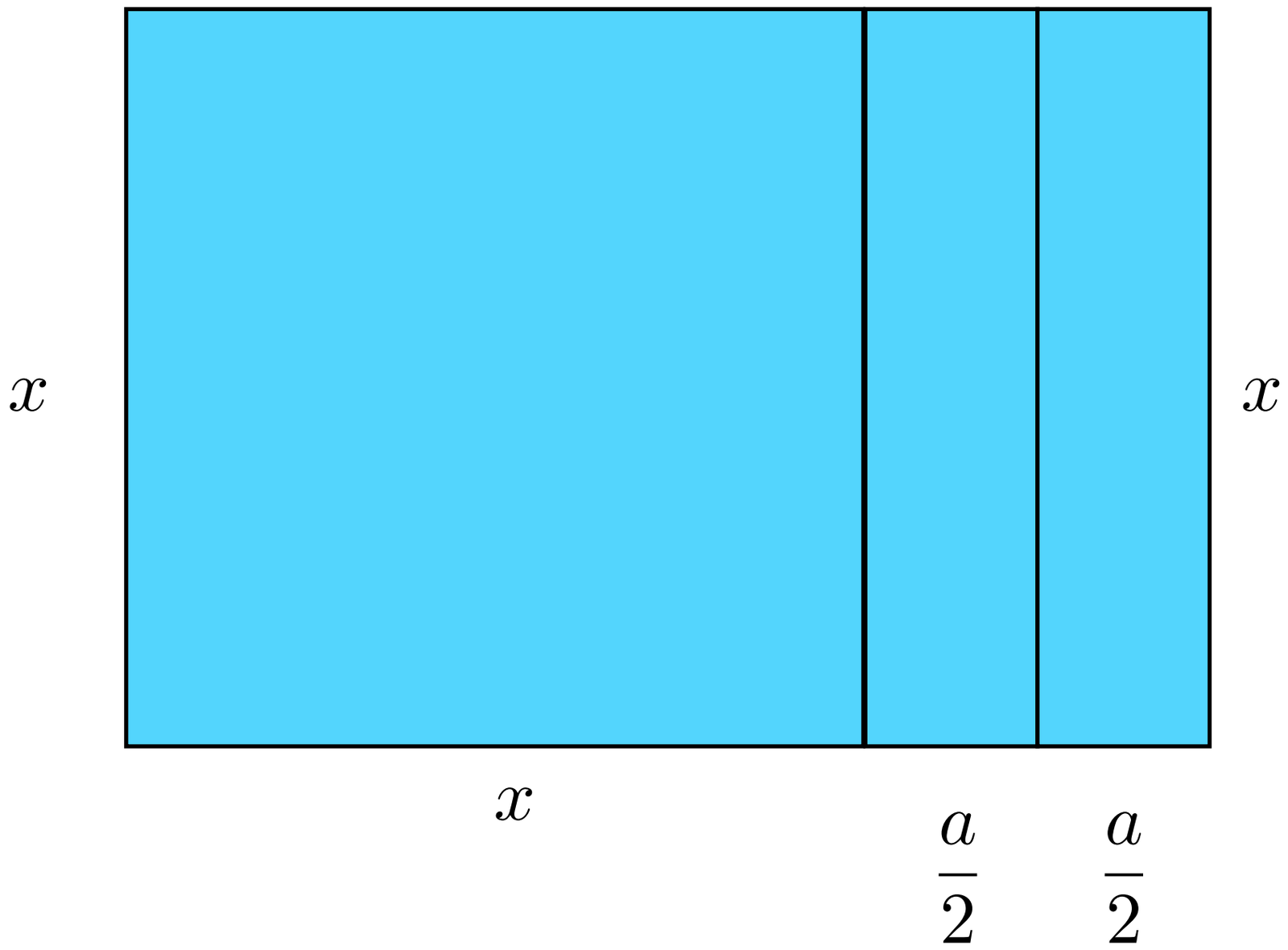


$$x^2 + ax = x^2 + \frac{a}{2}x + \frac{a}{2}x$$



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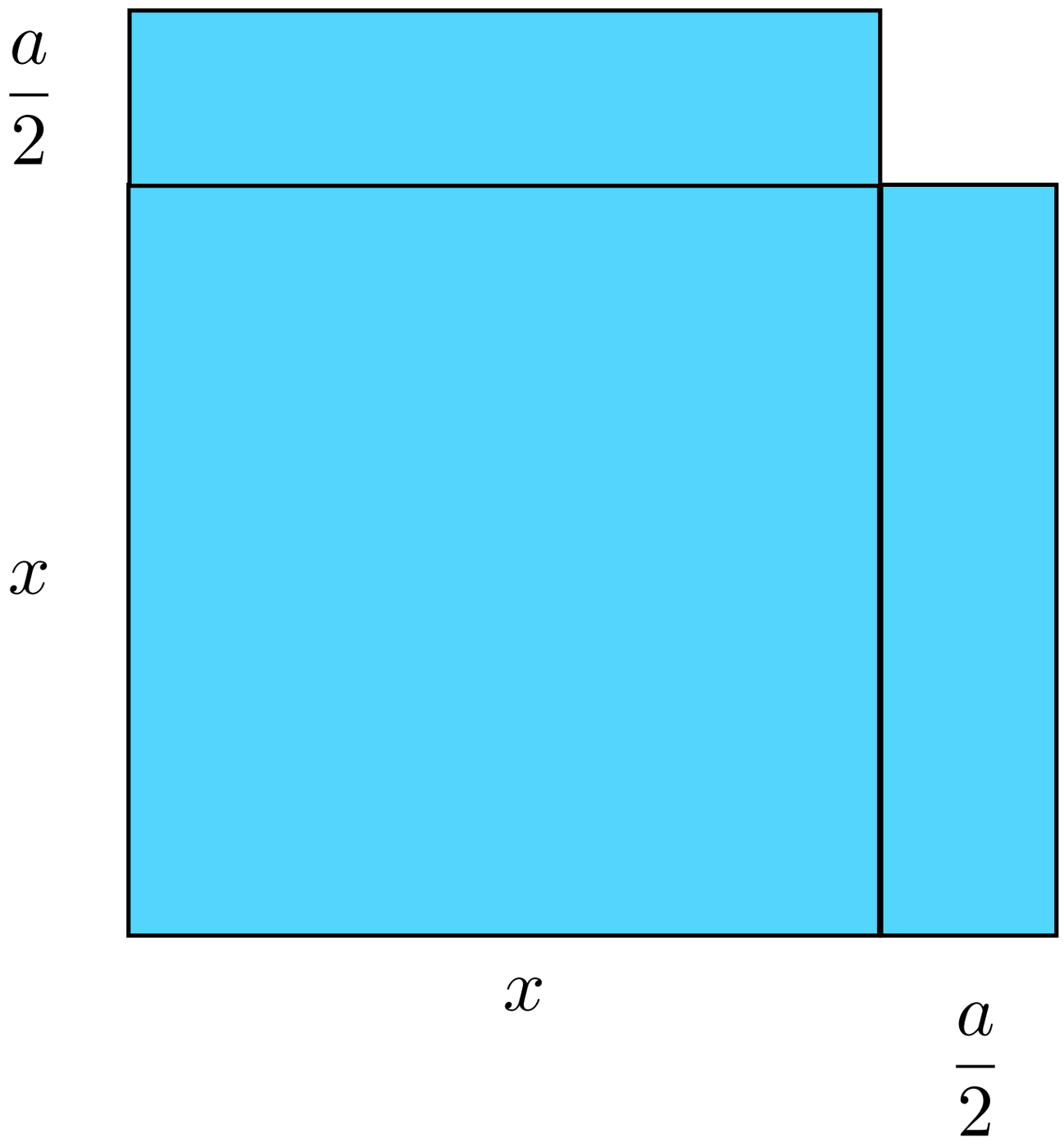

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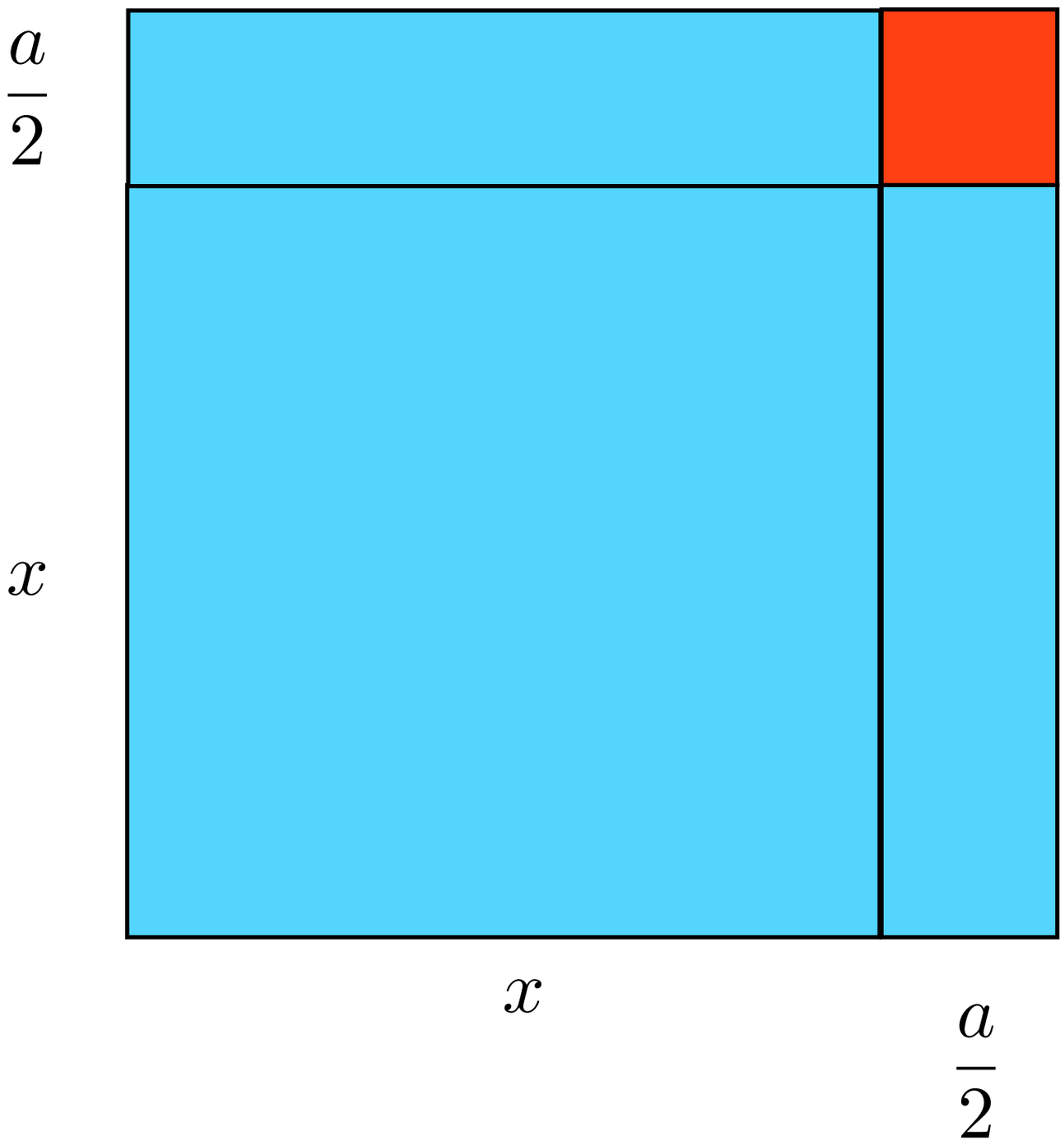

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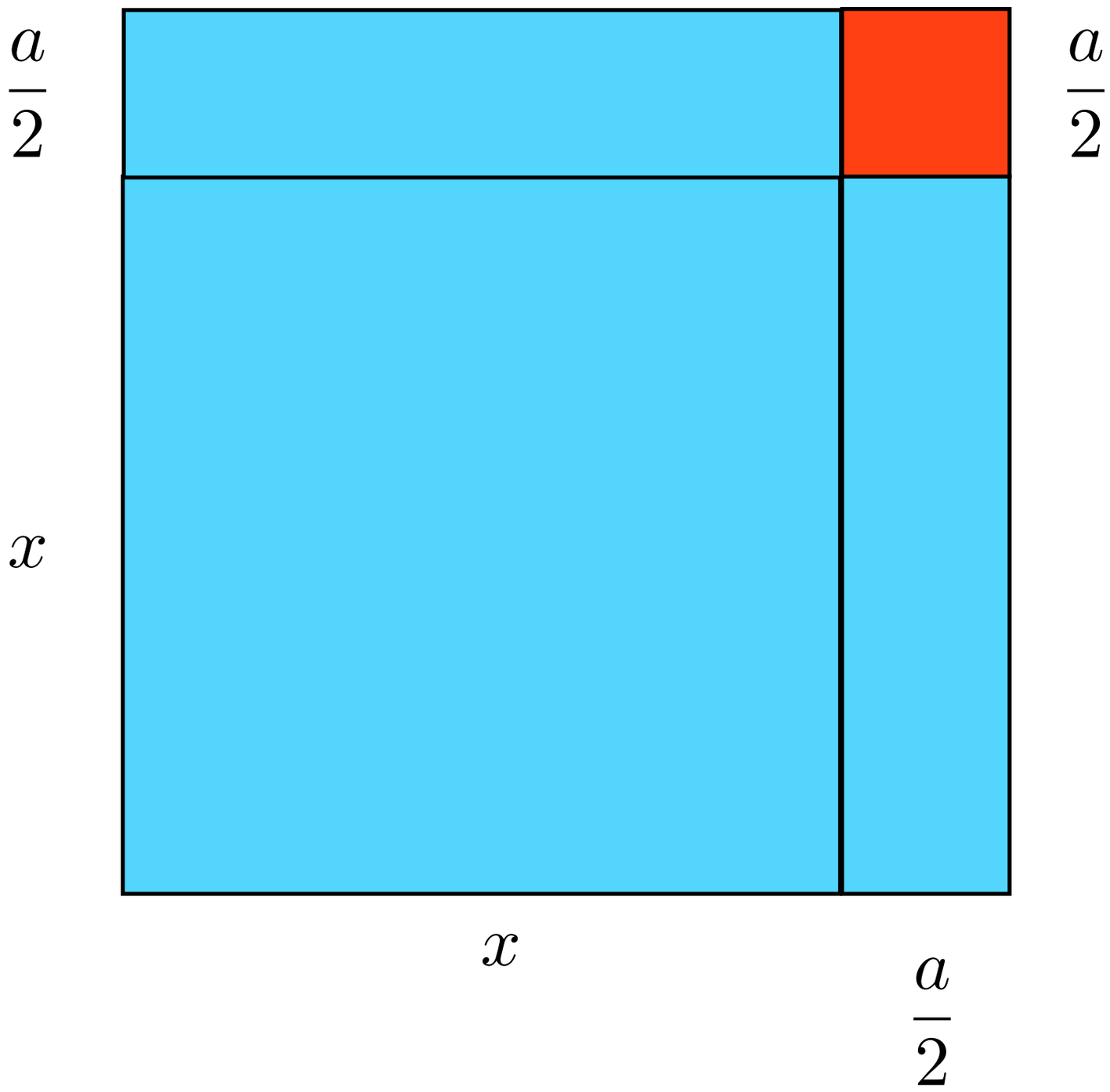
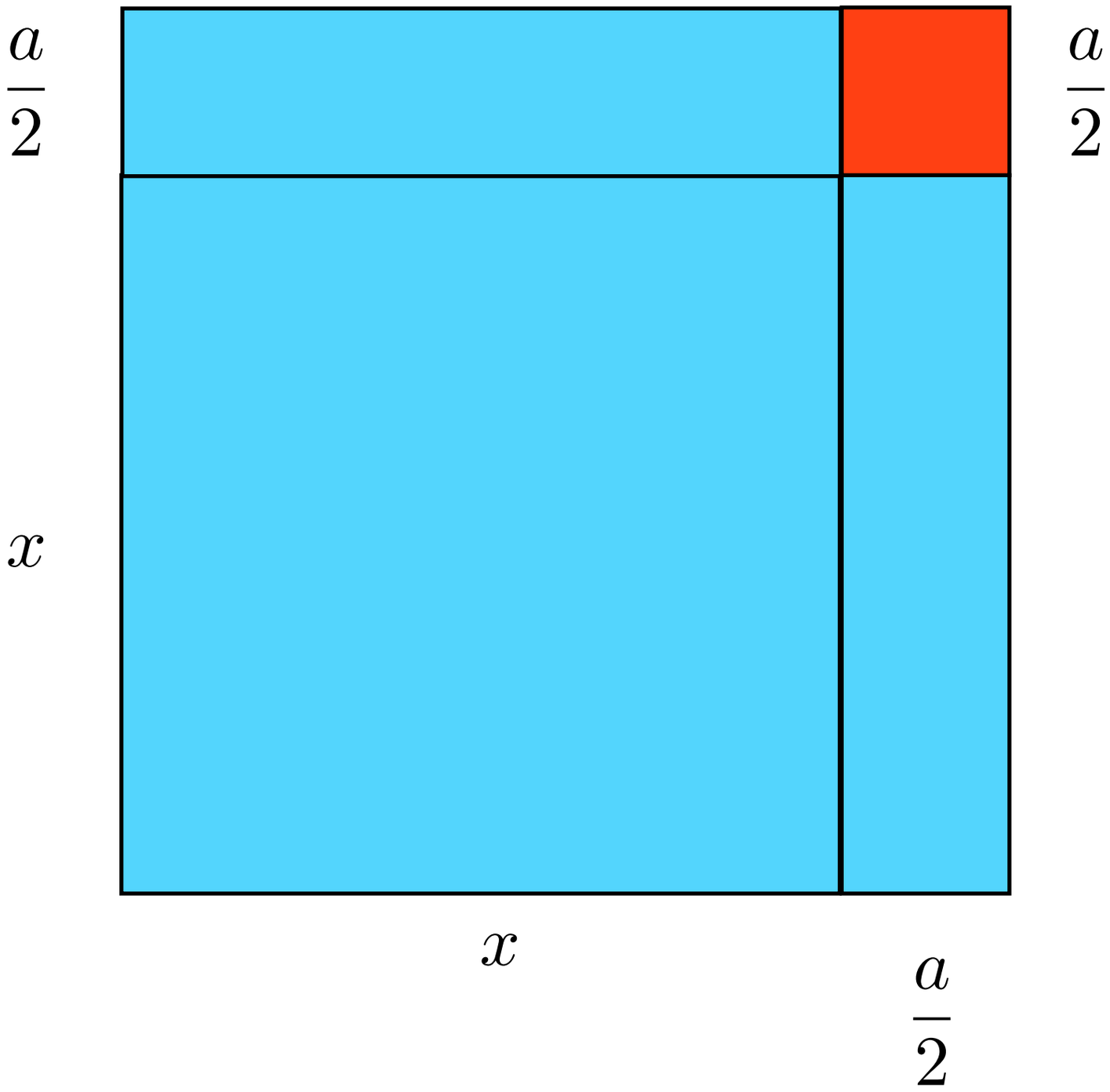


$$x^2 + ax = x^2 + \frac{a}{2}x + \frac{a}{2}x + \left(\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2$$



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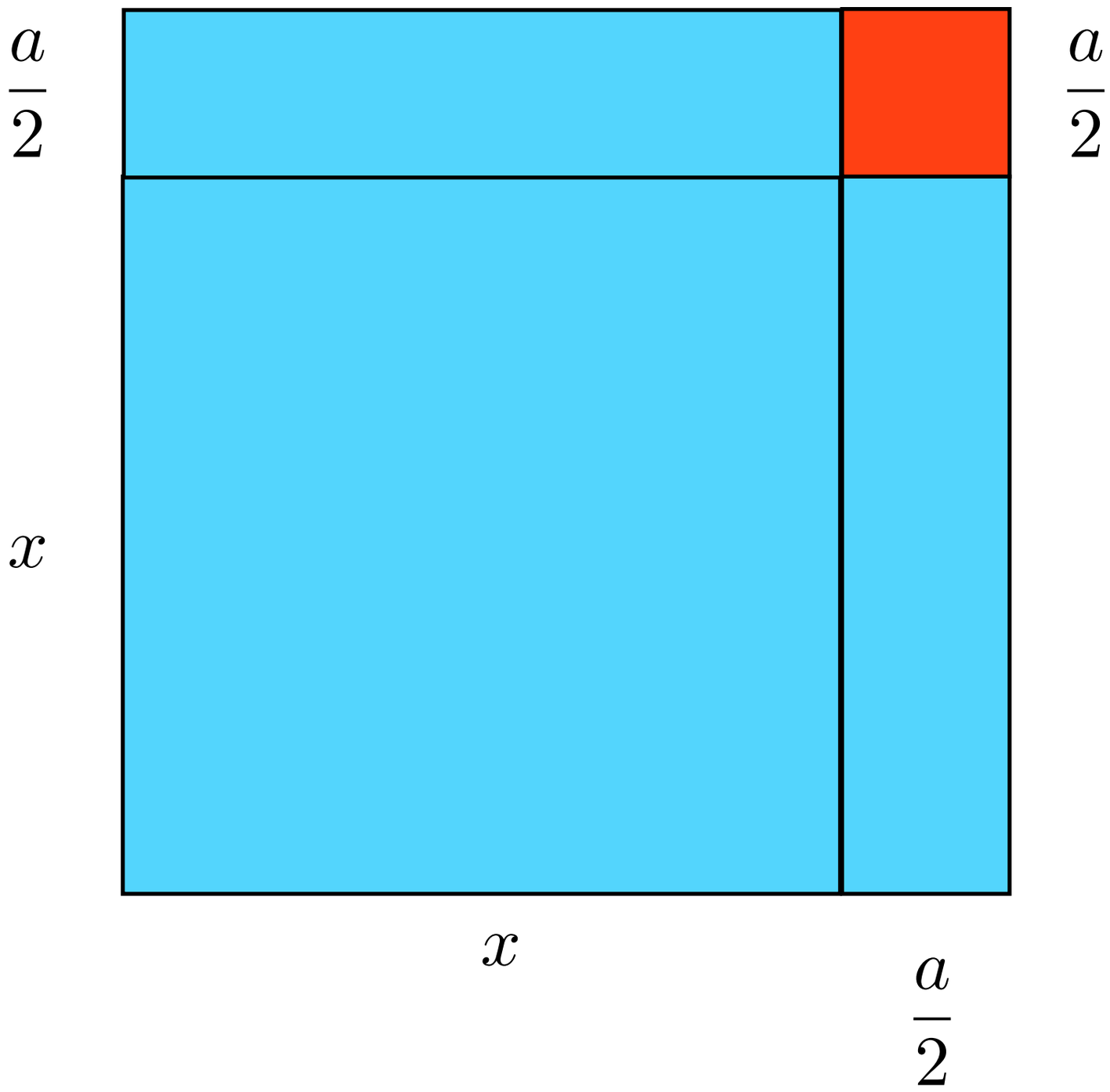

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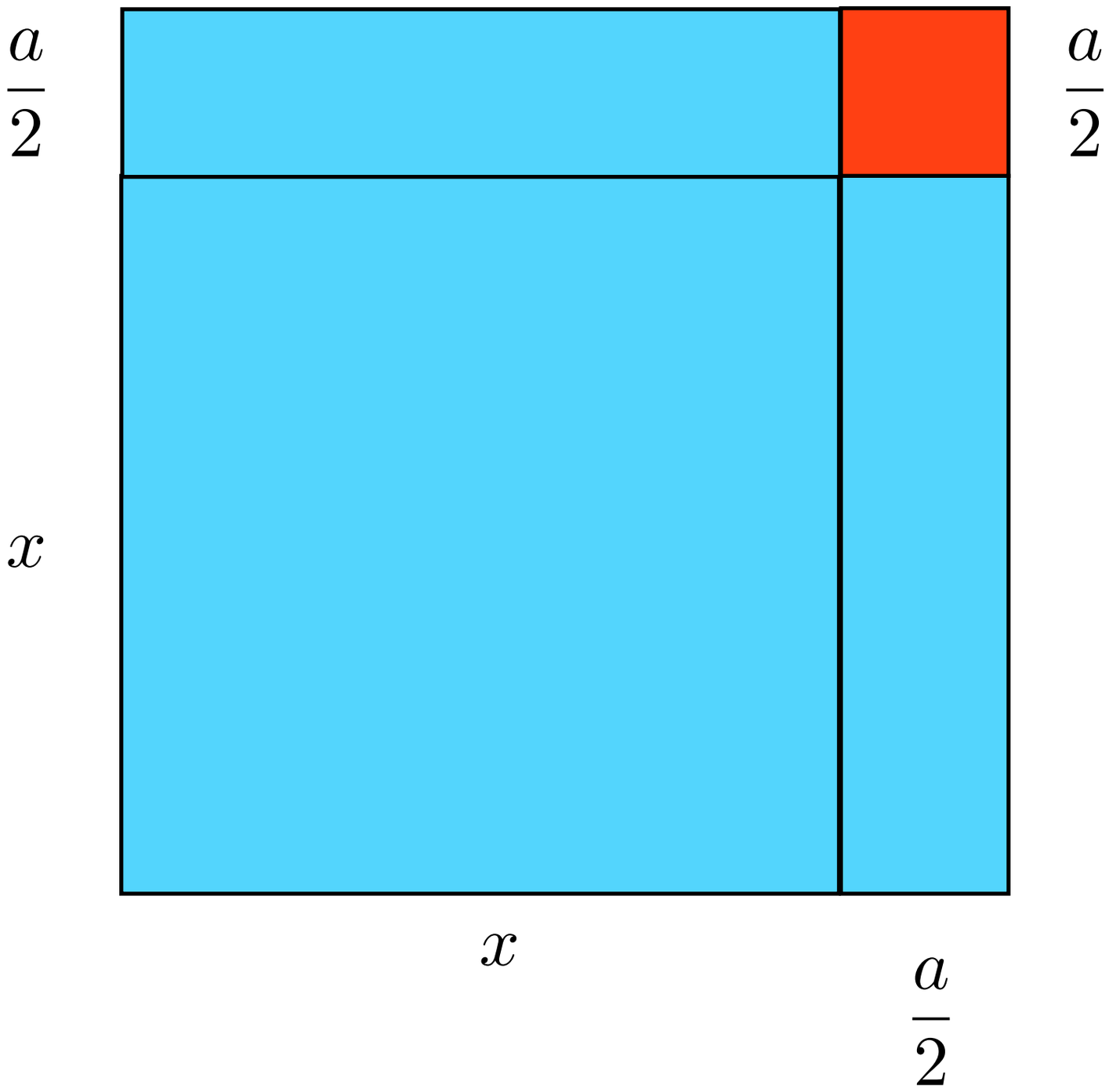

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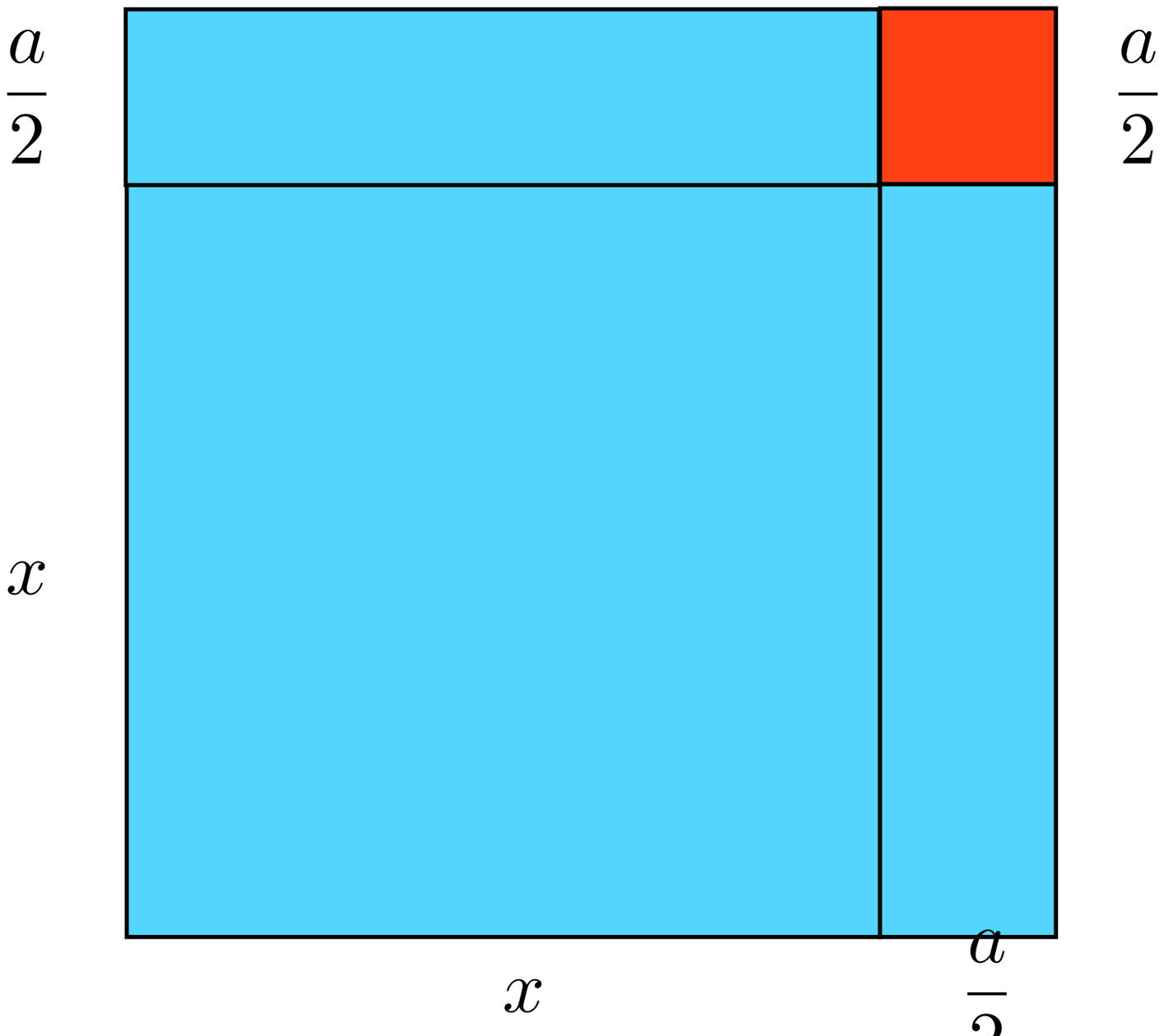

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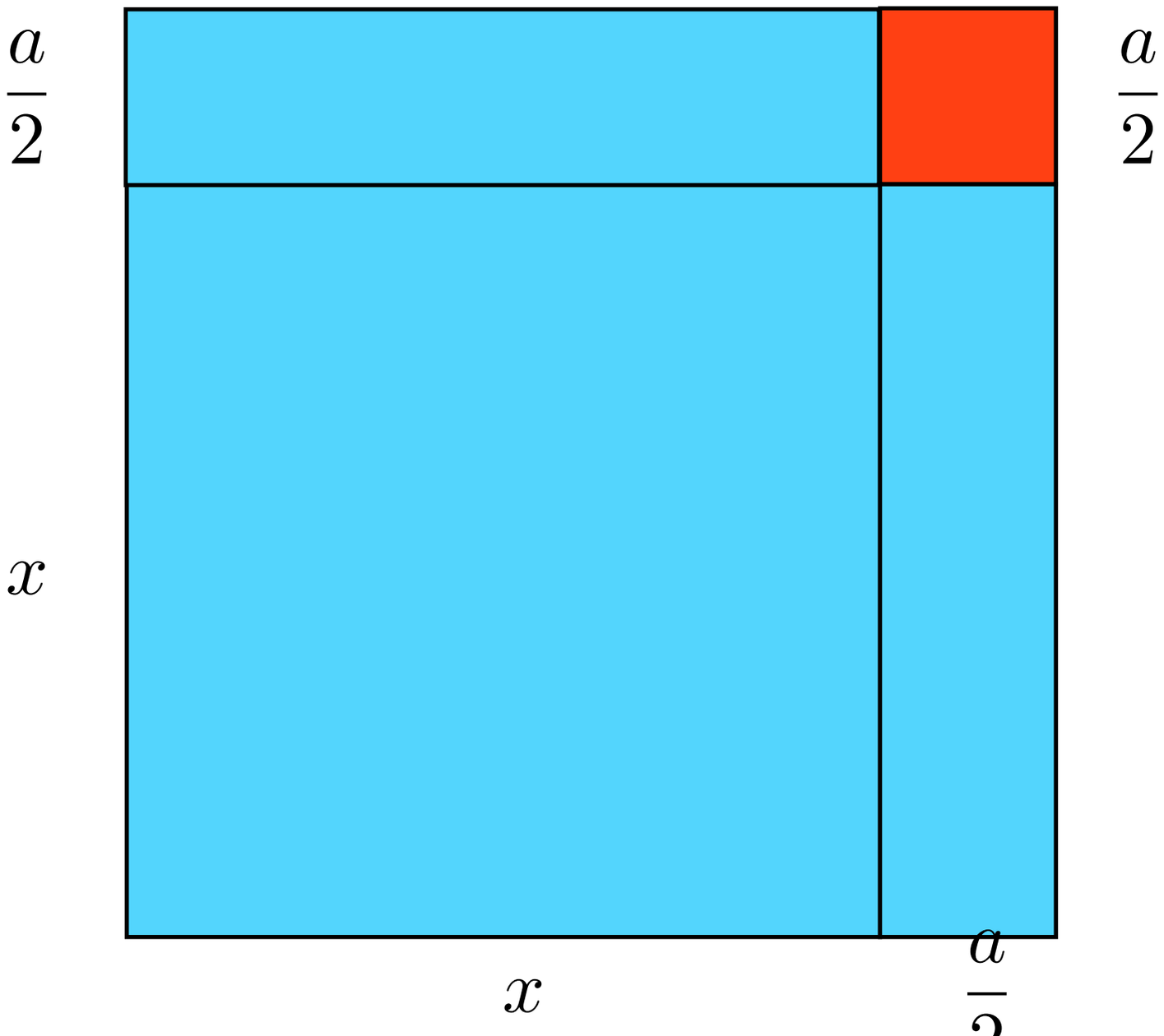

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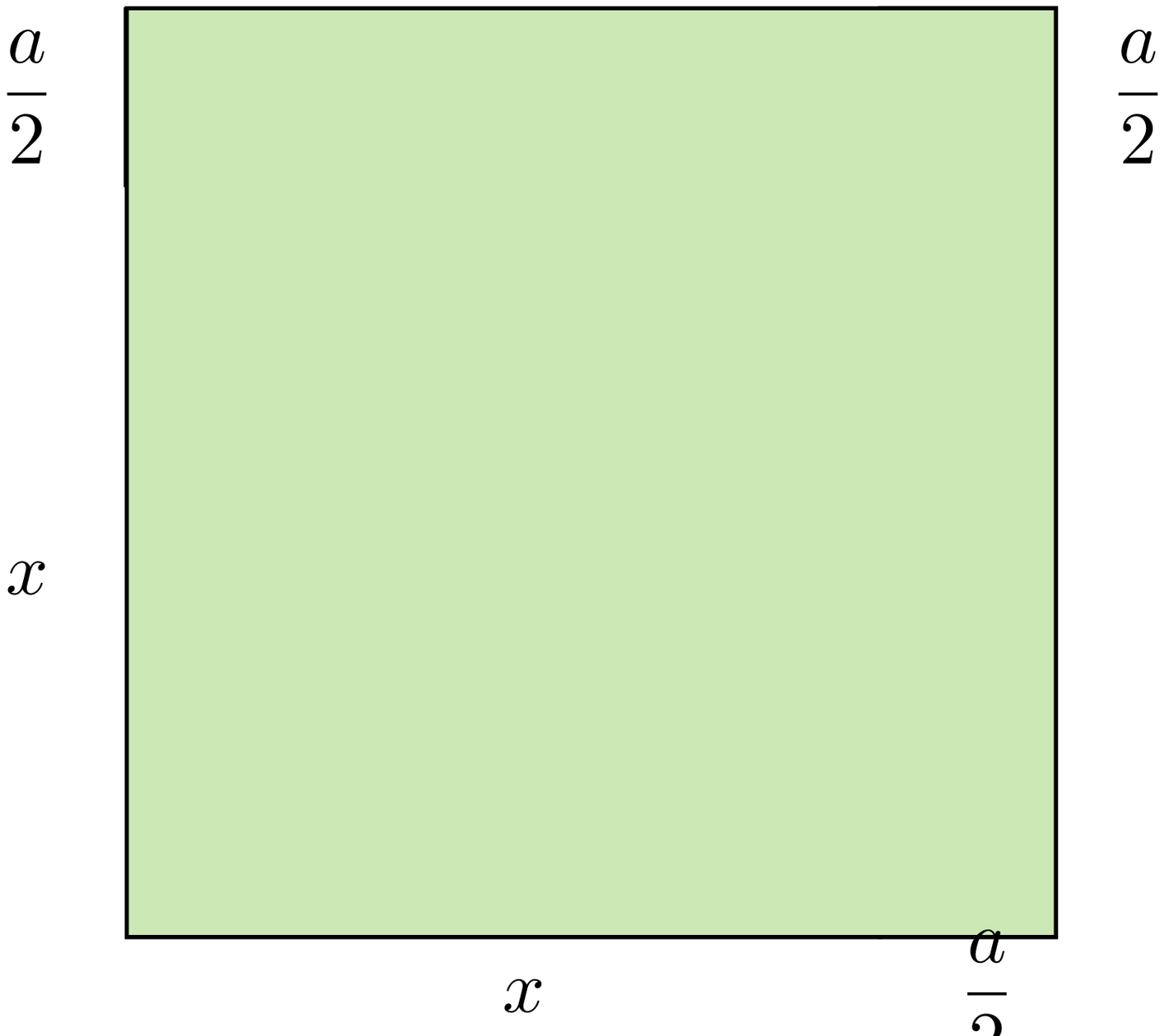

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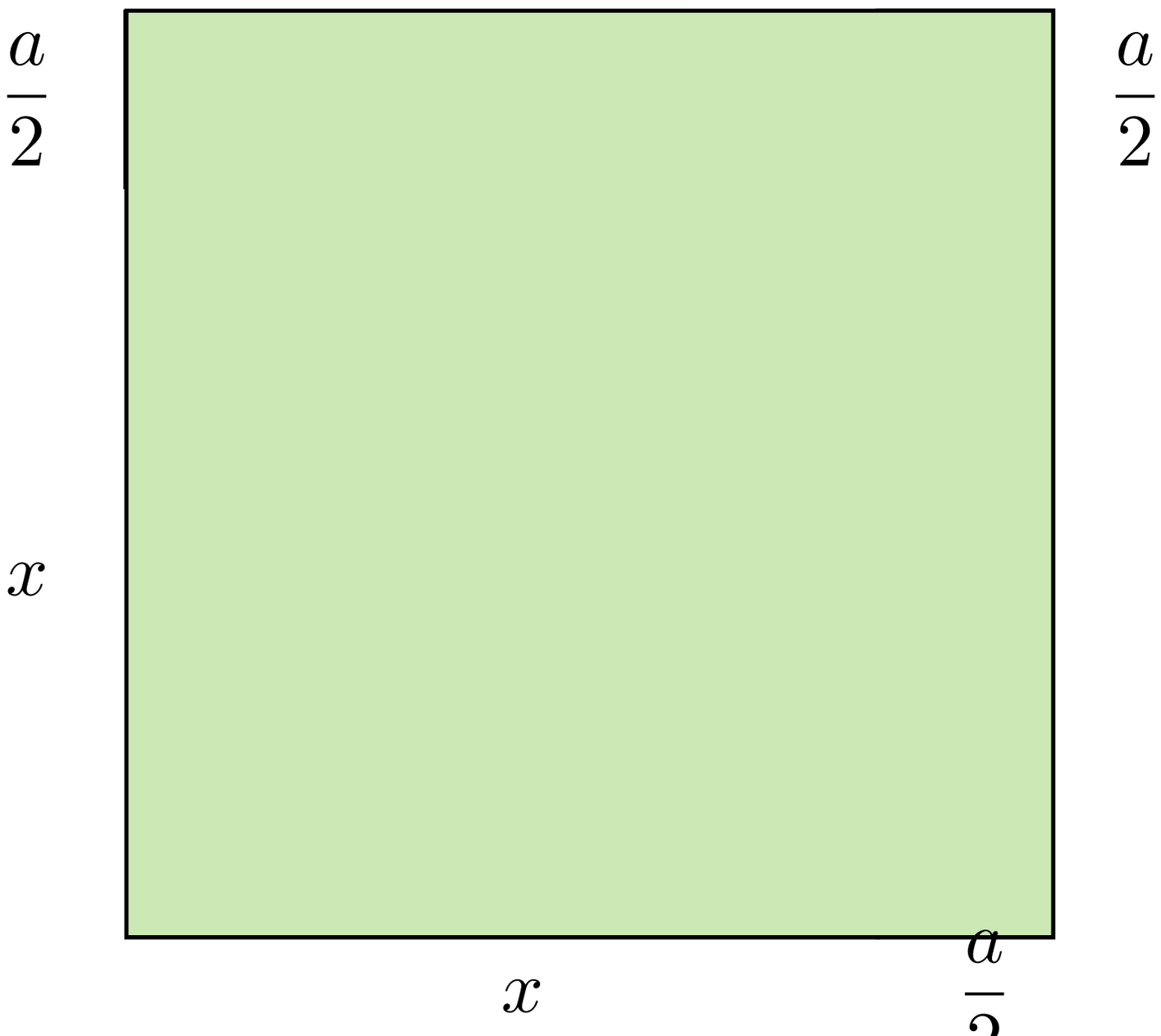
$$= \left(x^2 + ax + \frac{a^2}{2^2}\right) - \frac{a^2}{2^2} = \left(x + \frac{a}{2}\right)^2 - \frac{a^2}{2^2}$$



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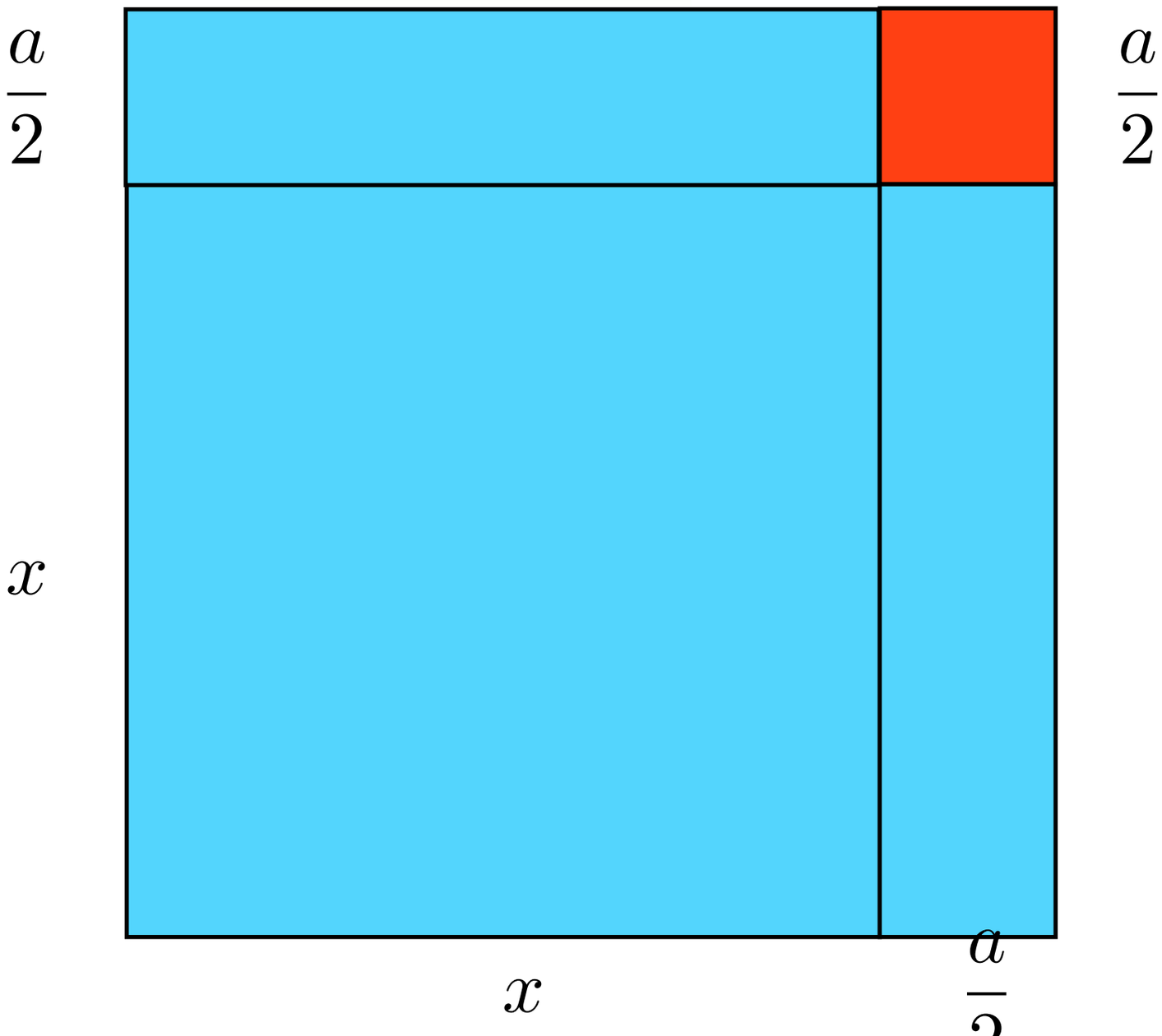

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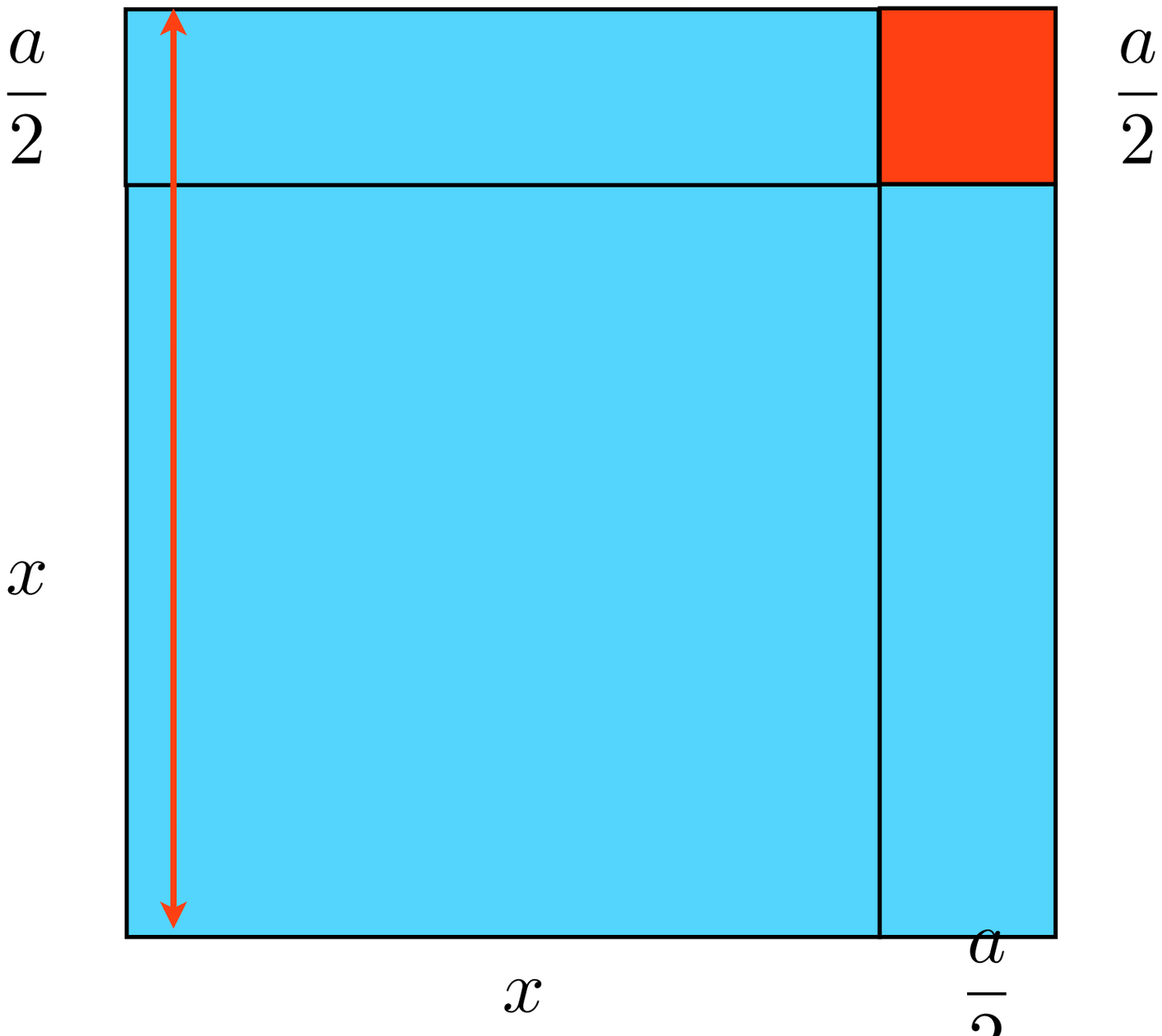

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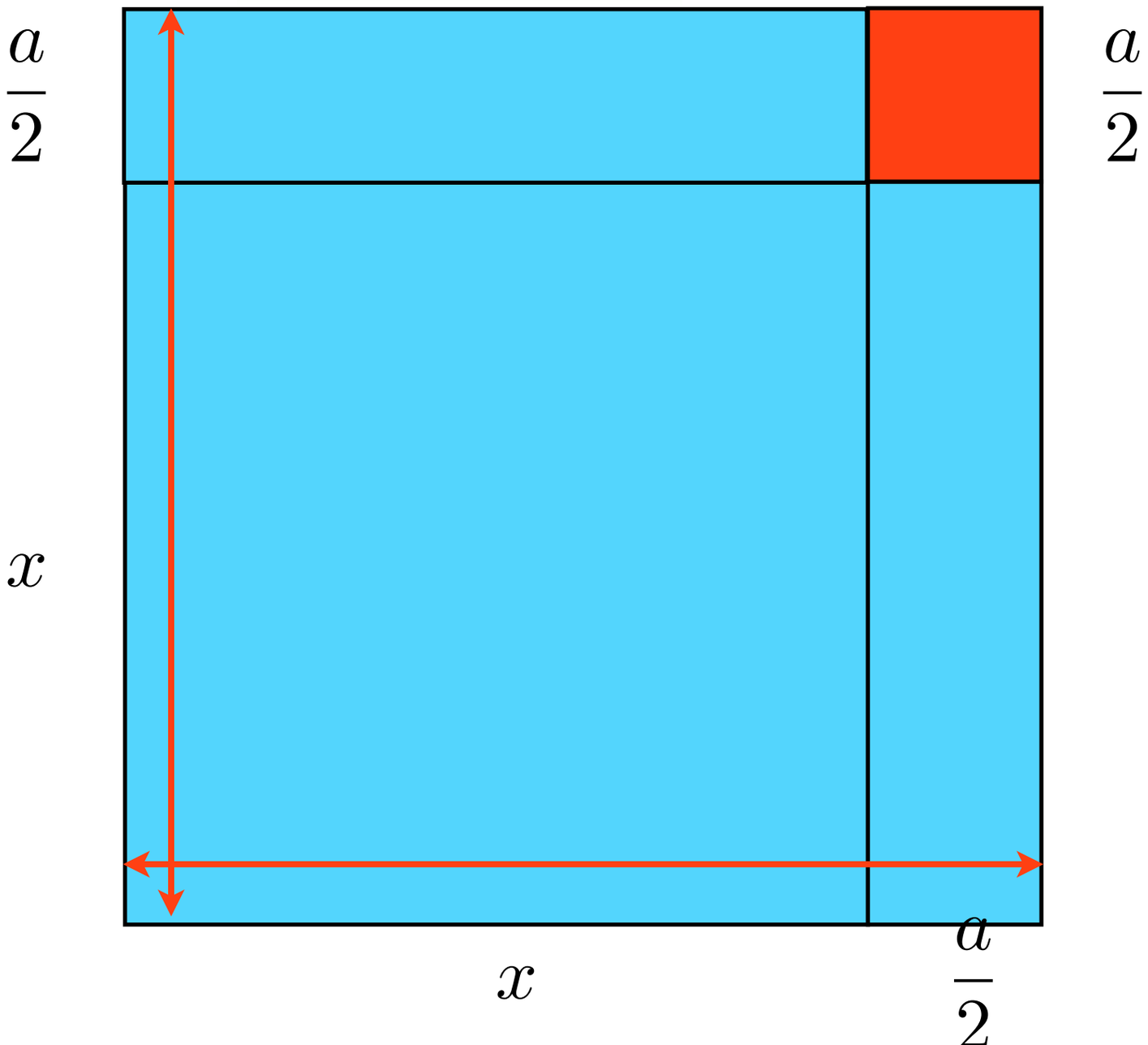

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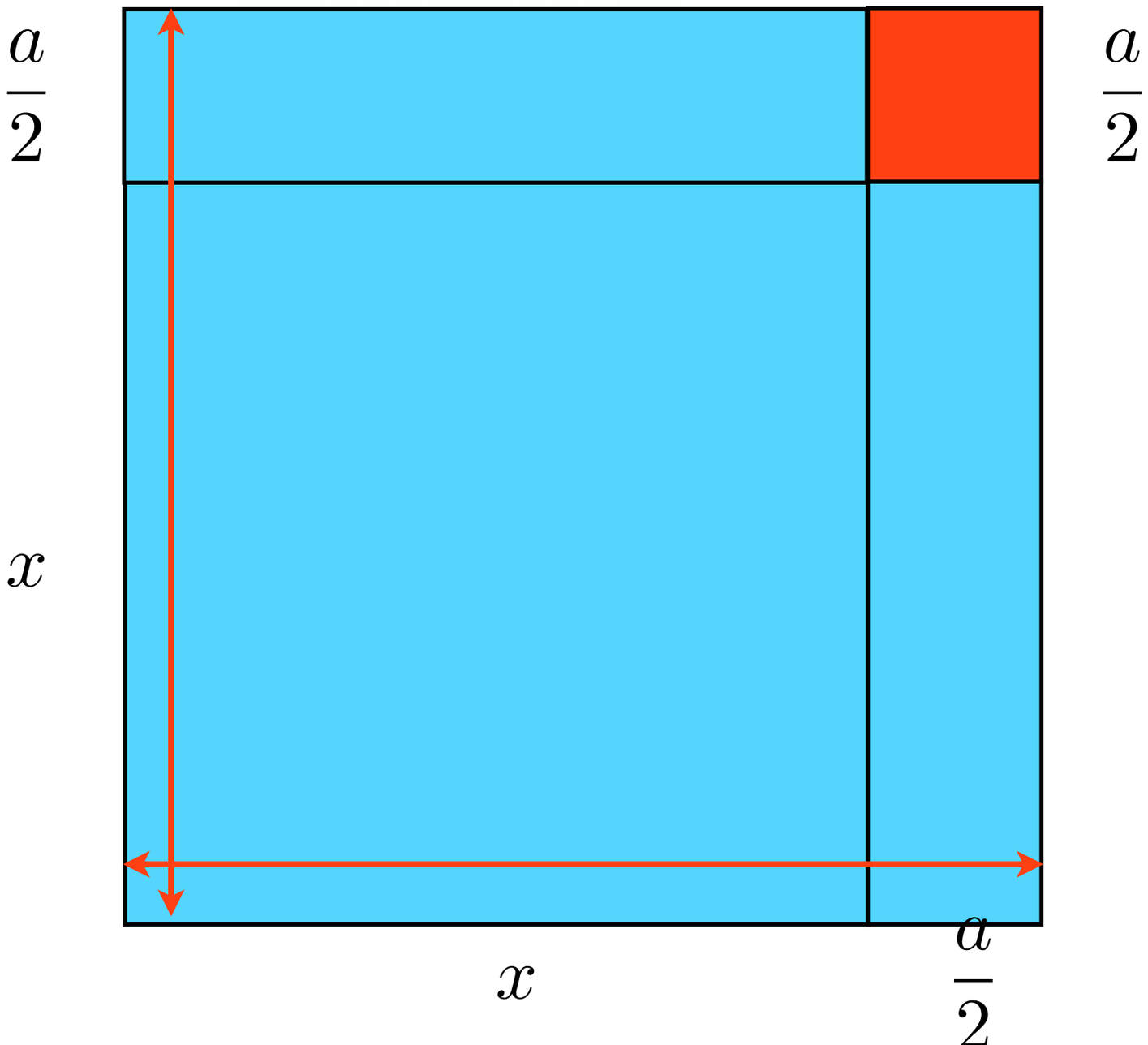

Exemple

$$2x^2 + 8x - 6$$

Exemple

$$2x^2 + 8x - 6 = 2(x^2 + 4x - 3)$$

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$$2x^2 + 8x - 6 = 2(x^2 + 4x - 3)$$

$$= 2(x^2 + 2x + 2x - 3)$$

$$= 2\left(x^2 + 2x + 2x + 2^2 - 2^2 - 3\right)$$

Exemple

$$2x^2 + 8x - 6 = 2(x^2 + 4x - 3)$$

$$= 2(x^2 + 2x + 2x - 3)$$

$$= 2\left(x^2 + 2x + 2x + 2^2 - 2^2 - 3\right)$$

$$= 2\left(x^2 + 4x + 4 - 7\right)$$

Exemple

$$2x^2 + 8x - 6 = 2(x^2 + 4x - 3)$$

$$= 2(x^2 + 2x + 2x - 3)$$

$$= 2\left(x^2 + 2x + 2x + 2^2 - 2^2 - 3\right)$$

$$= 2\left(x^2 + 4x + 4 - 7\right)$$

$$= 2\left((x+2)^2 - 7\right)$$

Faites les exercices suivants

Section 2 # 16

Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx$$

Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx$$

$$x^2 - 6x + 2$$

Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx$$

$$x^2 - 6x + 2 = x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 2$$

Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx$$

$$x^2 - 6x + 2 = x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 2$$

Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx$$

$$\begin{aligned} x^2 - 6x + 2 &= x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 2 \\ &= x^2 - 6x + 9 - 7 \end{aligned}$$

Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx$$

$$\begin{aligned} x^2 - 6x + 2 &= x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 2 \\ &= x^2 - 6x + 9 - 7 = (x-3)^2 - 7 \end{aligned}$$

Exemple

$$\int \frac{1}{x^2-6x+2}\,dx = \int \frac{1}{(x-3)^2-7}\,dx$$

$$x^2-6x+2 = x^2-6x+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+2$$

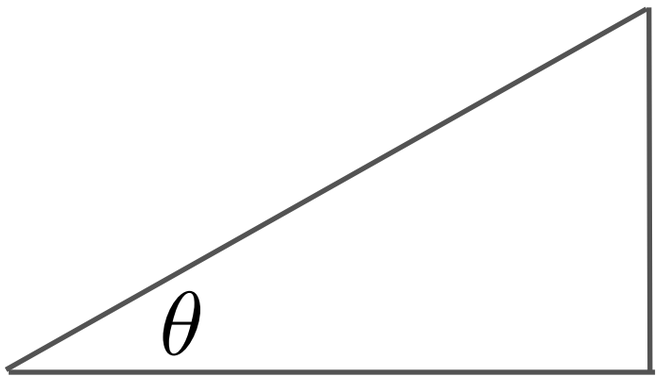
$$= x^2-6x+9-7 = (x-3)^2-7$$

Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx = \int \frac{1}{(x-3)^2 - 7}\, dx$$

$$x^2 - 6x + 2 = x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 2$$

$$= x^2 - 6x + 9 - 7 = (x-3)^2 - 7$$



Exemple

$$\int \frac{1}{x^2 - 6x + 2}\, dx = \int \frac{1}{(x-3)^2 - 7}\, dx$$

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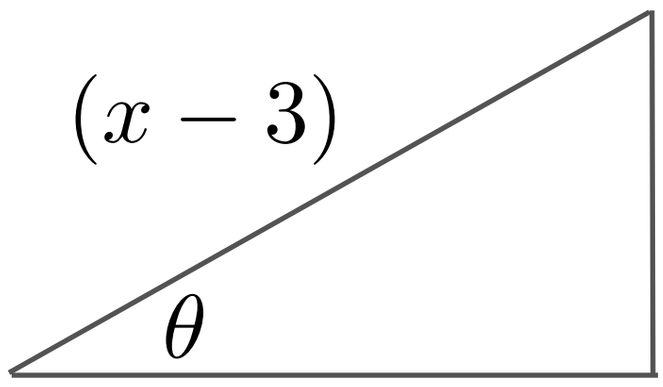

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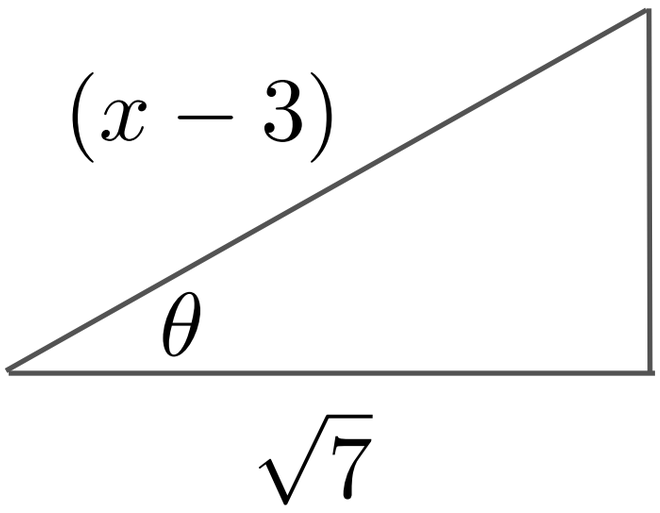

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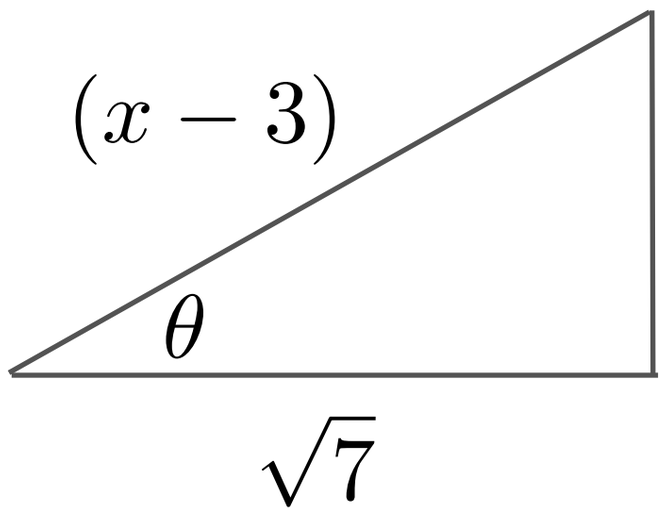

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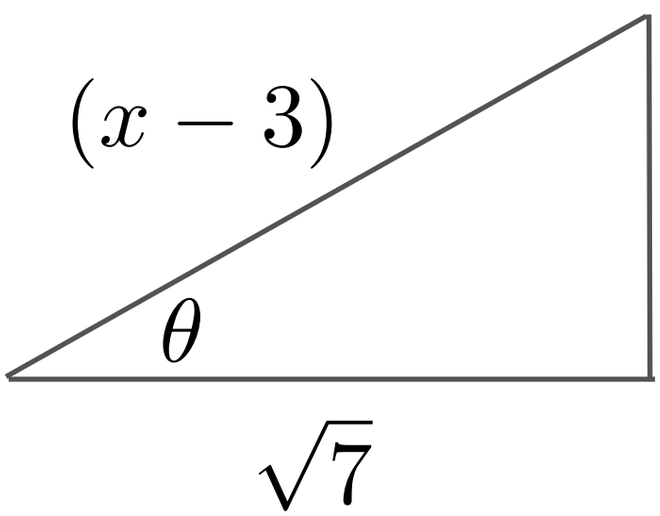

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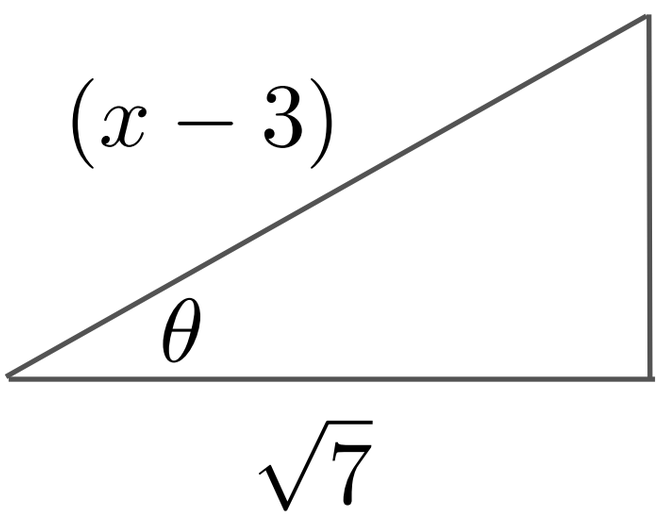

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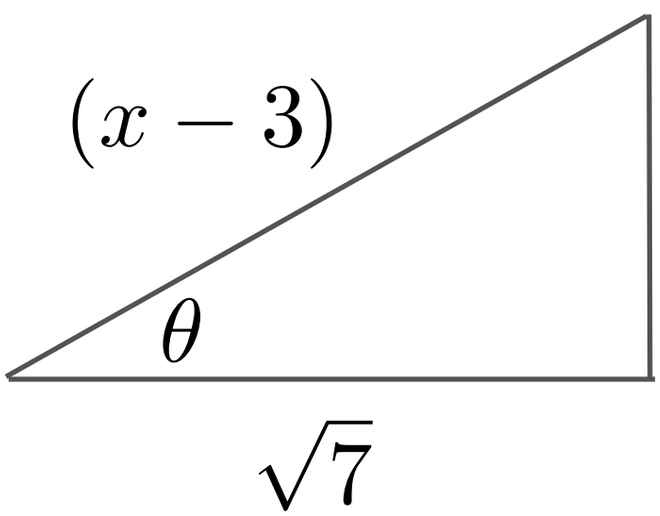

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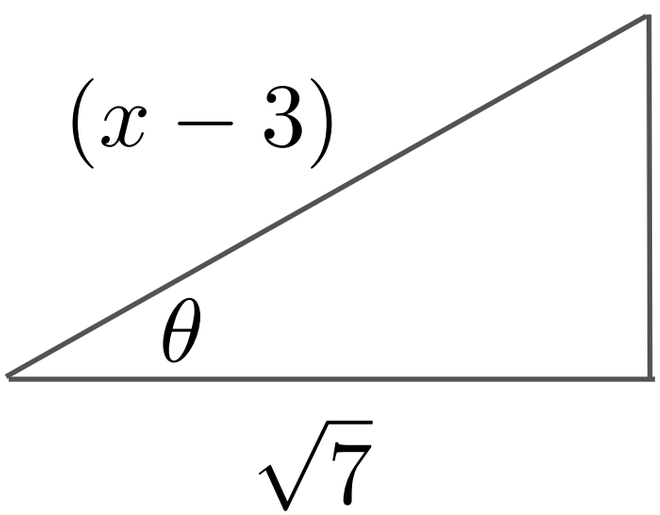

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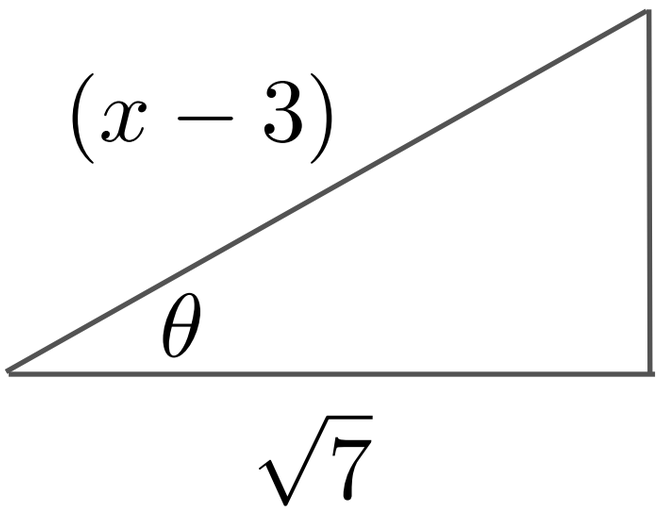

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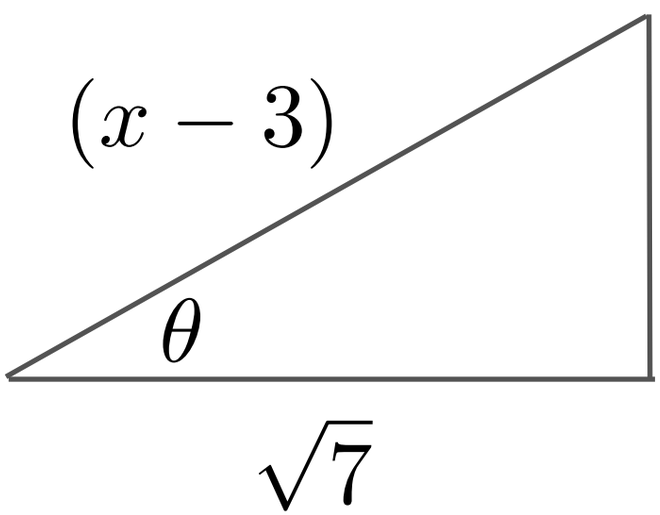

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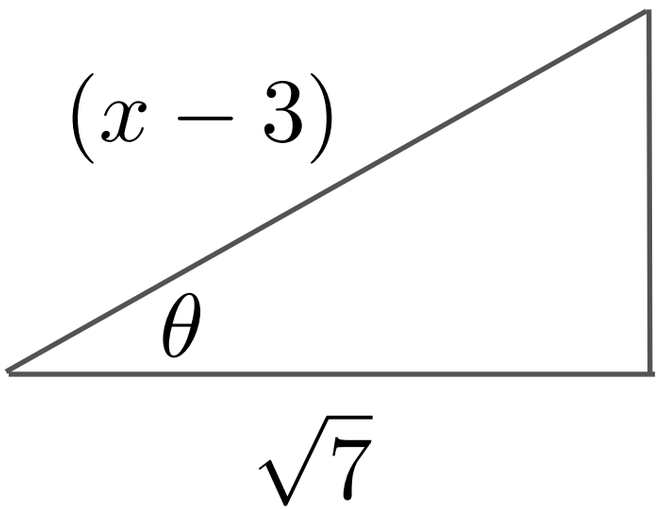

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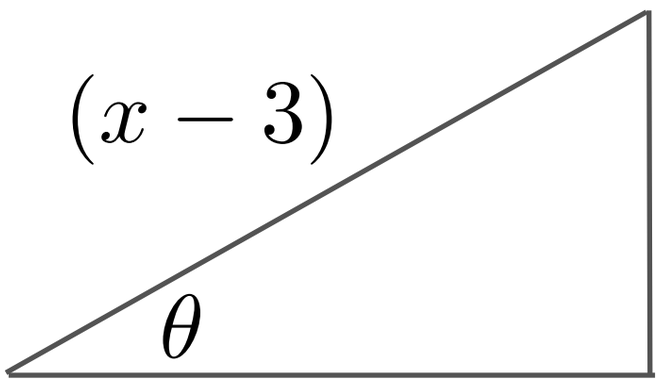

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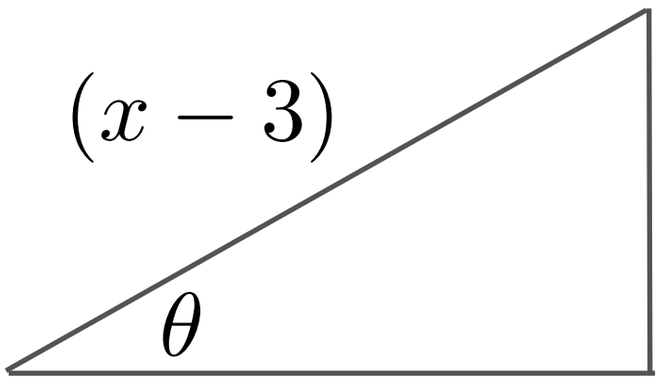

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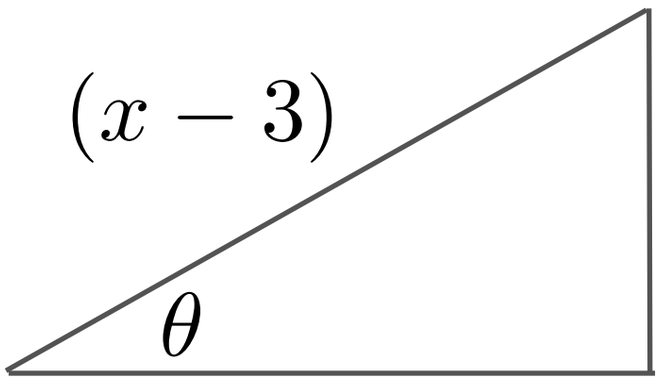

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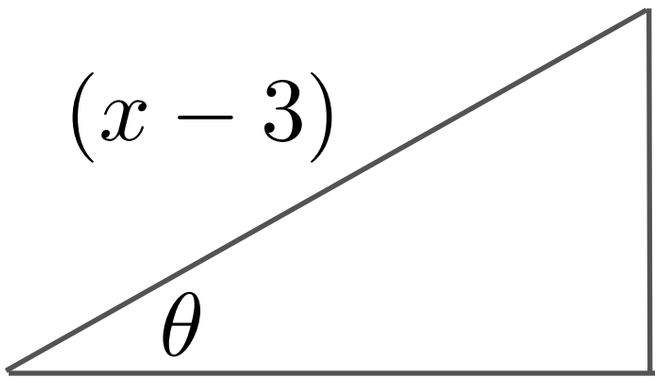

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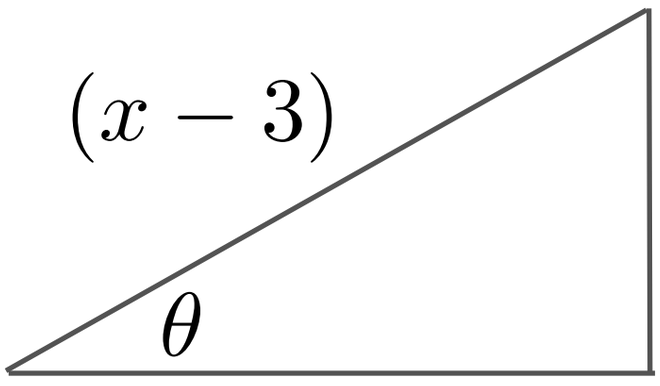

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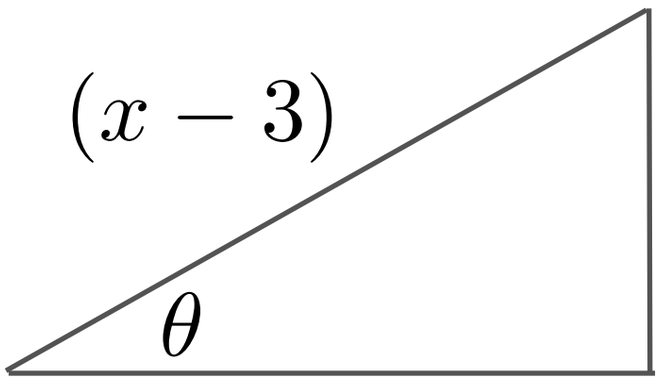

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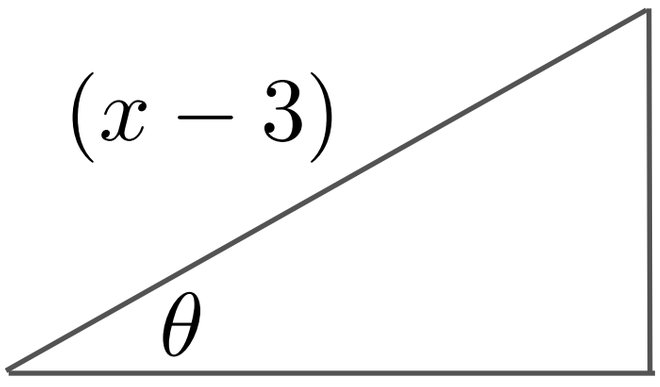

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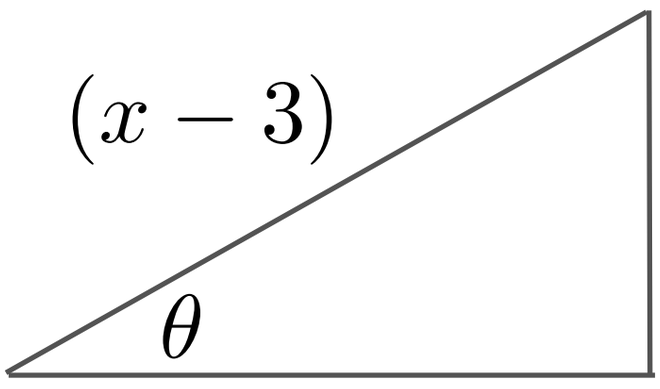

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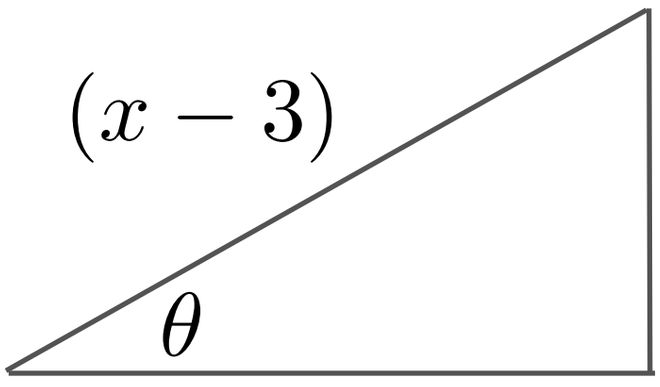

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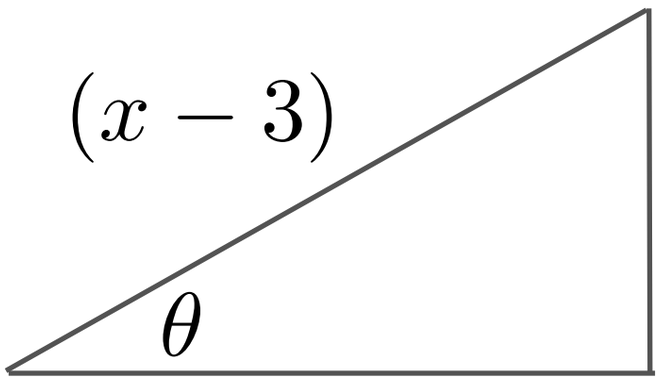

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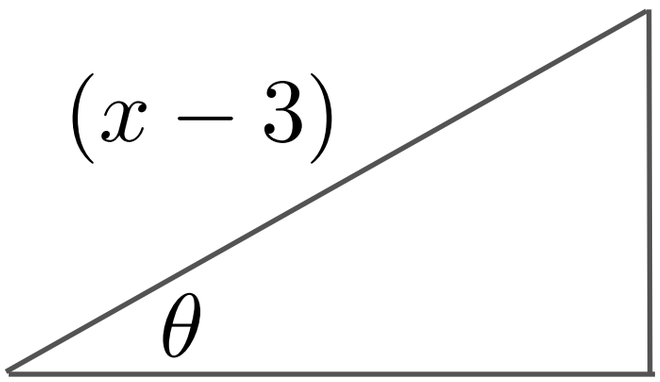

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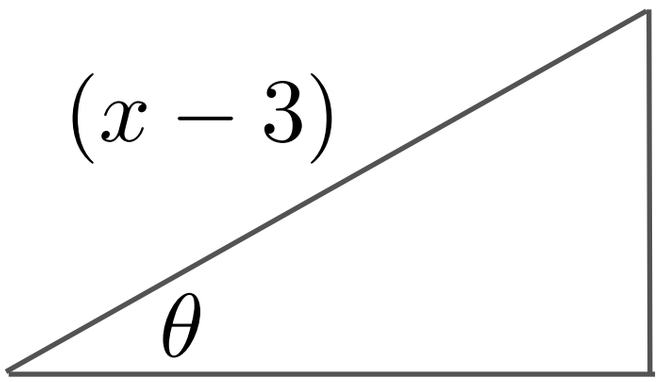

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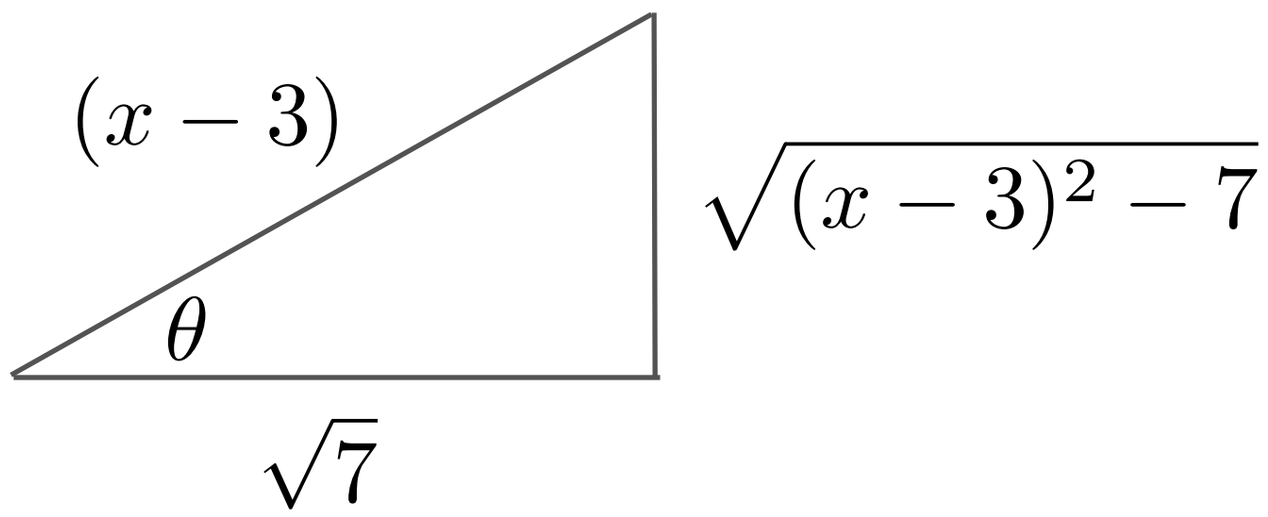

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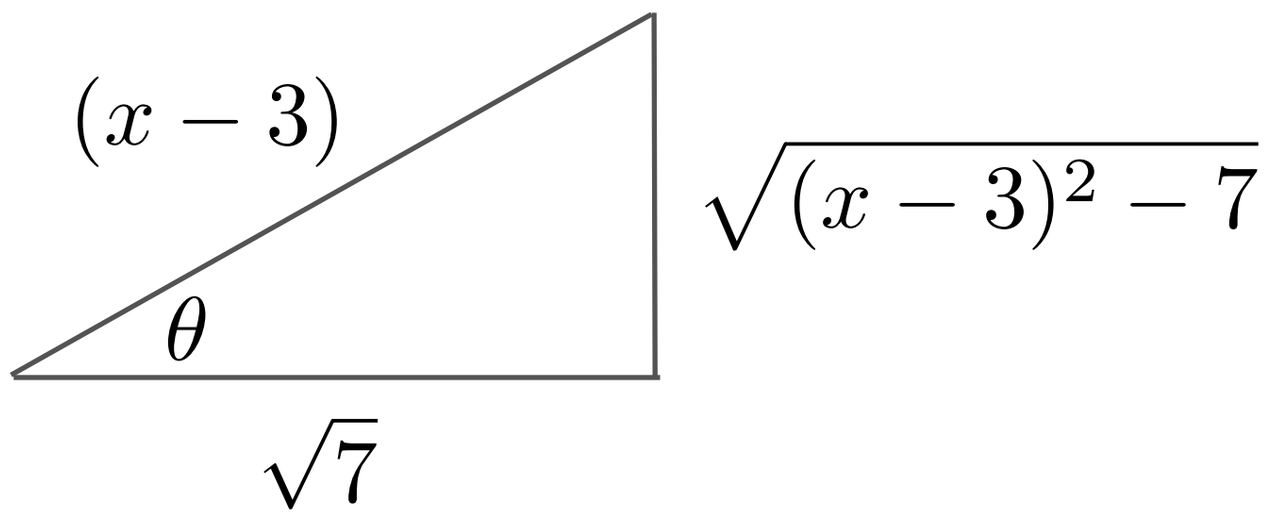
$$= \frac{1}{\sqrt{7}} \int \csc\theta \; d\theta = -\frac{1}{\sqrt{7}} \ln\left|\csc\theta + \cot\theta\right| + C$$



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$$= \frac{1}{\sqrt{7}}\int \csc\theta\;d\theta = -\frac{1}{\sqrt{7}}\ln\left|\csc\theta+\cot\theta\right| + C$$

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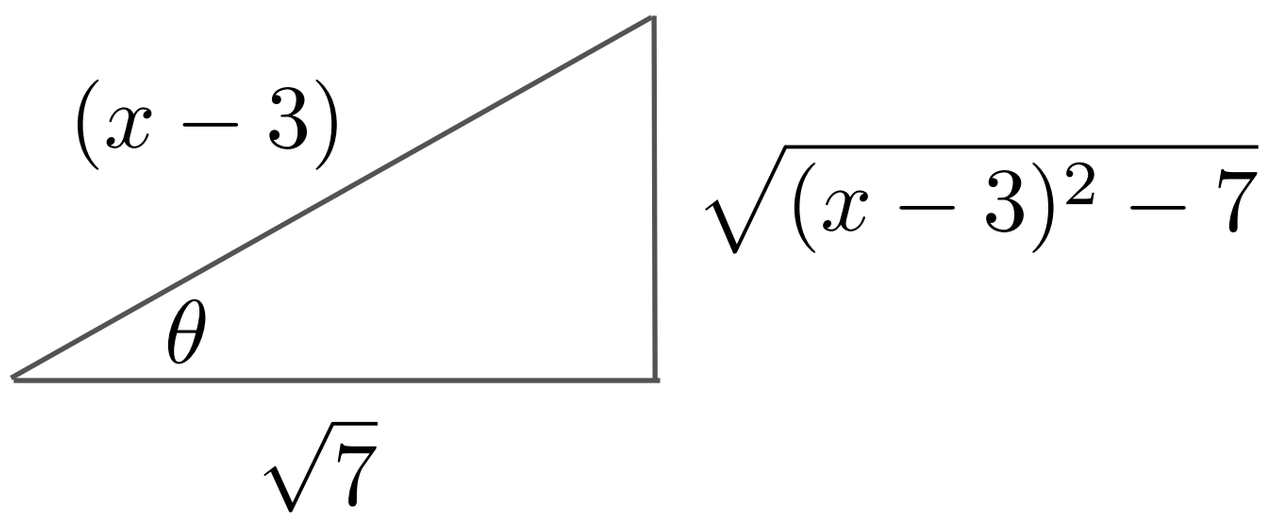

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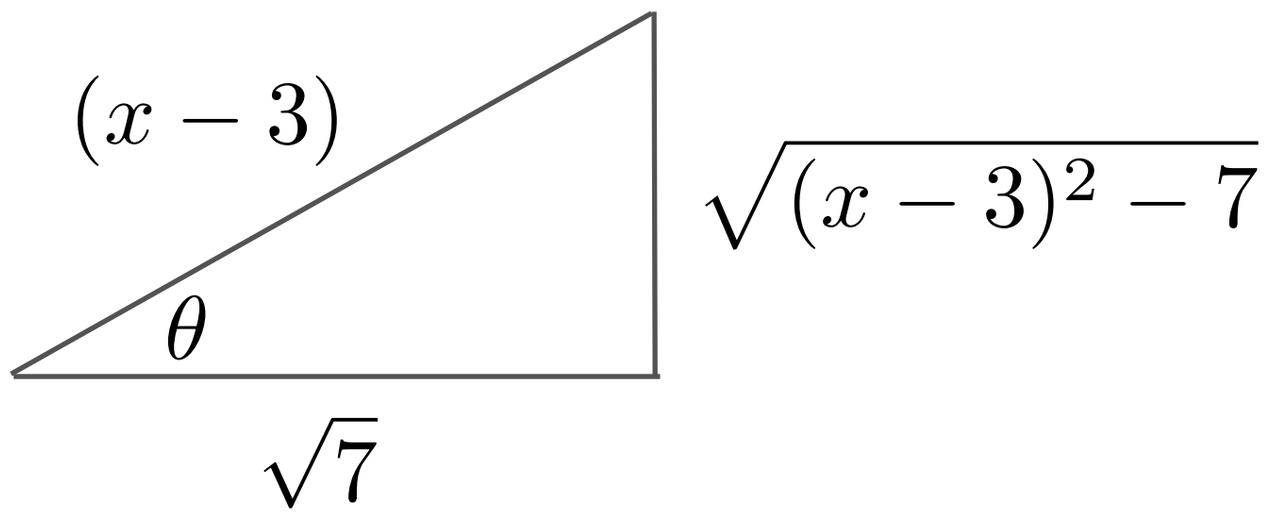

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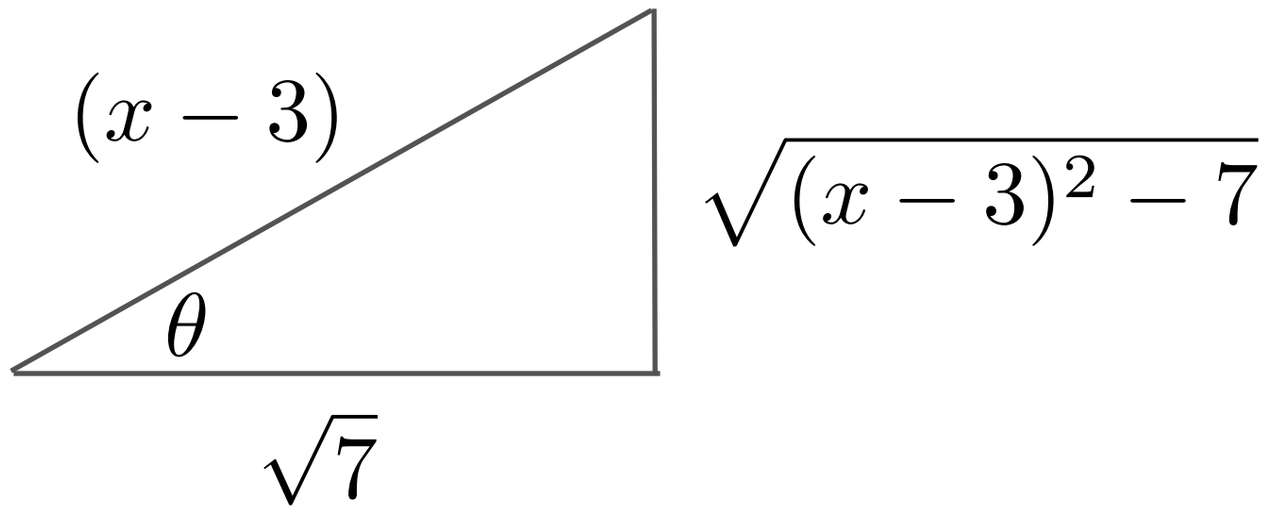

Faites les exercices suivants

Section 2 # 17, 18

Exemple

$$\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$$

Exemple

$$\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$$

$$= \int \frac{\sin x}{1 - \cos^2 x + \cos x}\, dx$$

Exemple

$$\int \frac{\sin x}{\sin^2 x + \cos x}\, dx \qquad u = \cos x$$

$$= \int \frac{\sin x}{1 - \cos^2 x + \cos x}\, dx$$

Exemple

$$\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$$

$$u = \cos x$$

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Exemple

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Exemple

$$\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$$

$$u = \cos x$$

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$$= \int \frac{\sin x}{1 - \cos^2 x + \cos x}\, dx$$

$$= -\int \frac{1}{1 - u^2 + u}\, du$$

$$= \int \frac{1}{u^2 - u - 1}\, du$$

Exemple

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$$u^2 - u - 1$$

$$= -\int \frac{1}{1 - u^2 + u}\, du$$

$$= u^2 - u + \frac{1}{4} - \frac{1}{4} - 1$$

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Exemple

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Exemple

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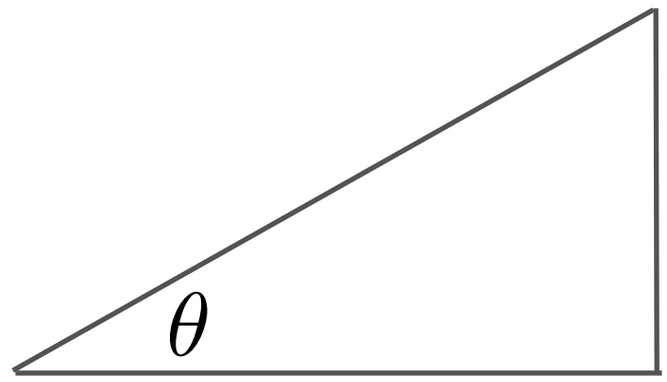

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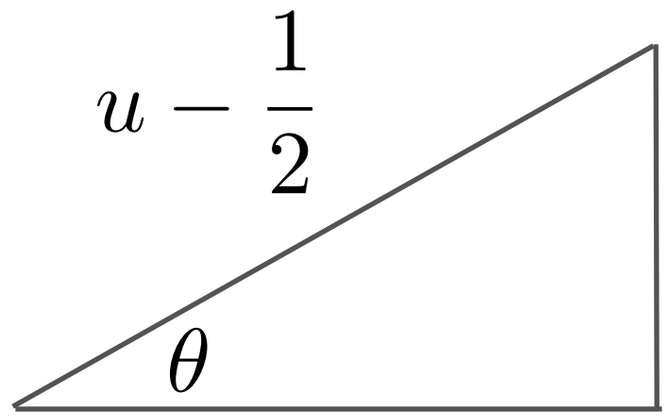

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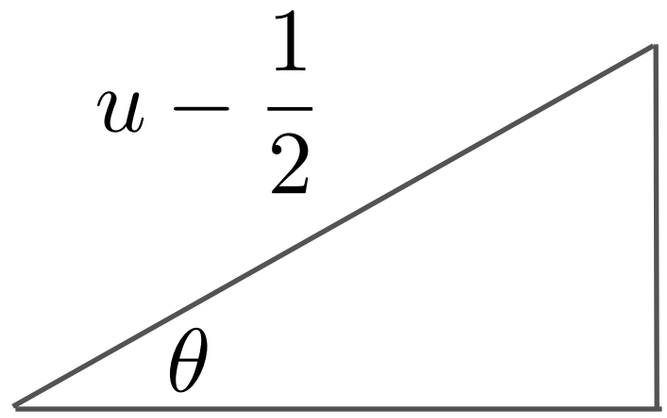

$$\sqrt{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}$$

Exemple $\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$

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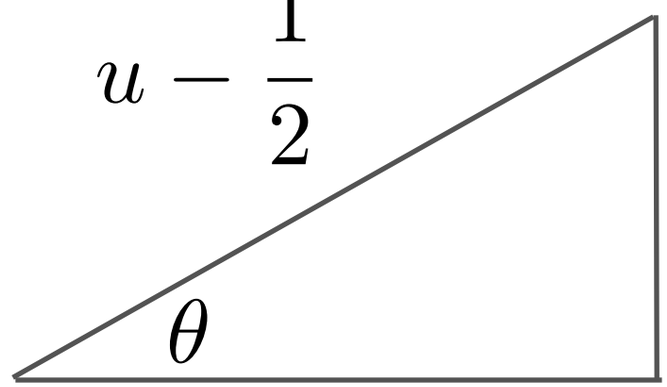

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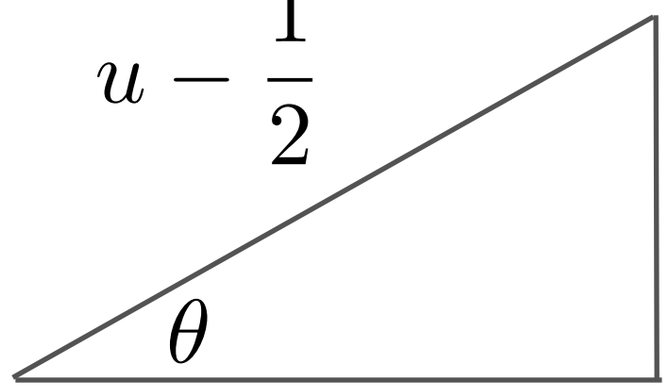

$$\sqrt{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}$$

$$\frac{u - \frac{1}{2}}{\frac{\sqrt{5}}{2}} = \sec\theta$$

Exemple

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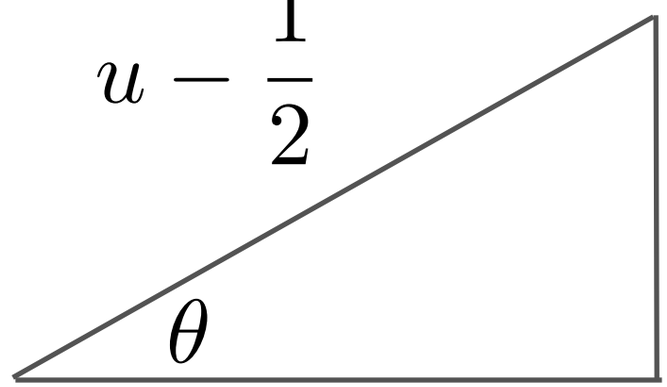

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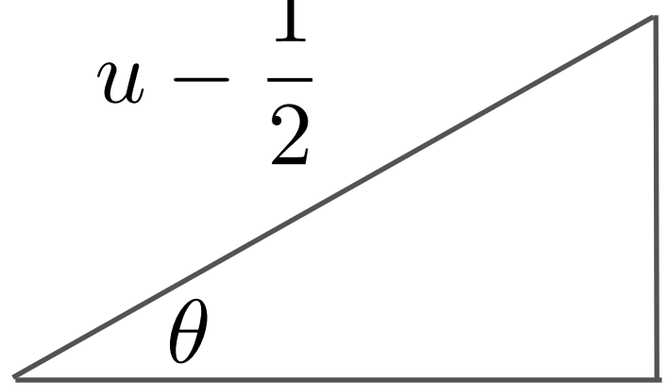

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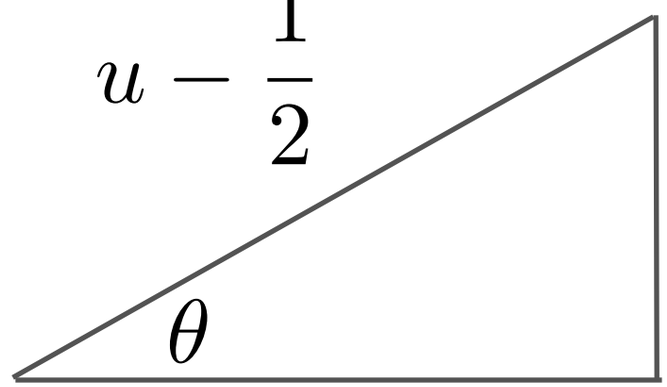

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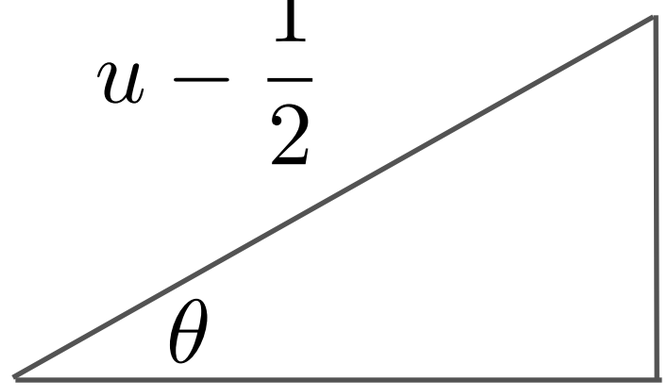

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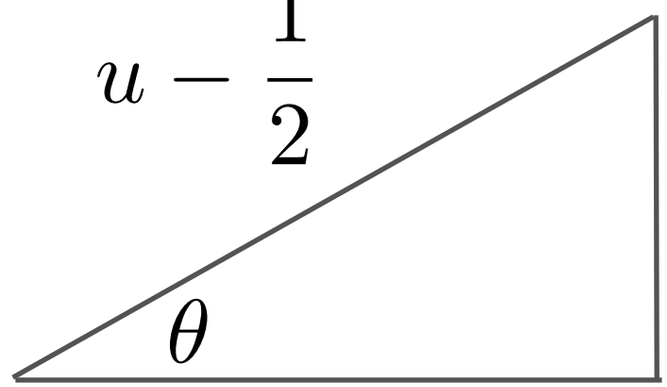

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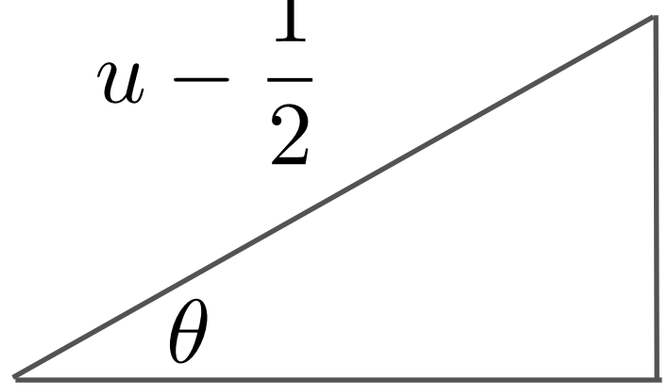

$$\sqrt{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}$$

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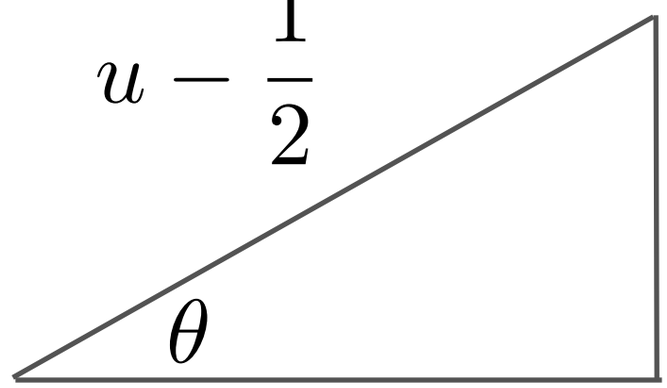

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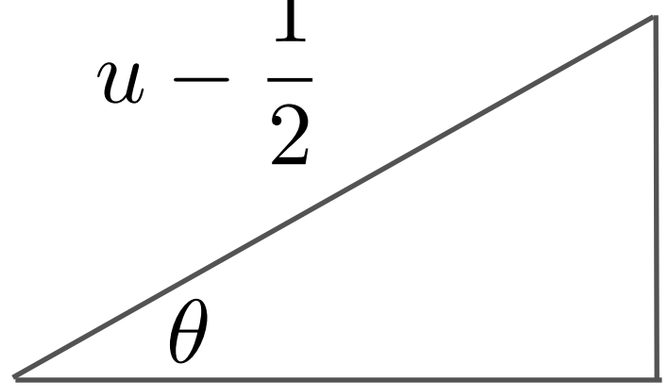

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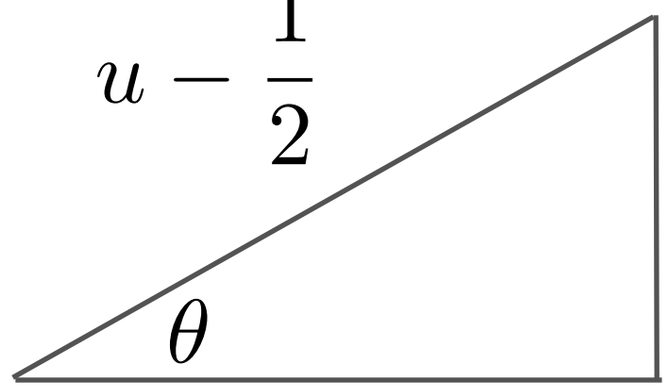

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$$\sqrt{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}} = \frac{\sqrt{5}}{2}\tan\theta \qquad \left(u - \frac{1}{2}\right)^2 - \frac{5}{4} = \frac{5}{4}\tan^2\theta$$

Exemple

$$\int \frac{\sin x}{\sin^2 x + \cos x}\, dx$$

$$u = \cos x$$
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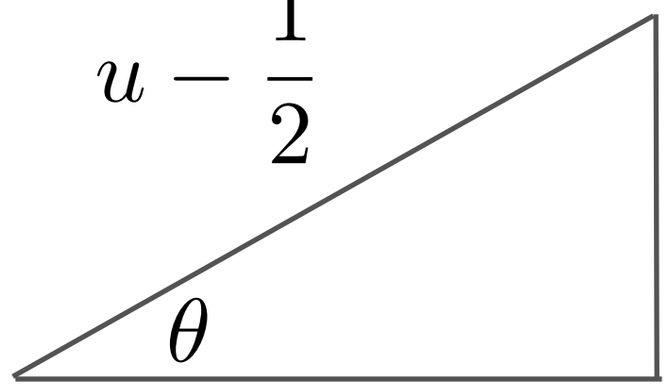

$$u = \frac{\sqrt{5}}{2}\sec\theta + \frac{1}{2}$$

$$du = \frac{\sqrt{5}}{2}\sec\theta\tan\theta\ d\theta$$

$$\left(u - \frac{1}{2}\right)^2 - \frac{5}{4} = \frac{5}{4}\tan^2\theta$$

Exemple

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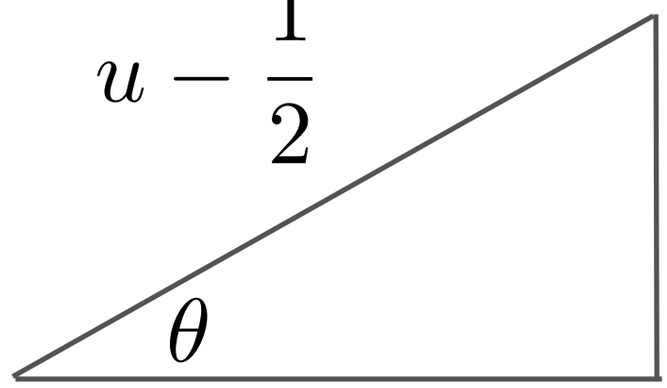

$$= \int \frac{\frac{\sqrt{5}}{2}\sec\theta\tan\theta}{\frac{5}{4}\tan^2\theta}\, d\theta$$

$$u = \frac{\sqrt{5}}{2}\sec\theta + \frac{1}{2}$$

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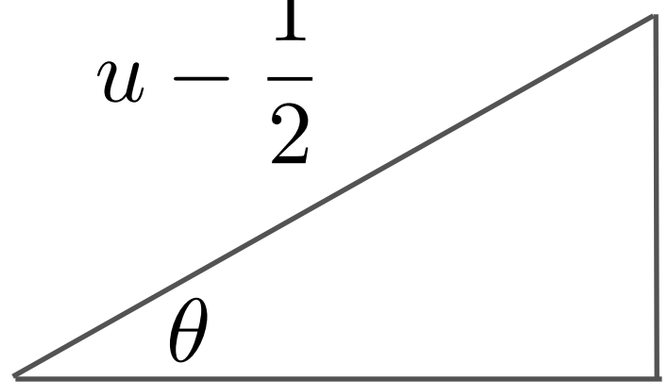

$$\sqrt{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}$$

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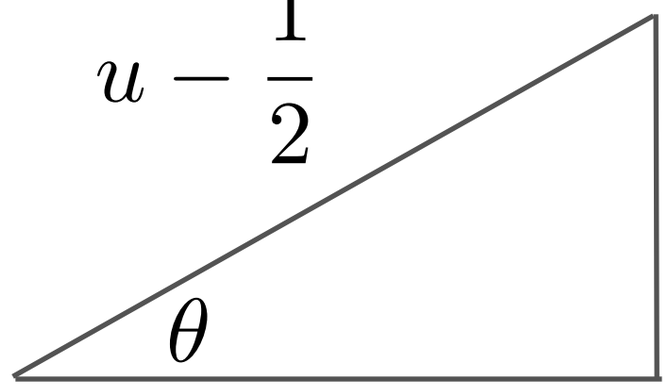

$$\sqrt{\left(u - \frac{1}{2}\right)^2 - \frac{5}{4}}$$

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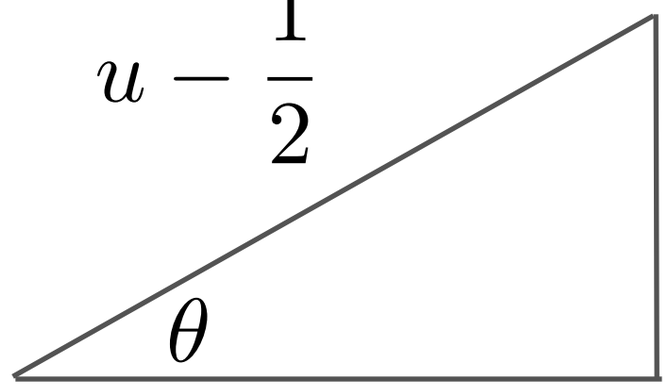

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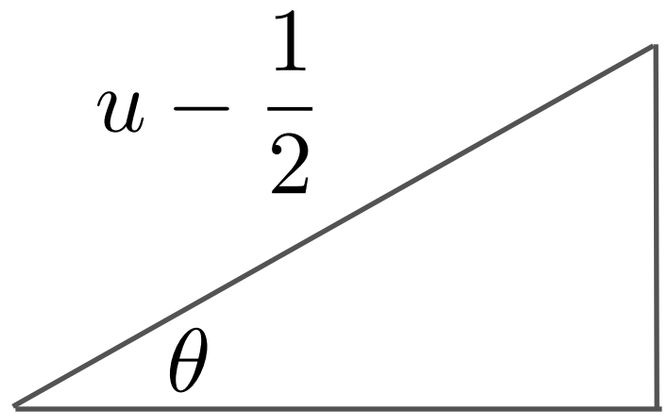

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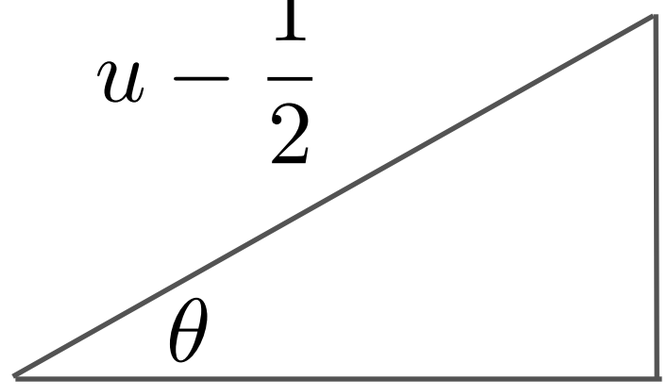

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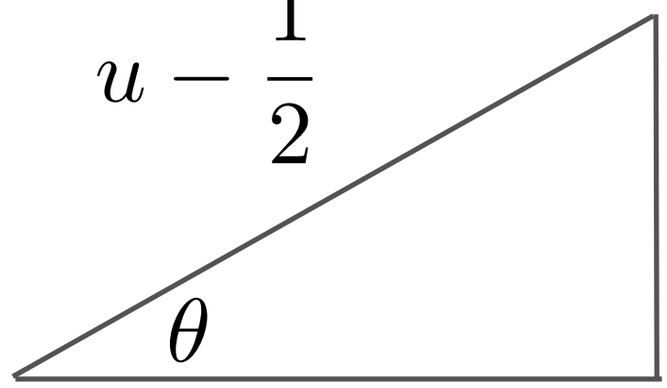

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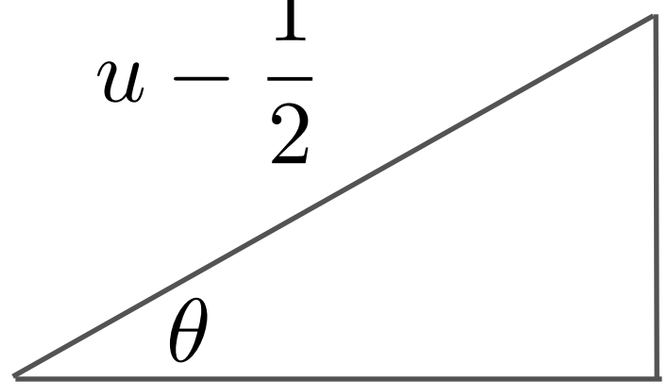

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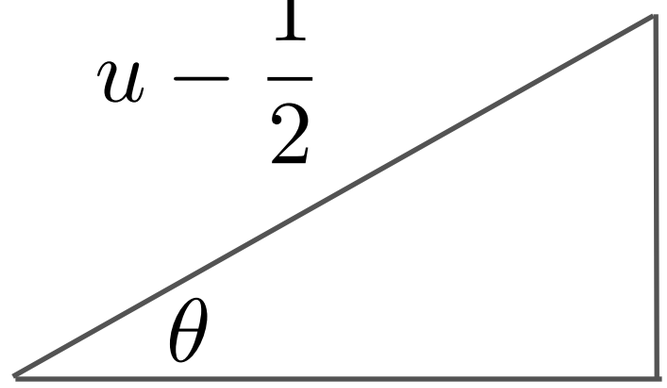

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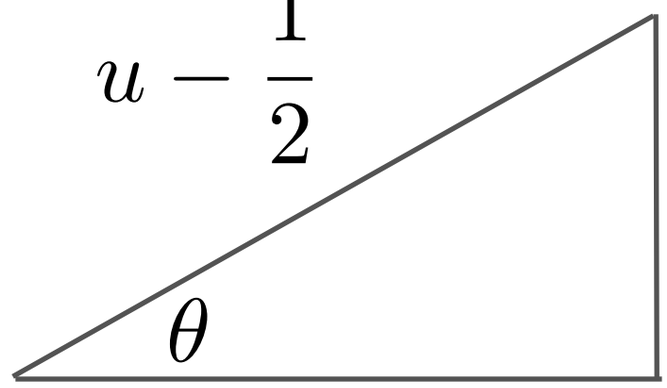

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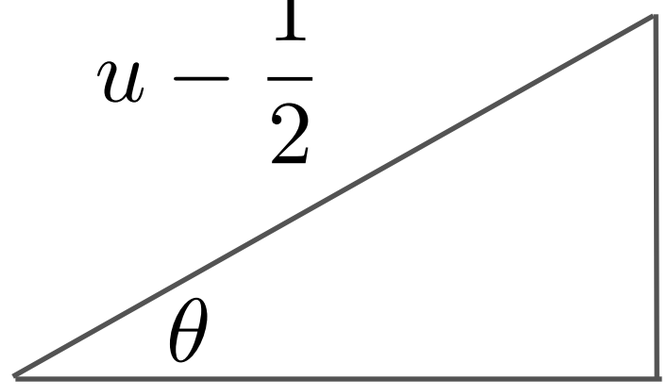

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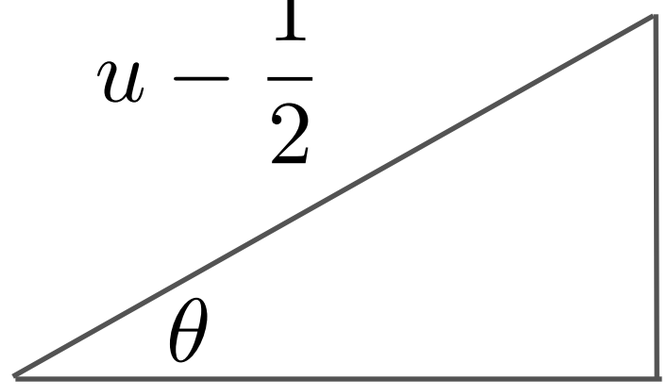

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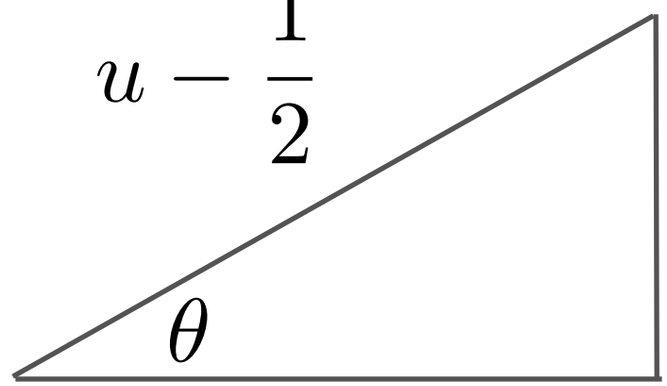

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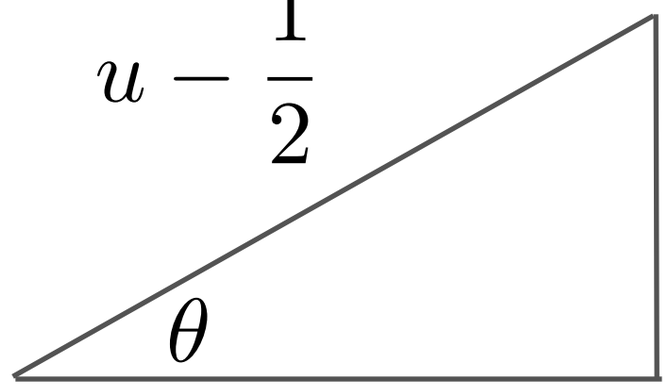

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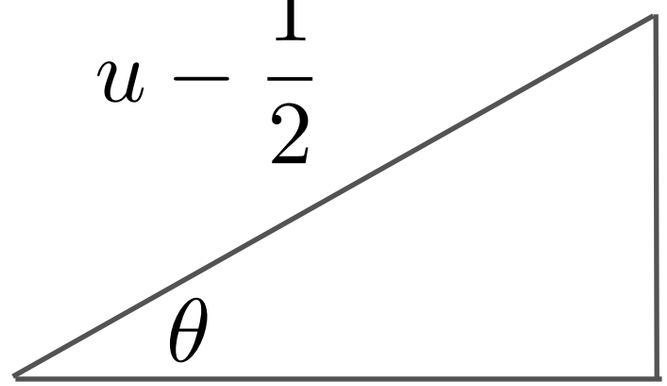

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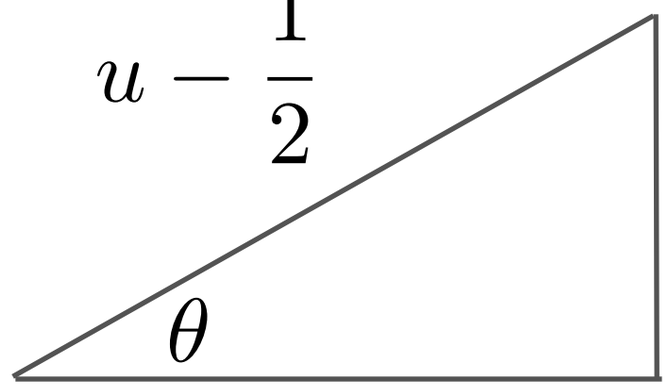

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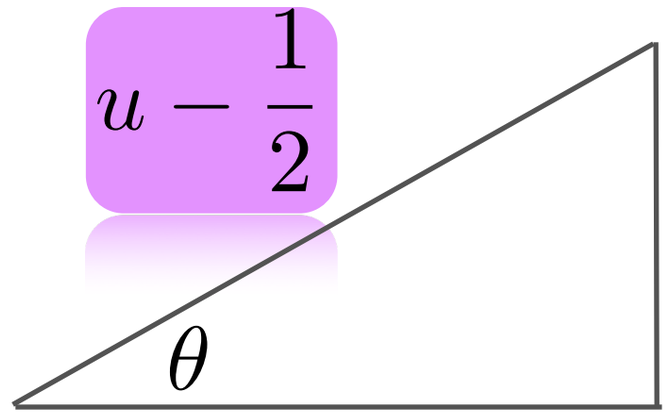

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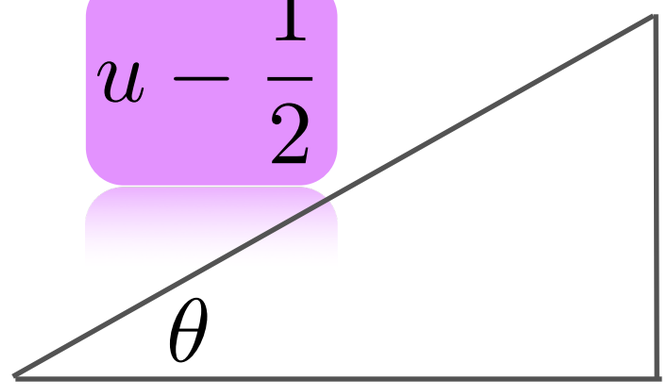

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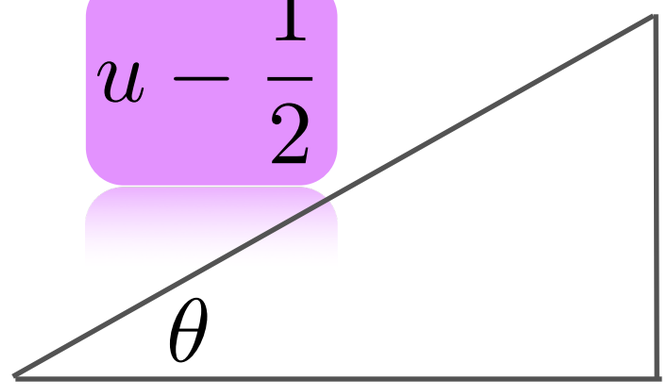

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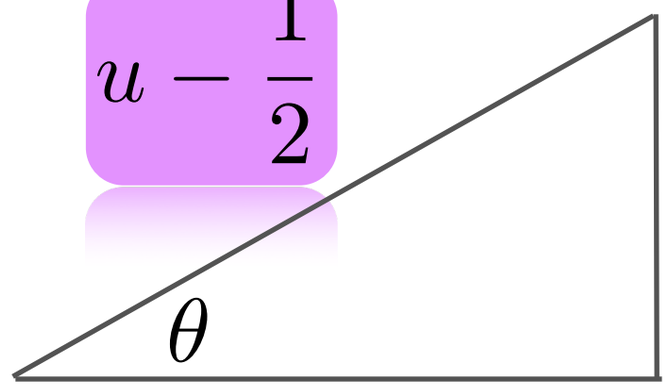

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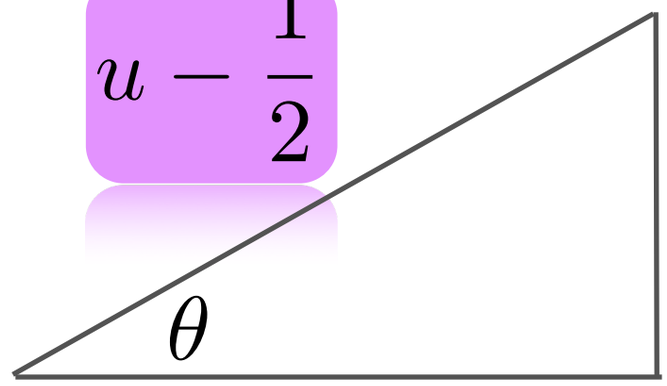

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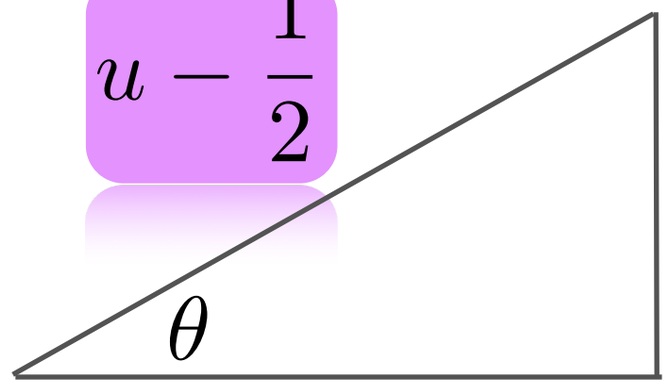

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Faites les exercices suivants

Section 2 # 17, 18

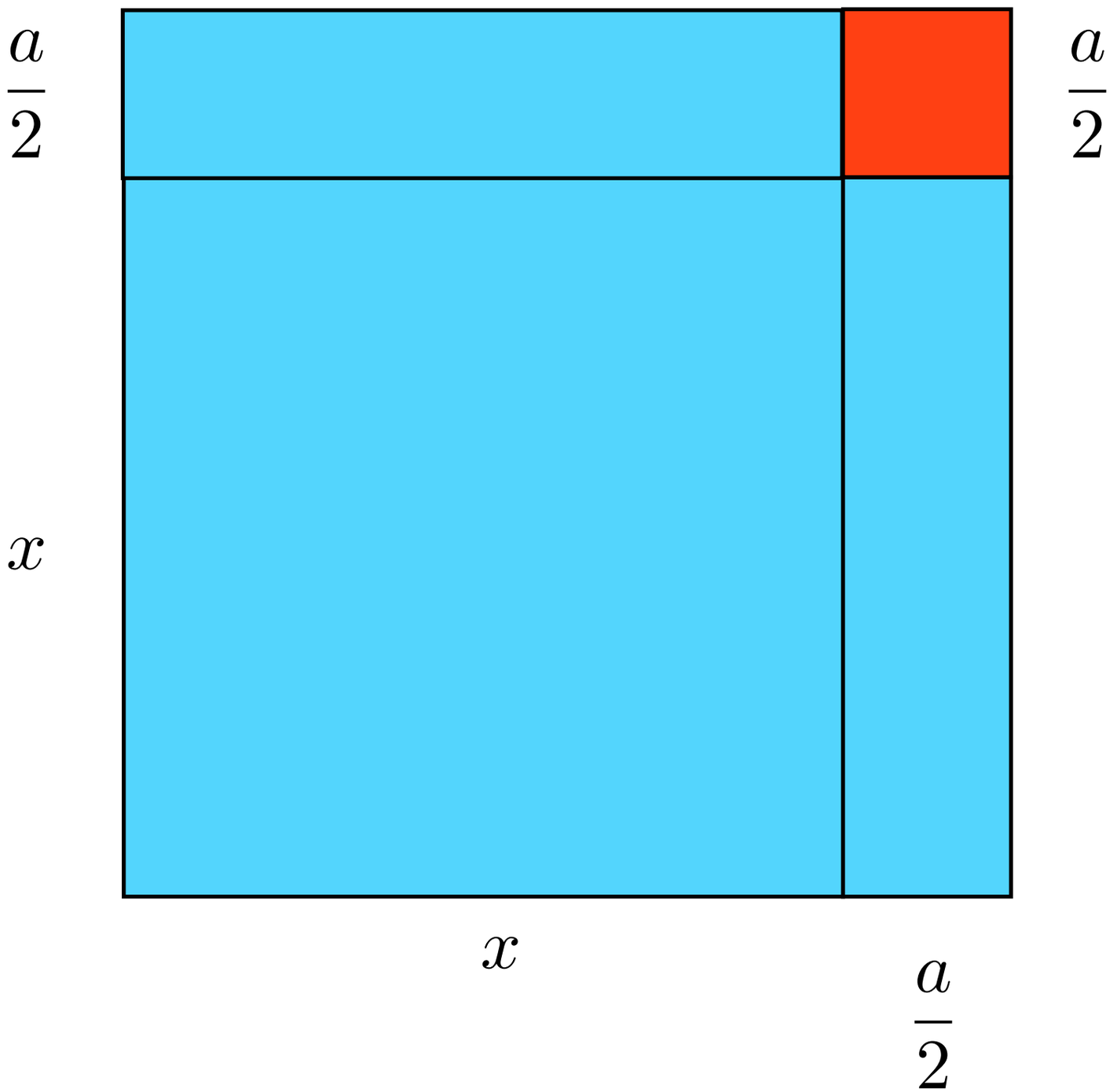
Aujourd'hui, nous avons vu

Aujourd'hui, nous avons vu

- ✓ Complétion de carré

Aujourd'hui, nous avons vu

- ✓ Complétion de carré



Devoir: Section 2.4