

3.2 VOLUME DE RÉVOLUTION (TUBES)

(LOBES)

cours 17

Aujourd'hui, nous allons voir

- ✓ Calcul de solide de révolution à l'aide de la méthode des tubes

Au dernier cours on a vu que si

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on tourne autour de l'axe des x ,

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$$\pi \int_a^b f^2(x) - g^2(x) dx$$

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et si on tourne autour de l'axe des y ,

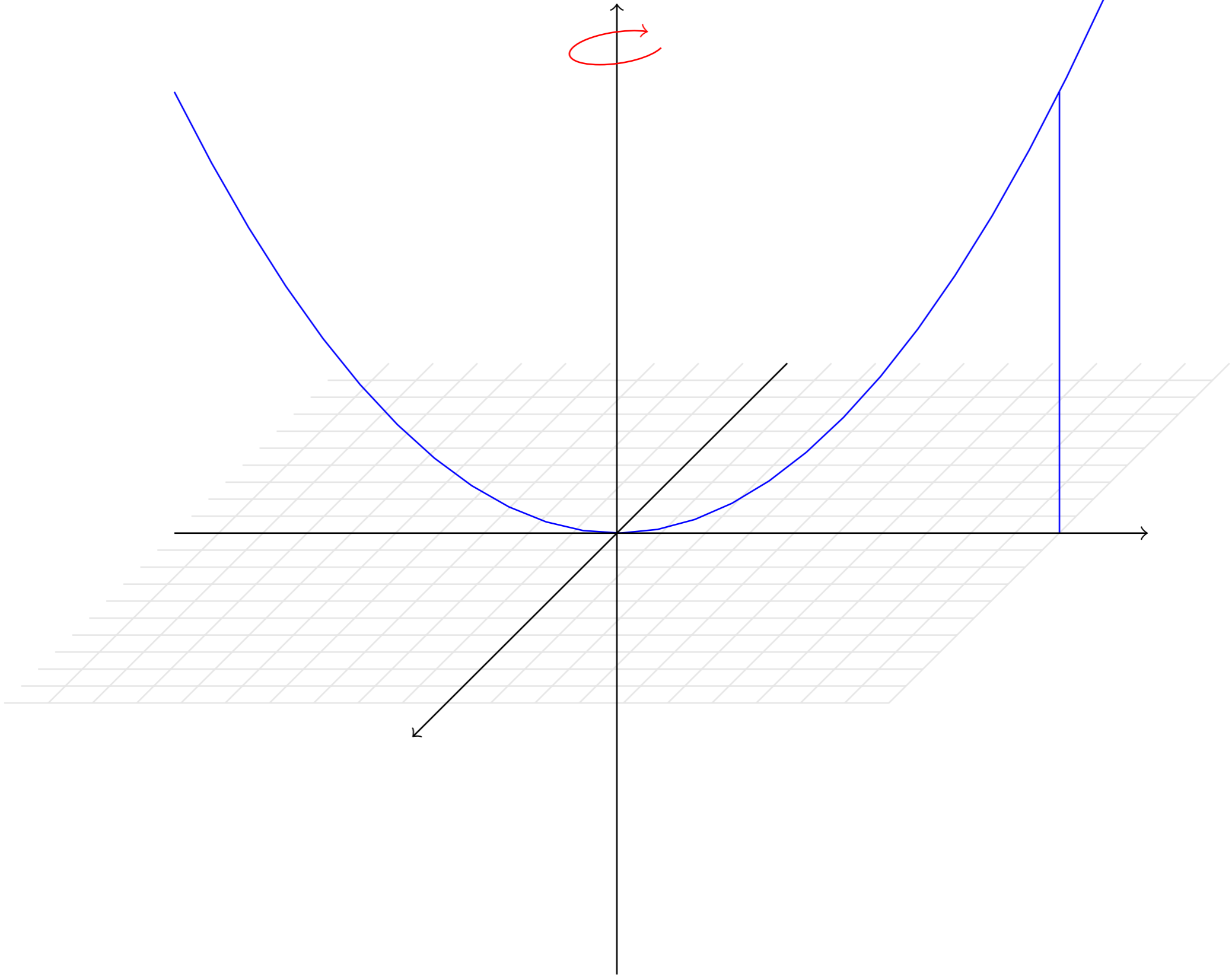
Au dernier cours on a vu que si

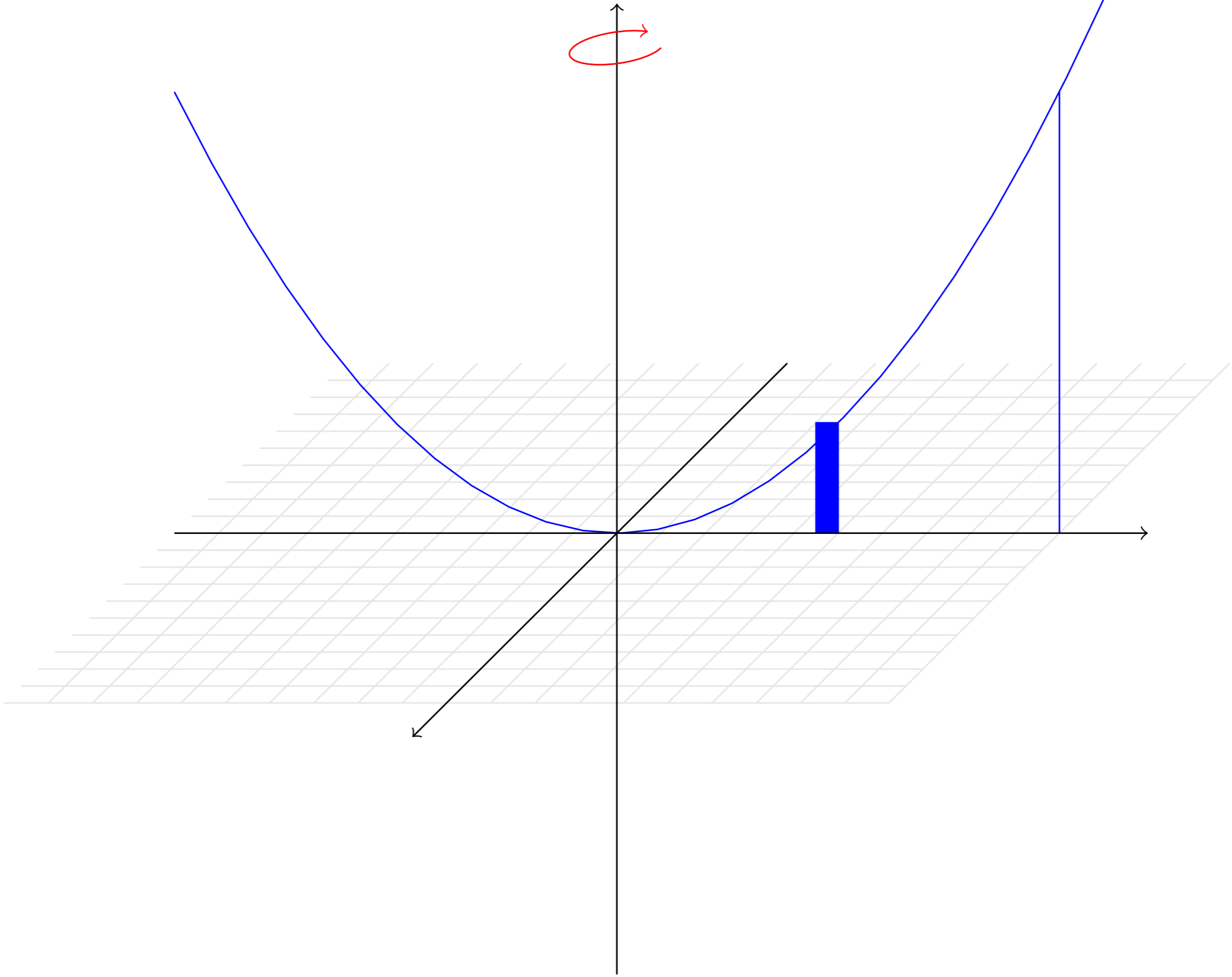
on tourne autour de l'axe des x ,

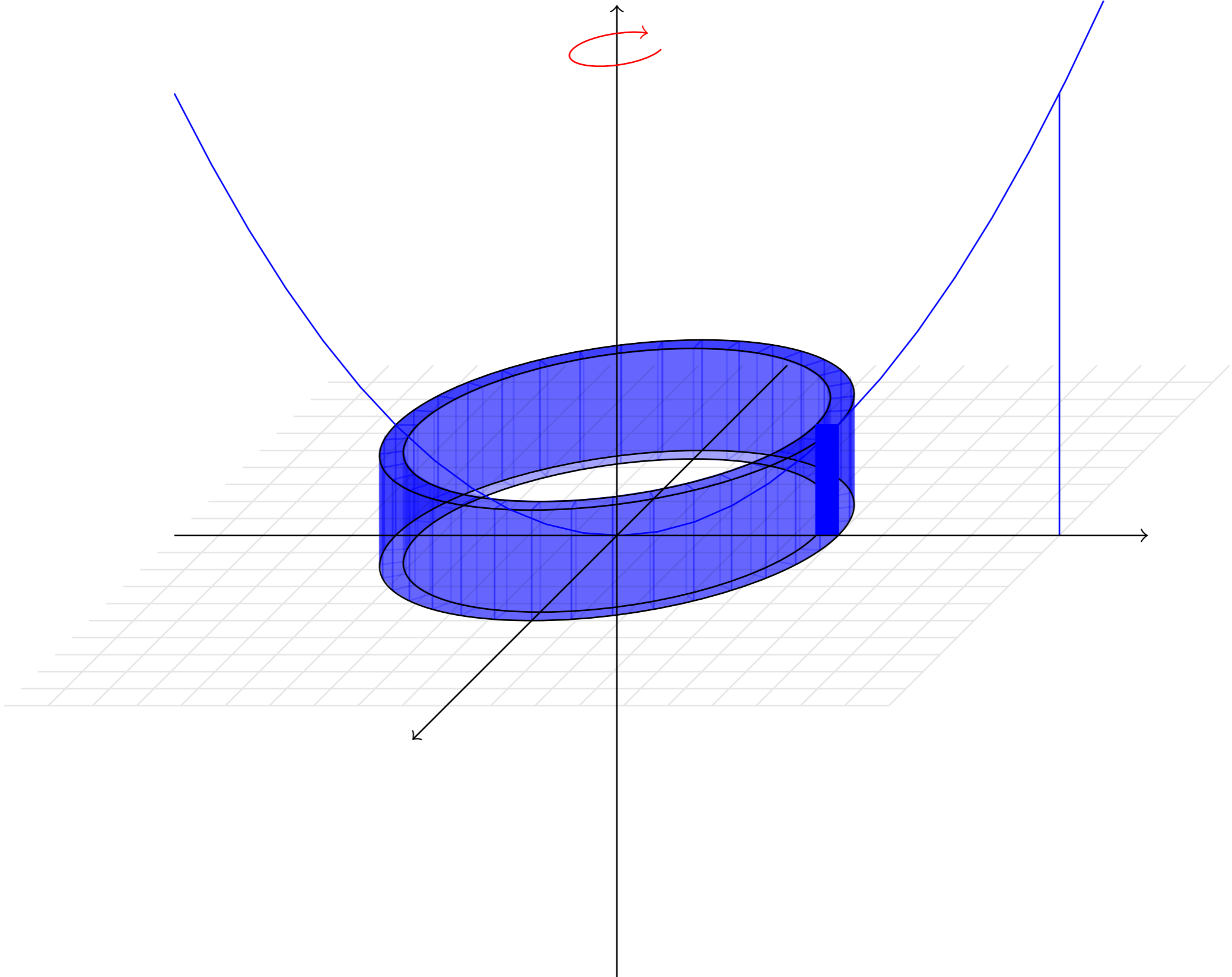
$$\pi \int_a^b f^2(x) - g^2(x) dx$$

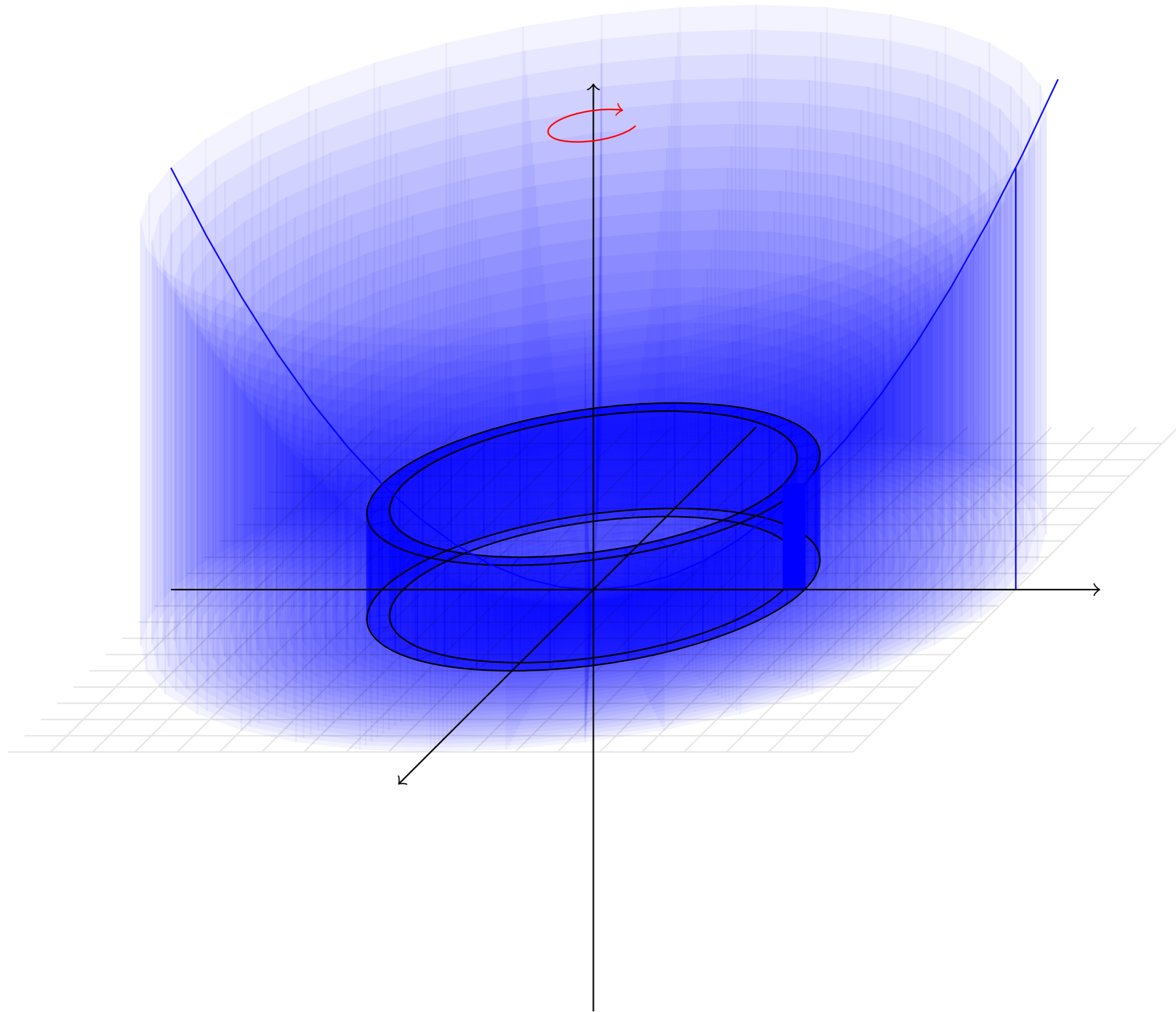
et si on tourne autour de l'axe des y ,

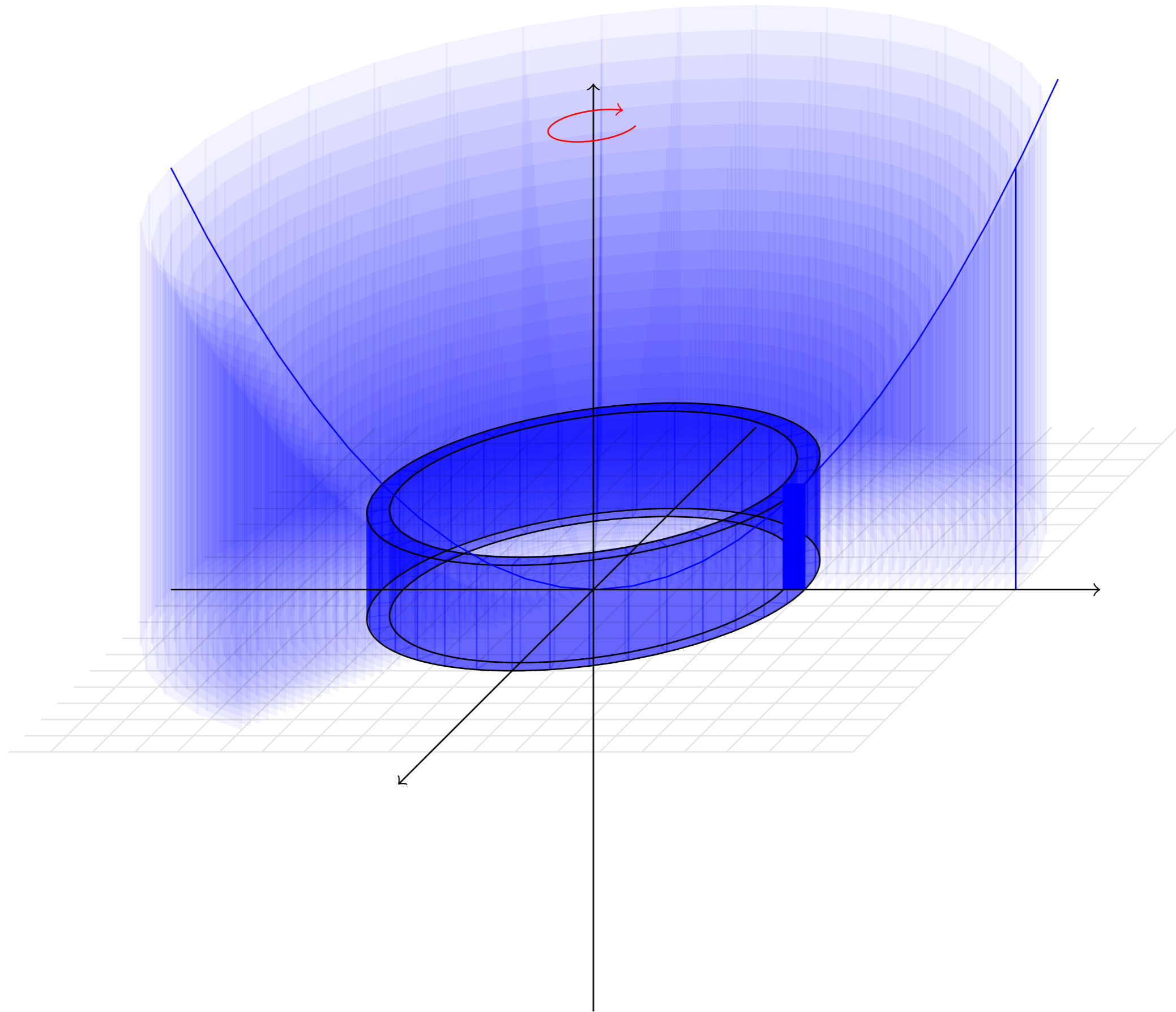
$$\pi \int_{f(a)}^{f(b)} (f^{-1}(x))^2 - (g^{-1}(y))^2 dy$$

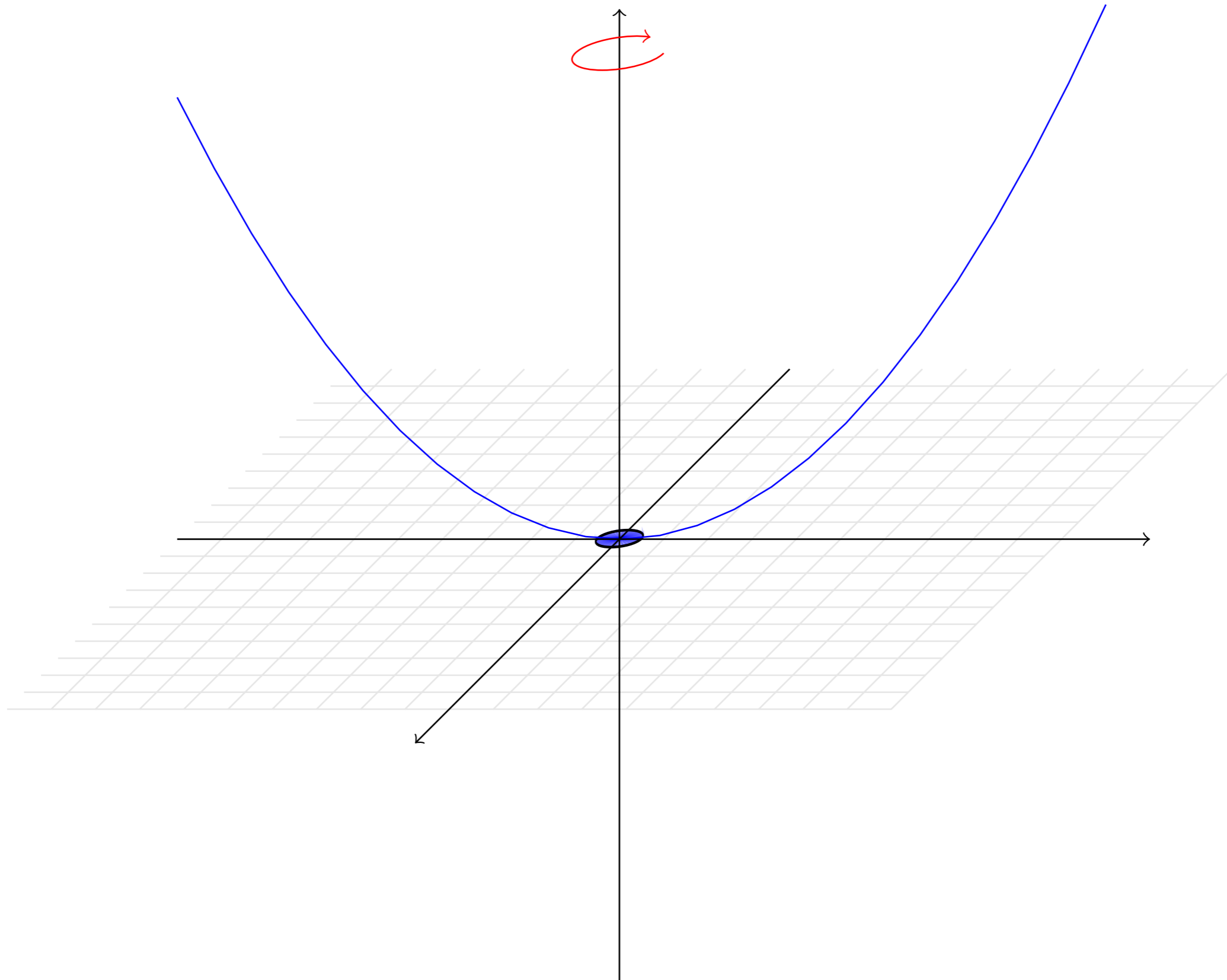


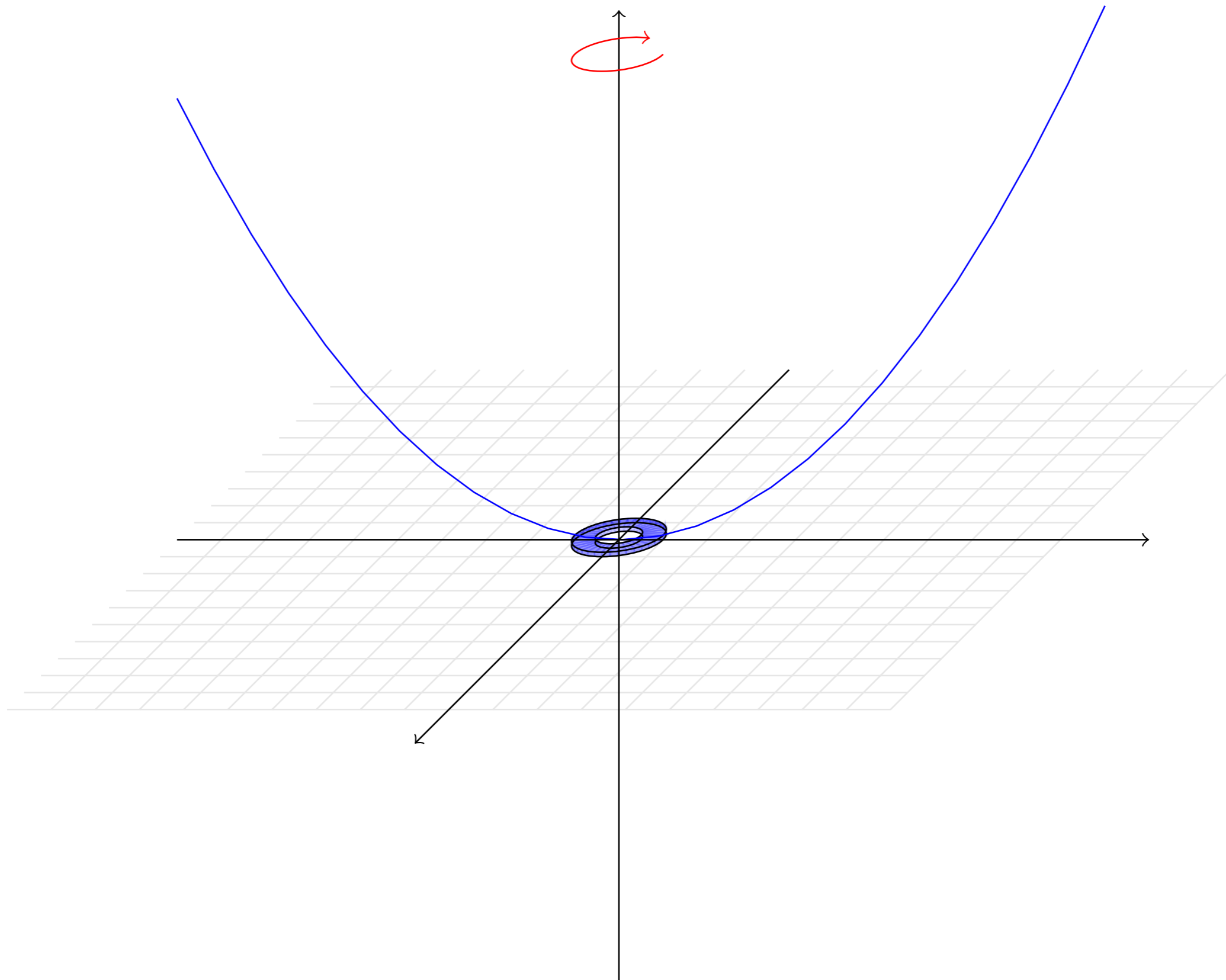


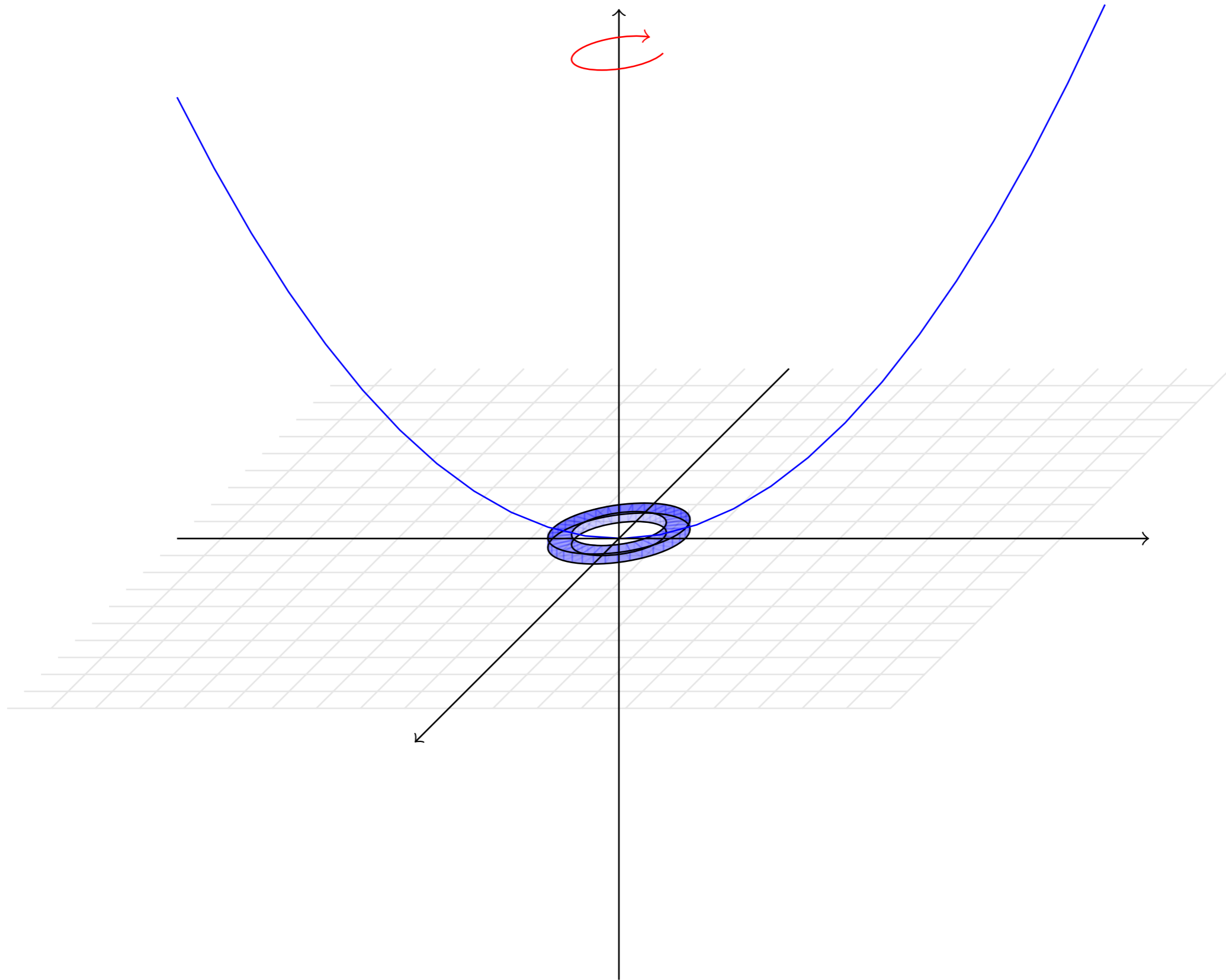


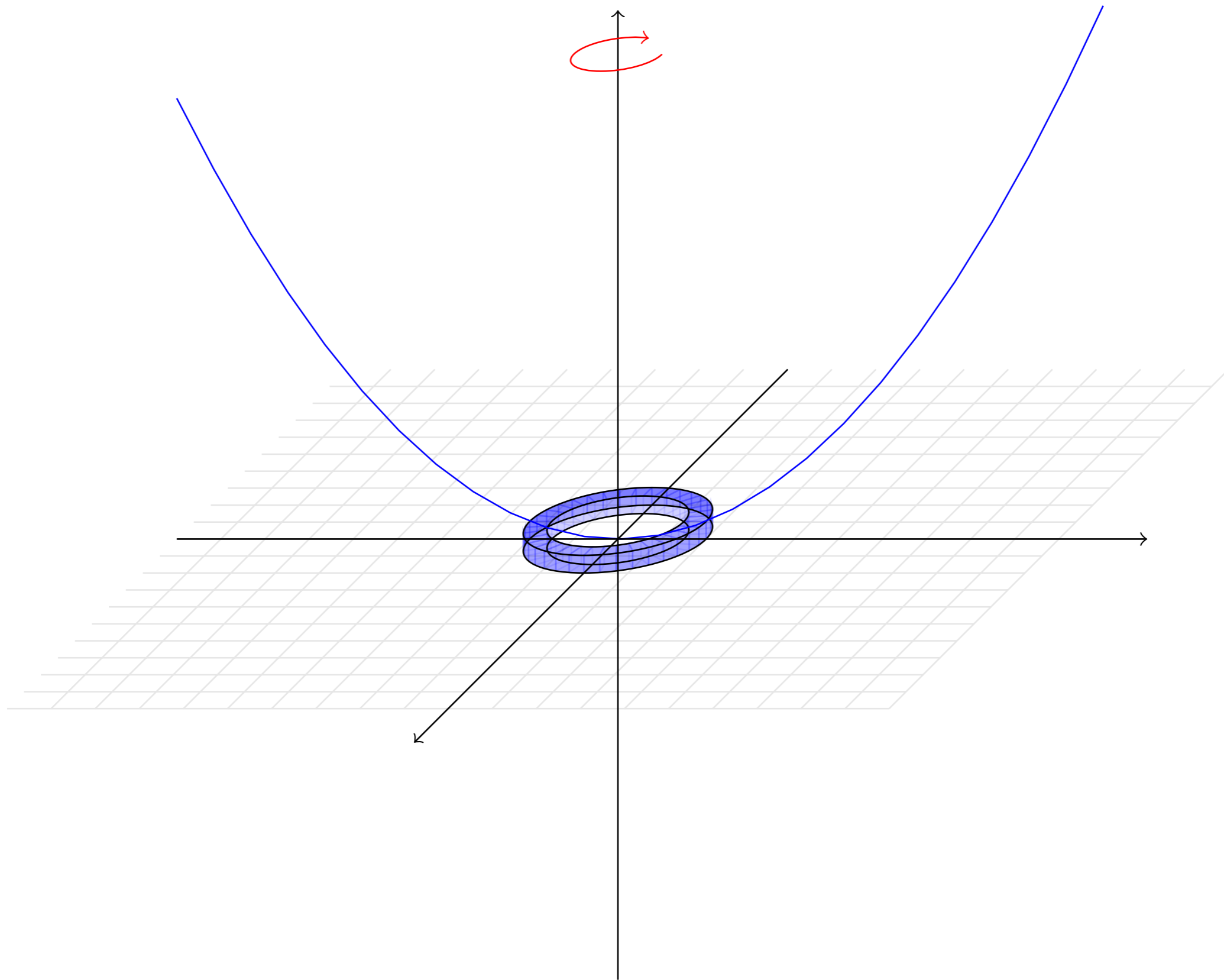


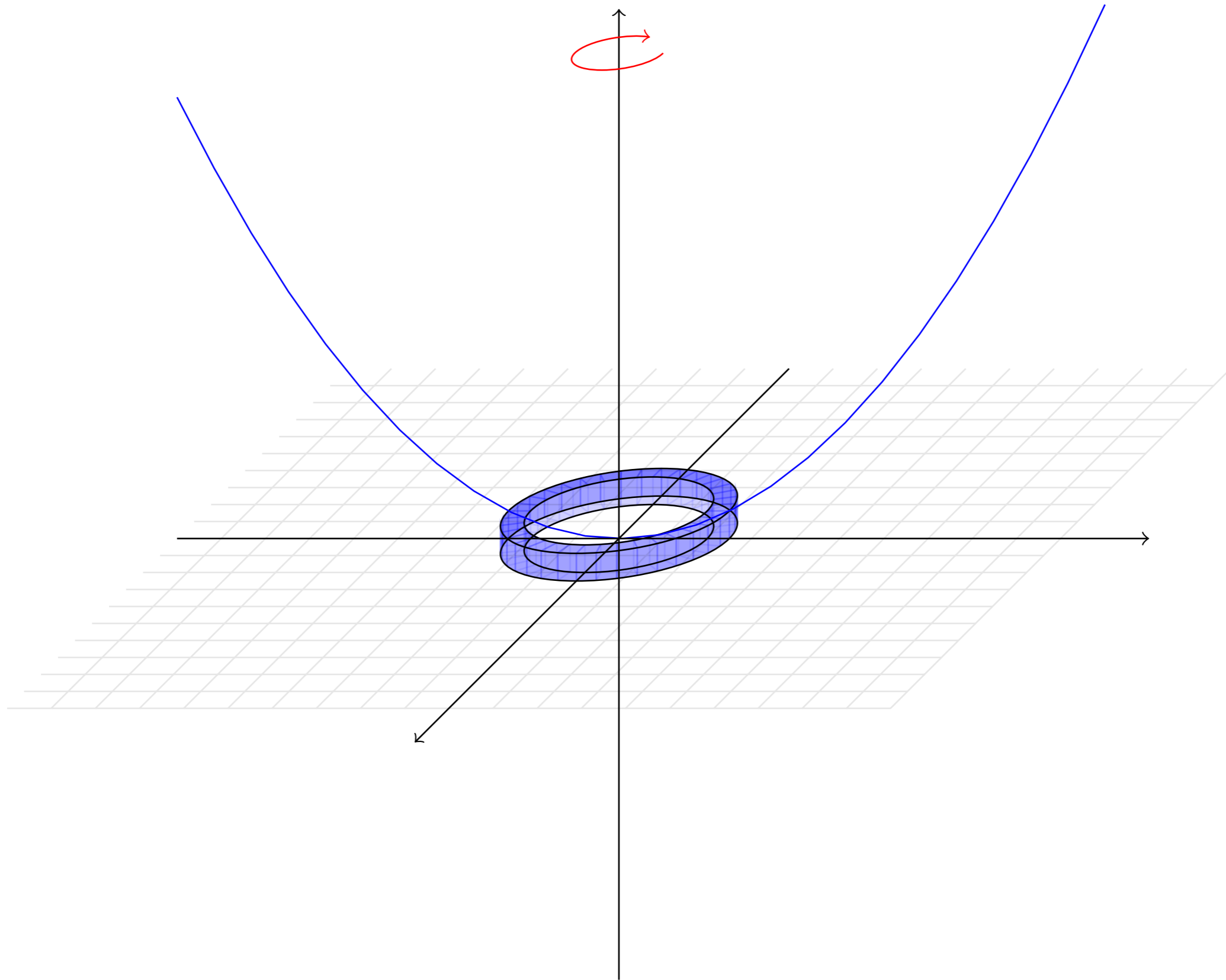


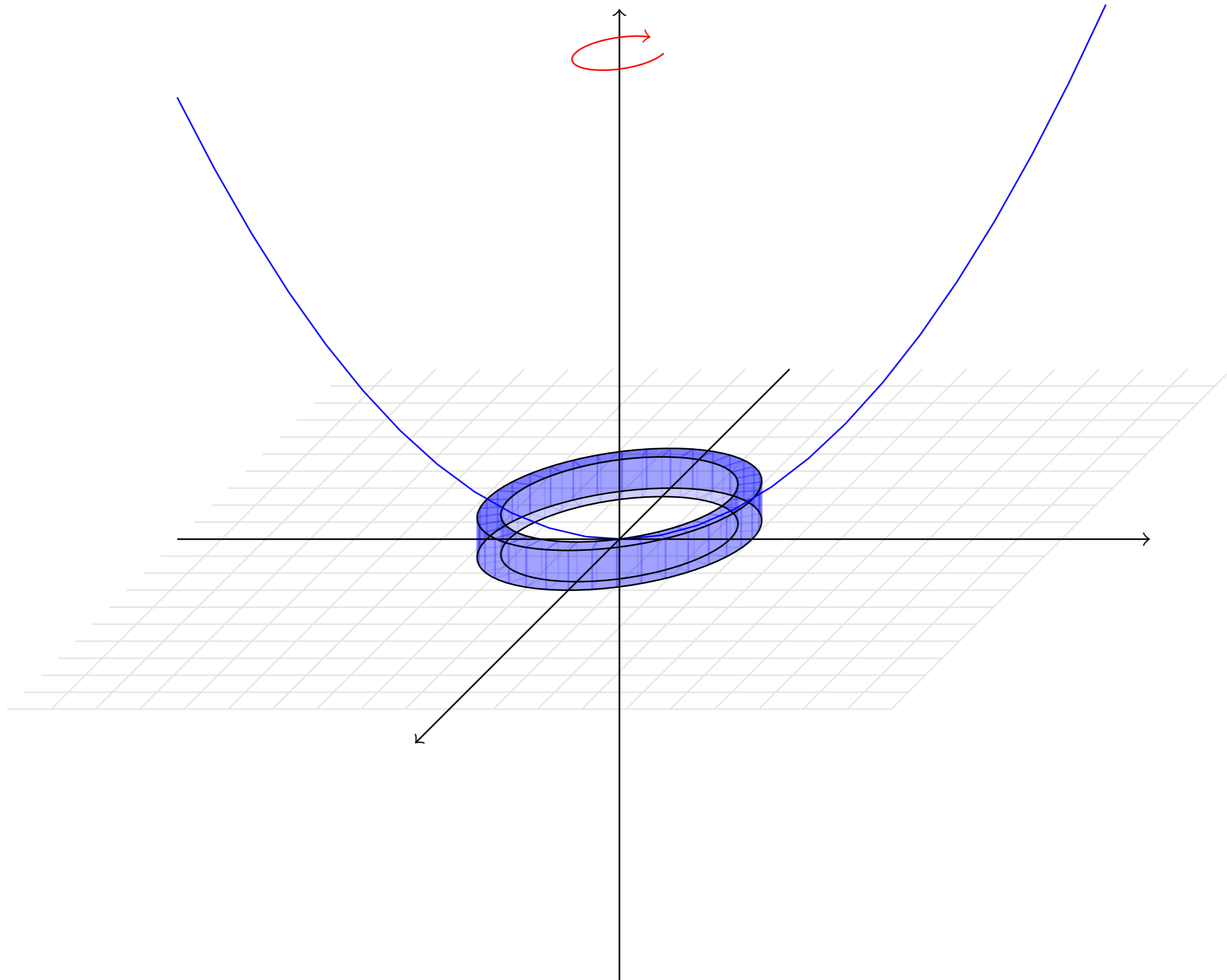


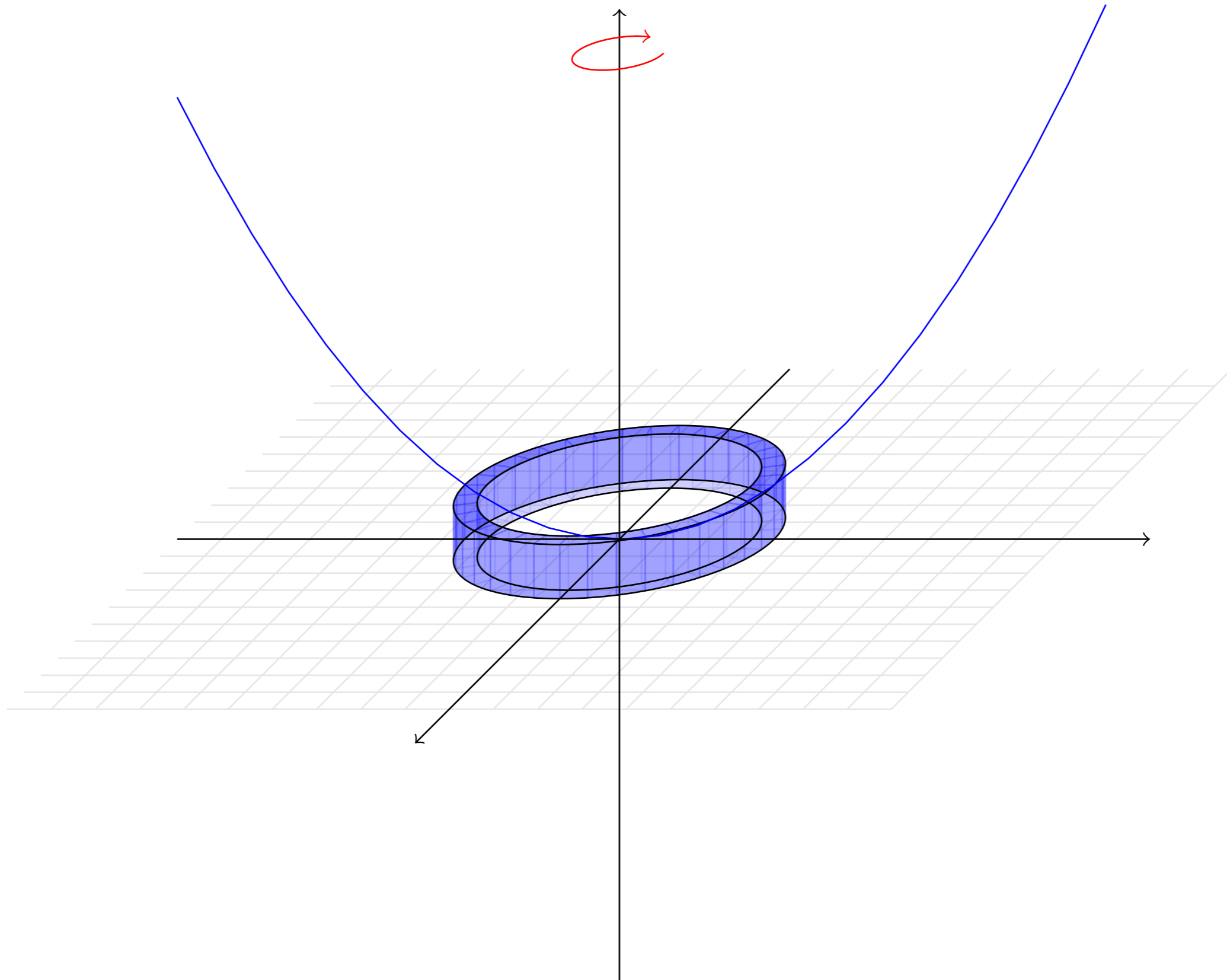


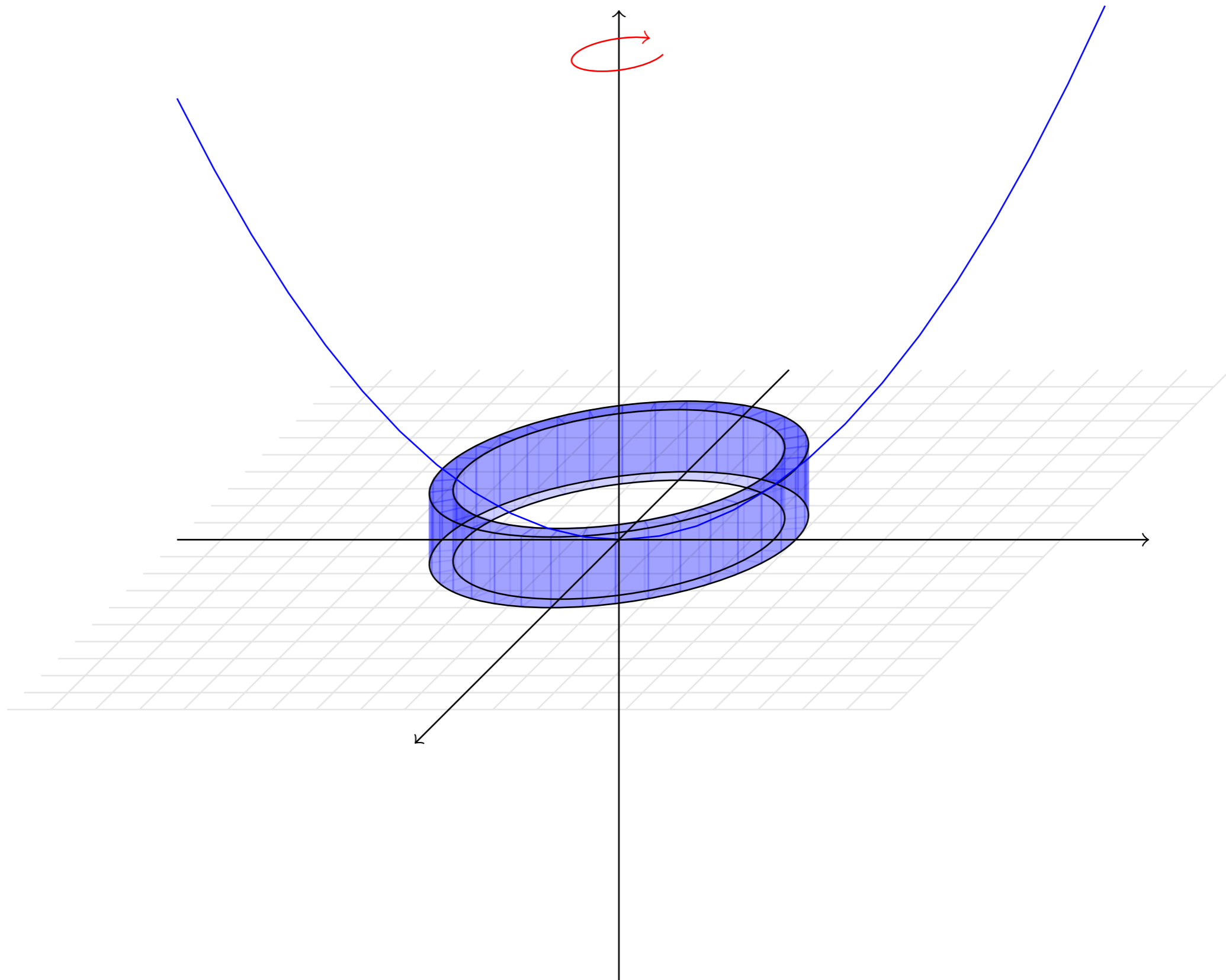


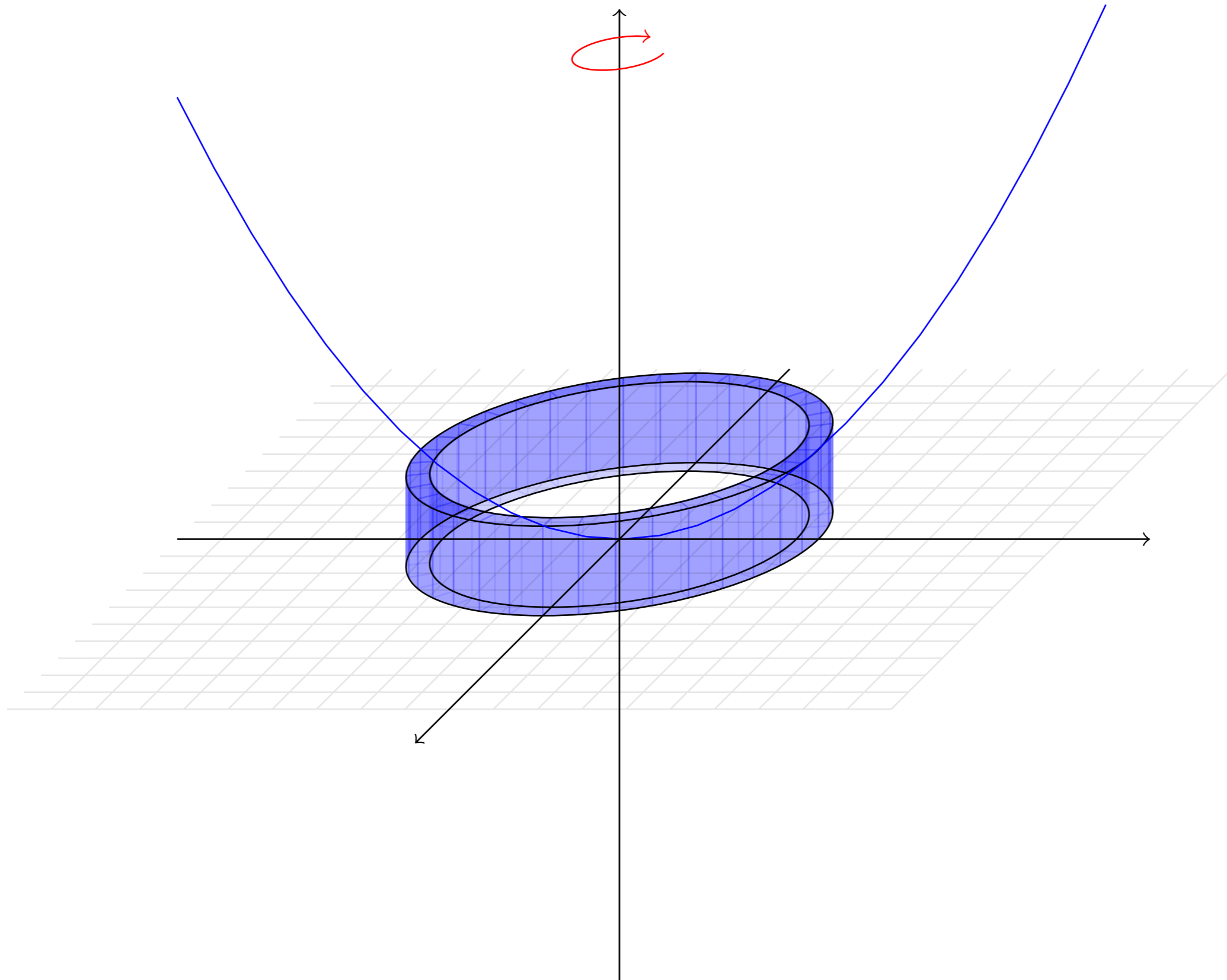


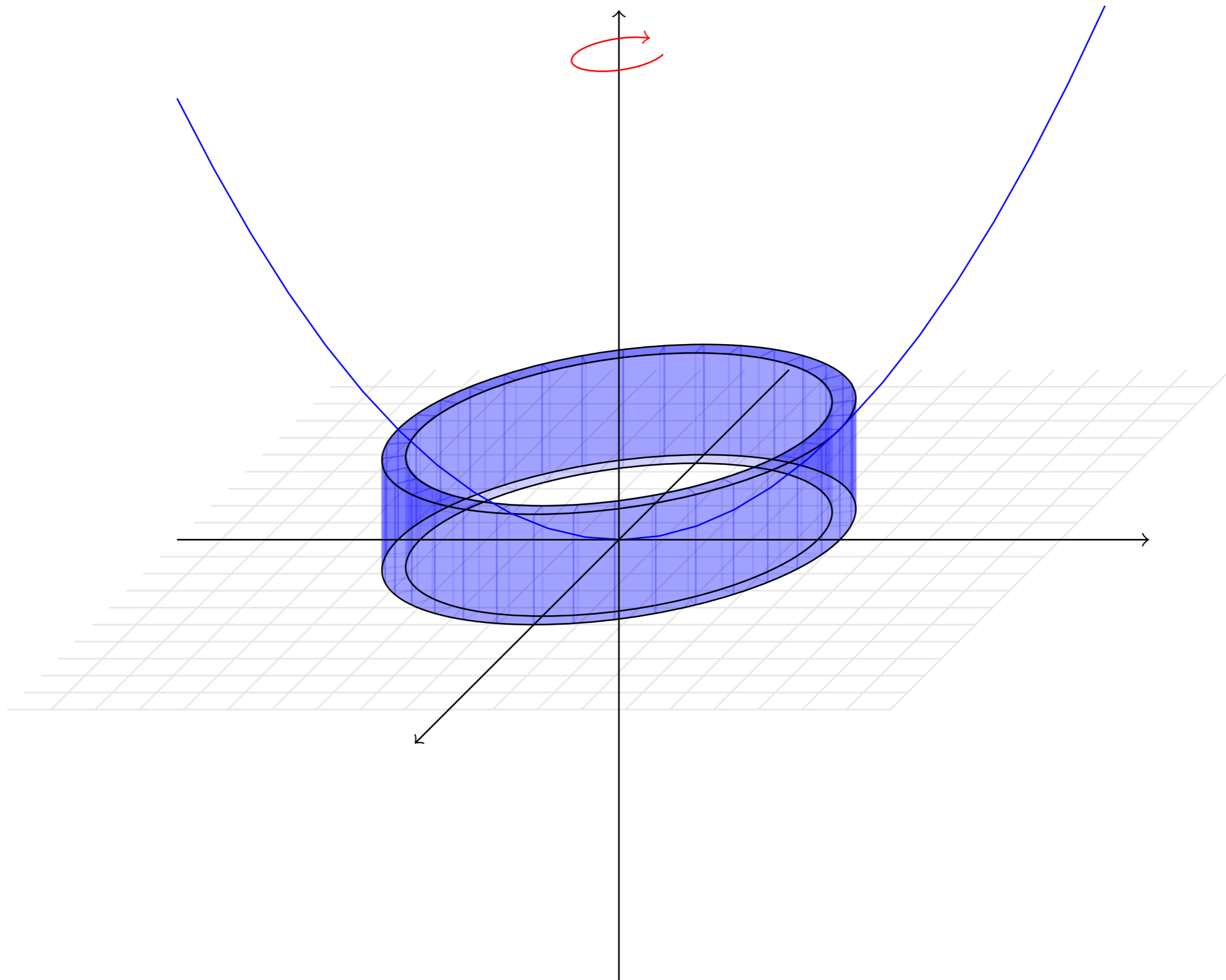


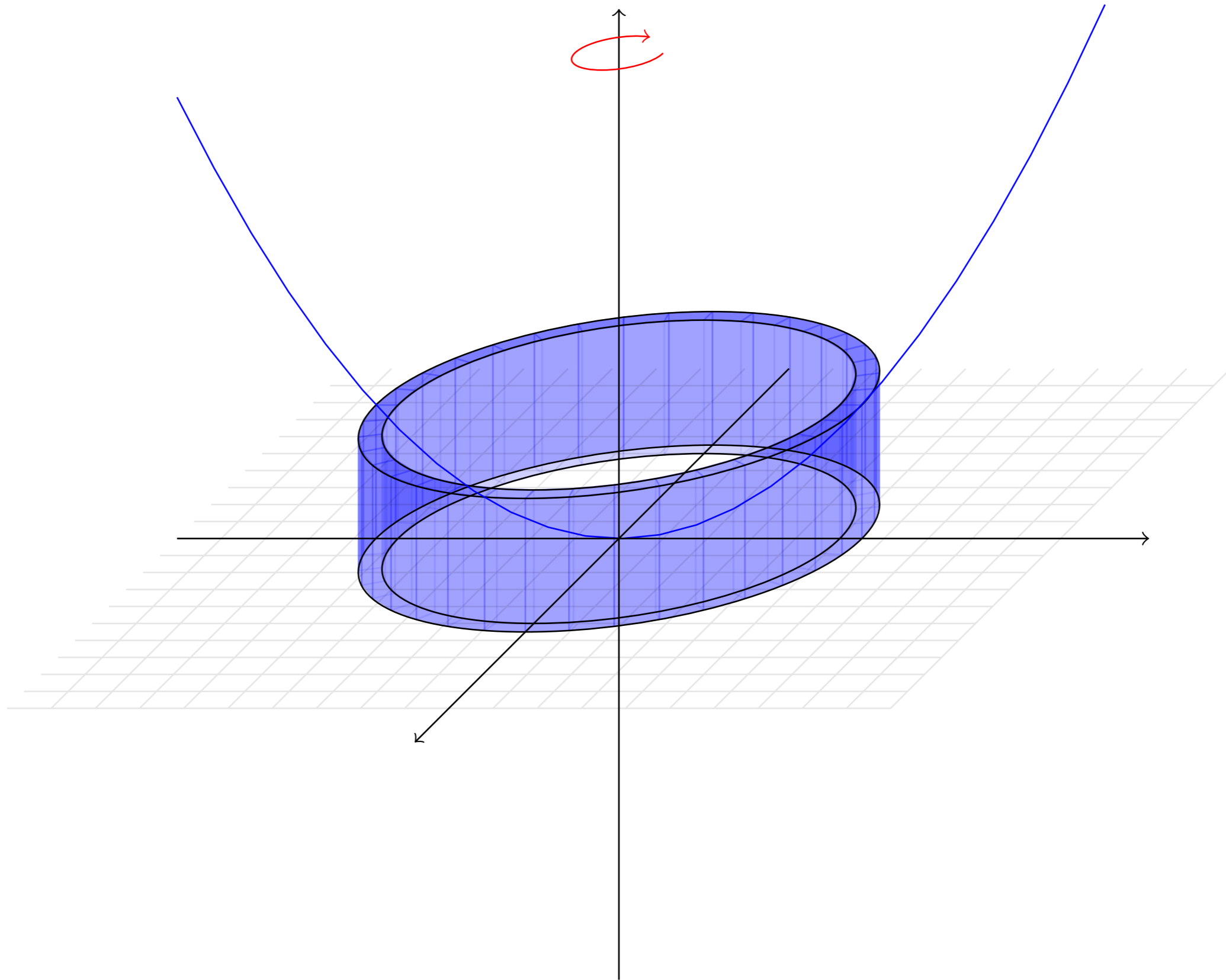


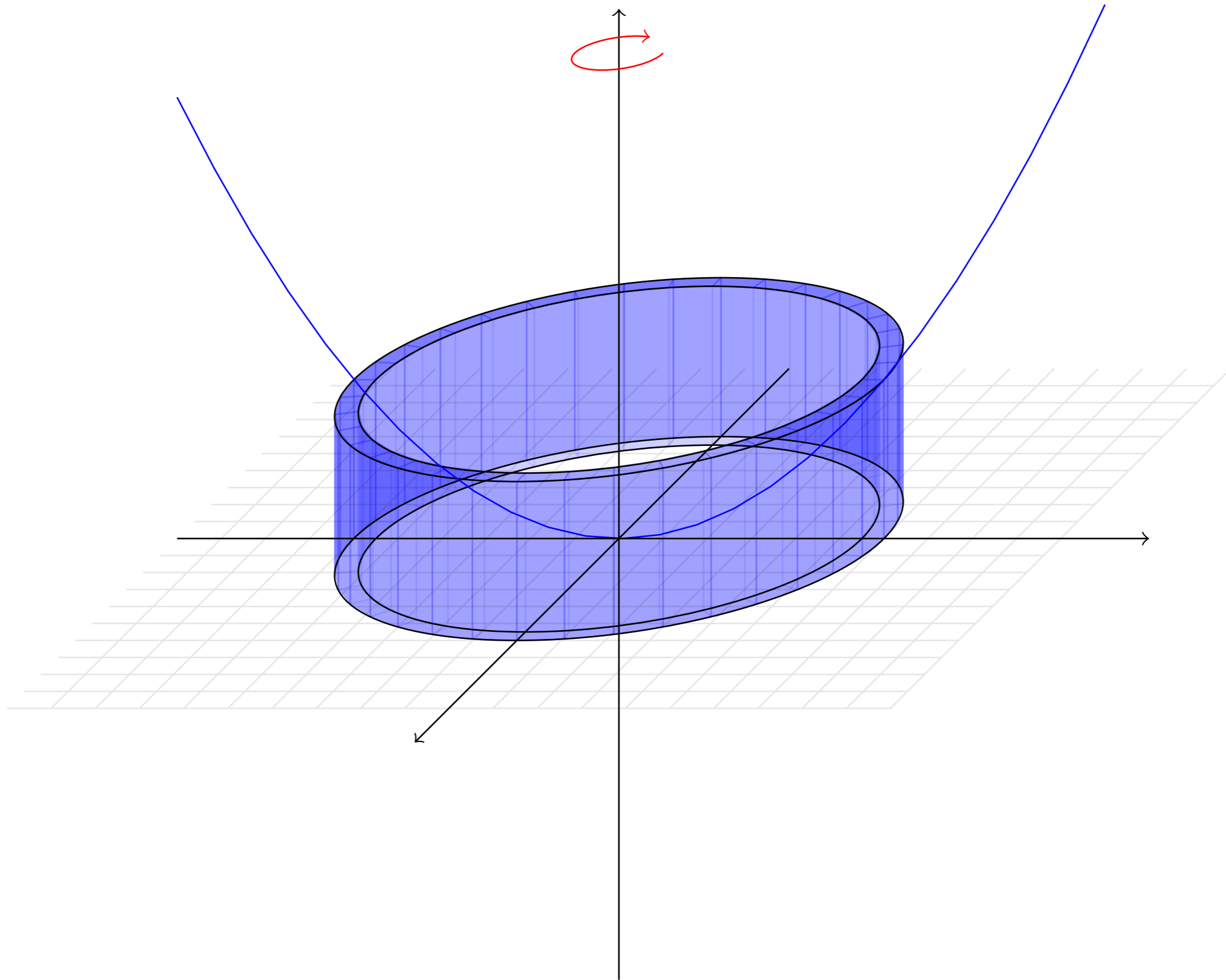


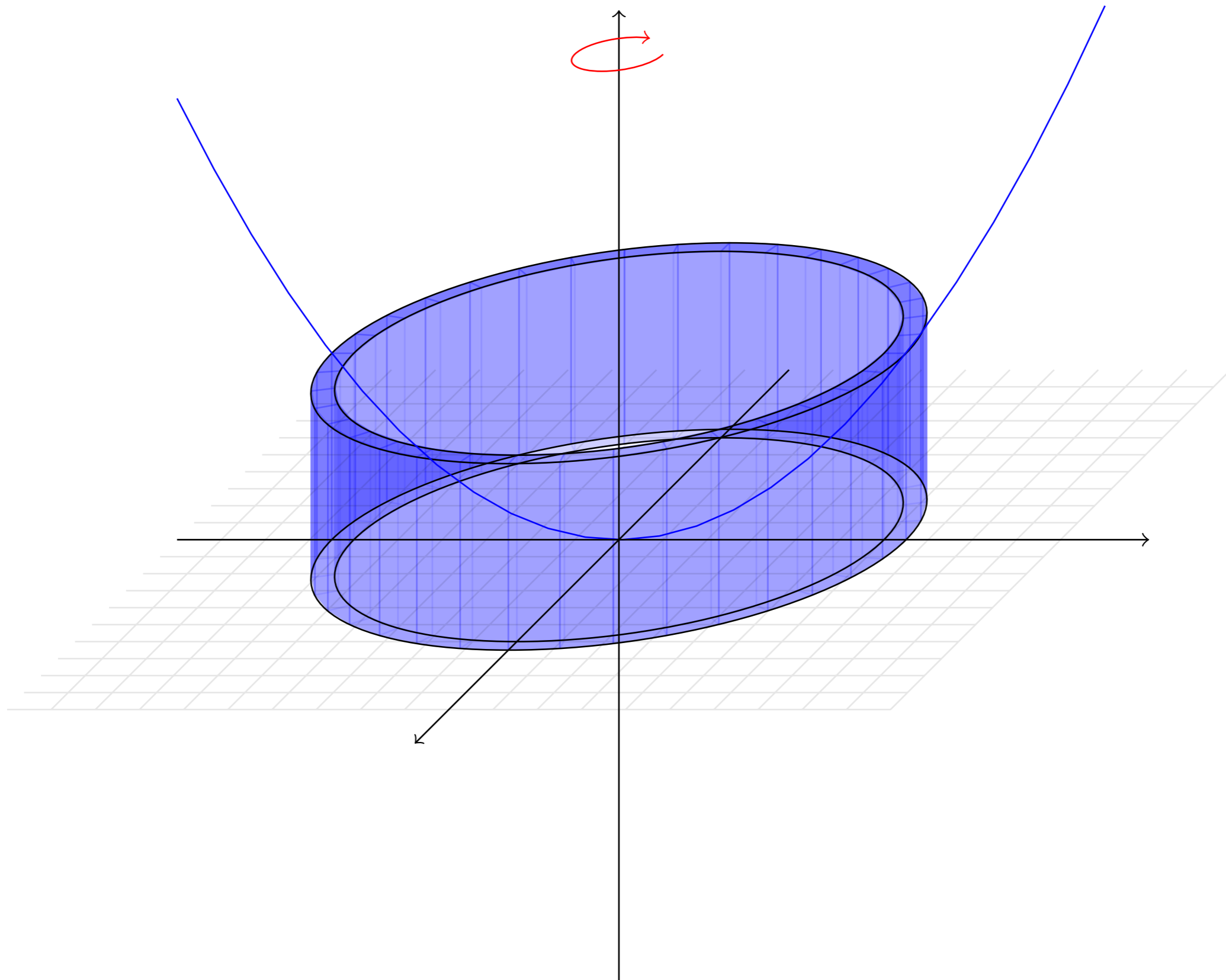


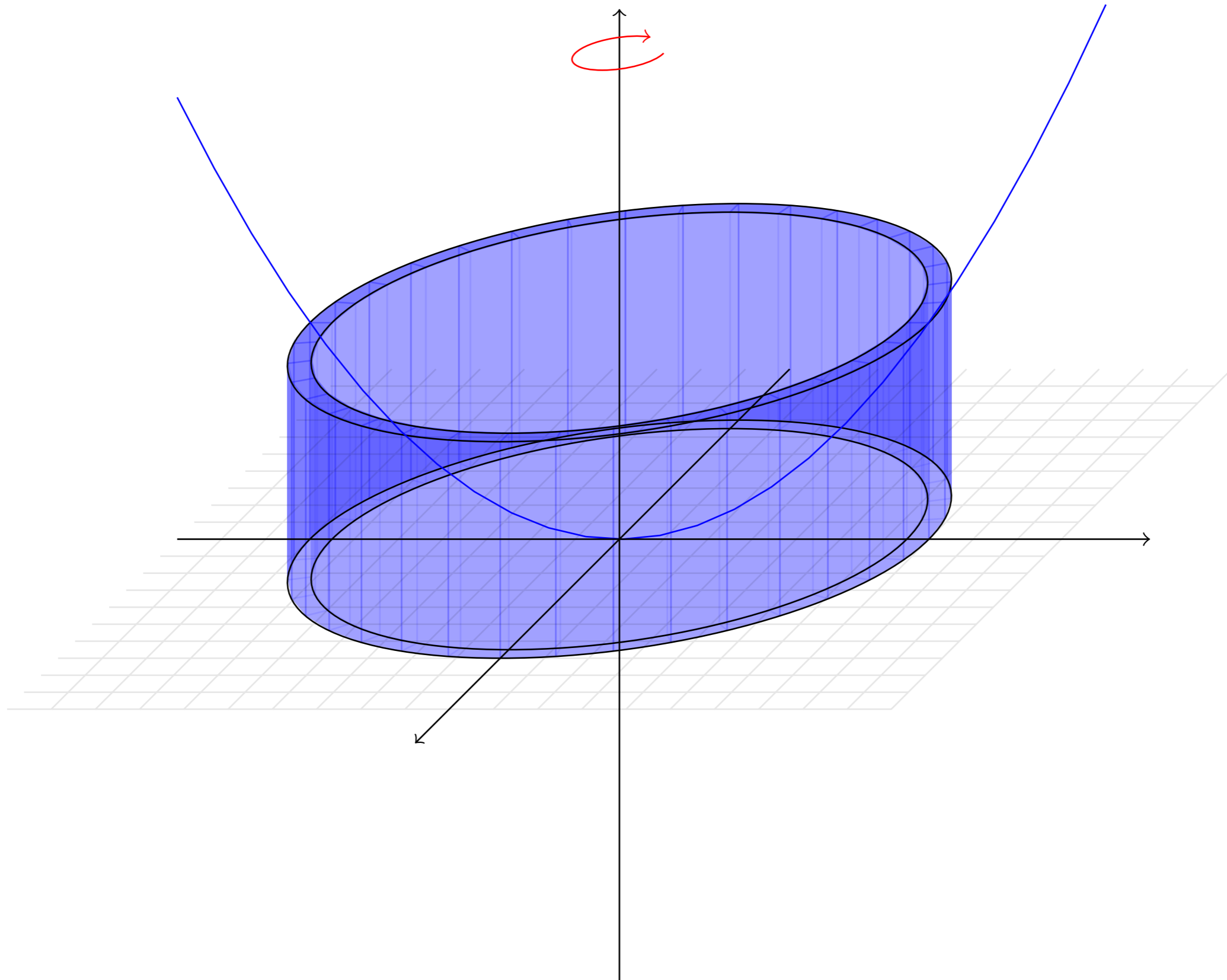


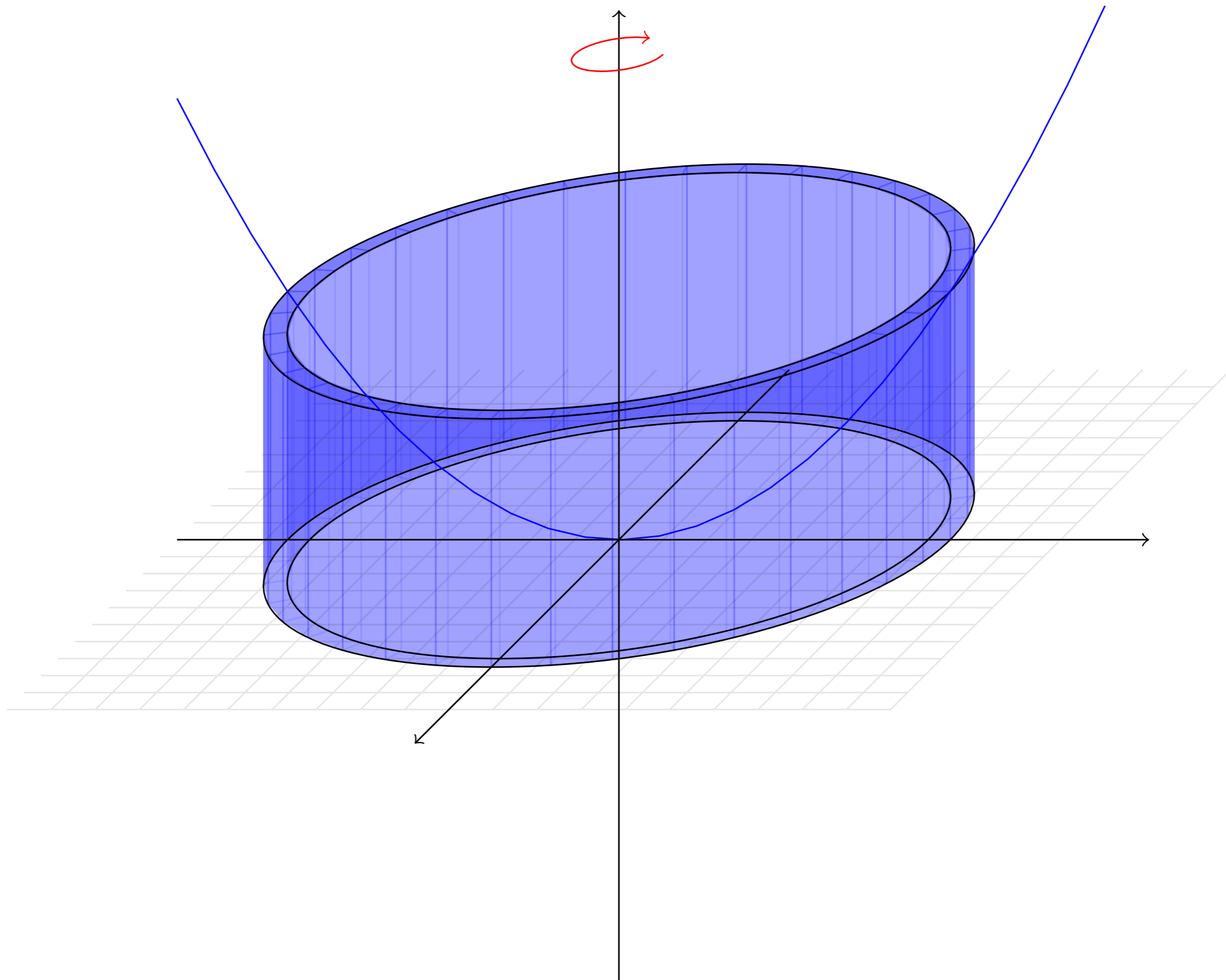


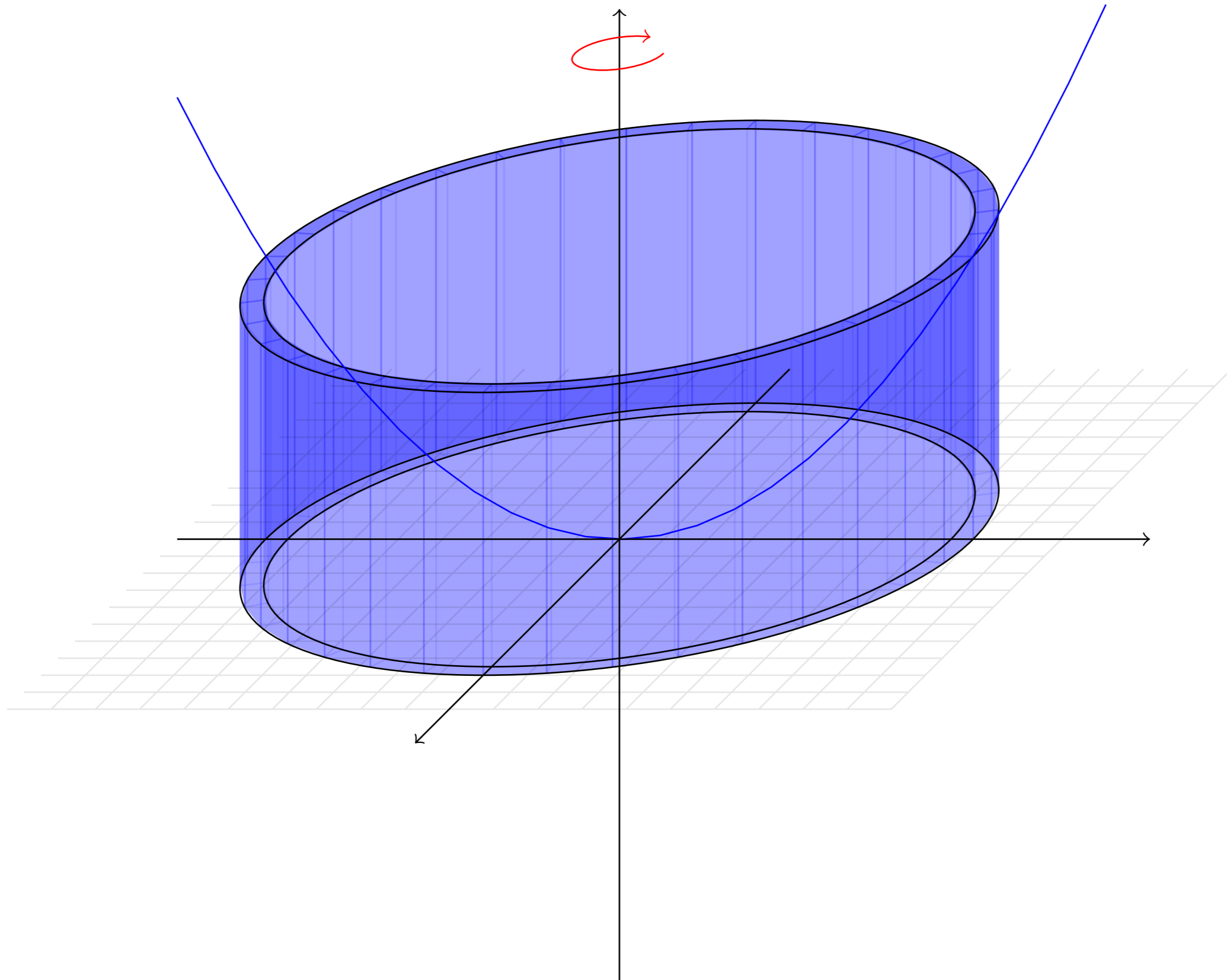


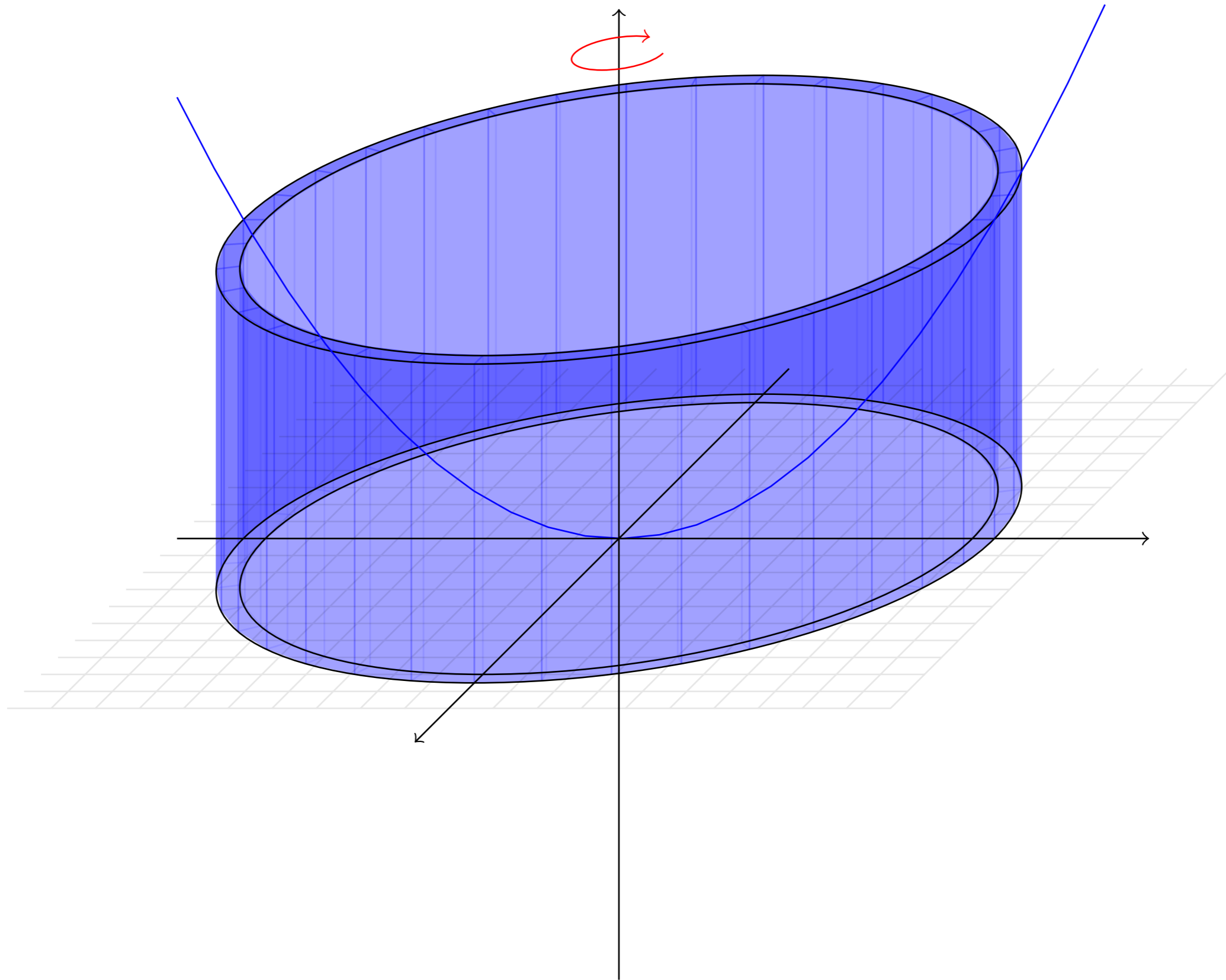


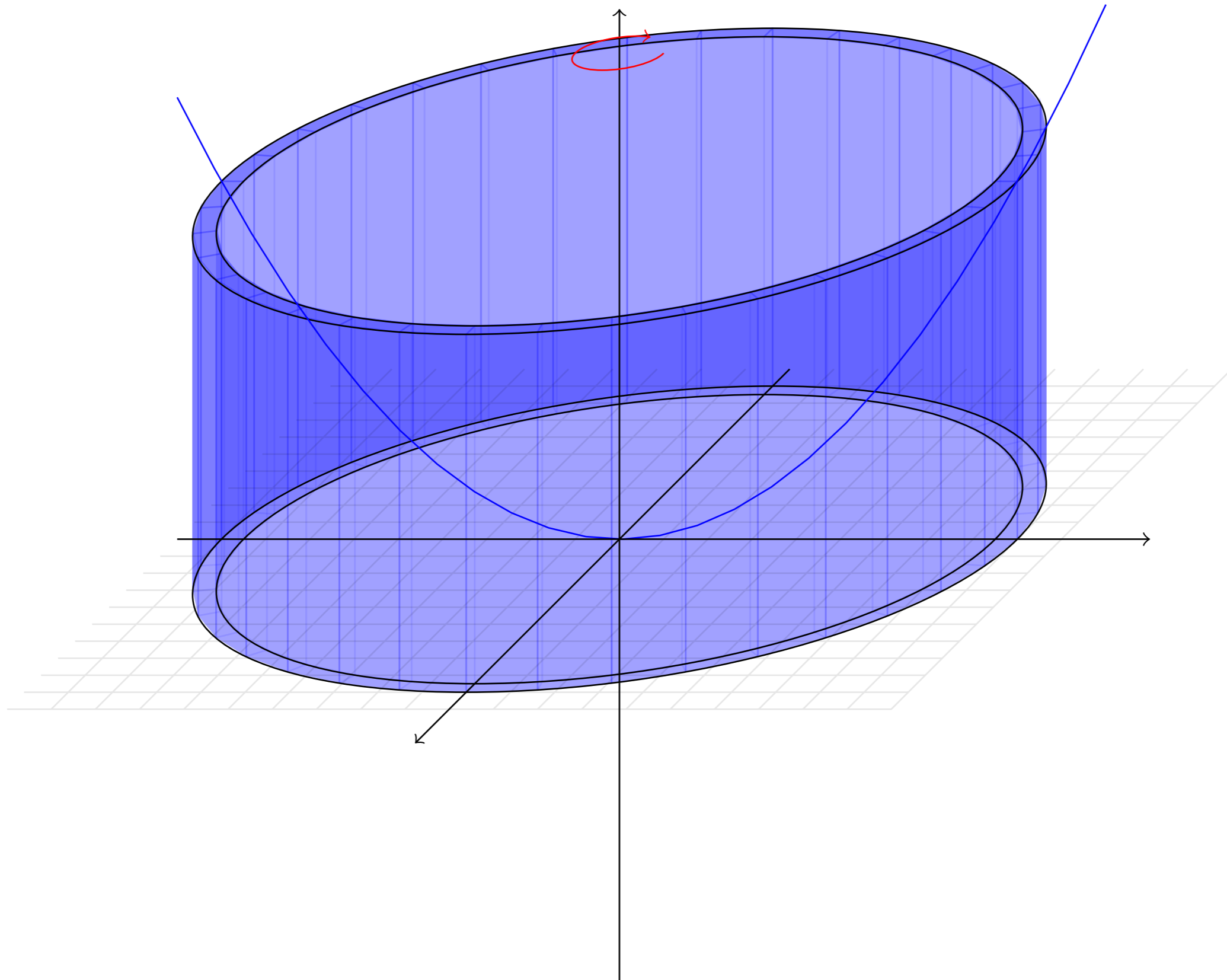


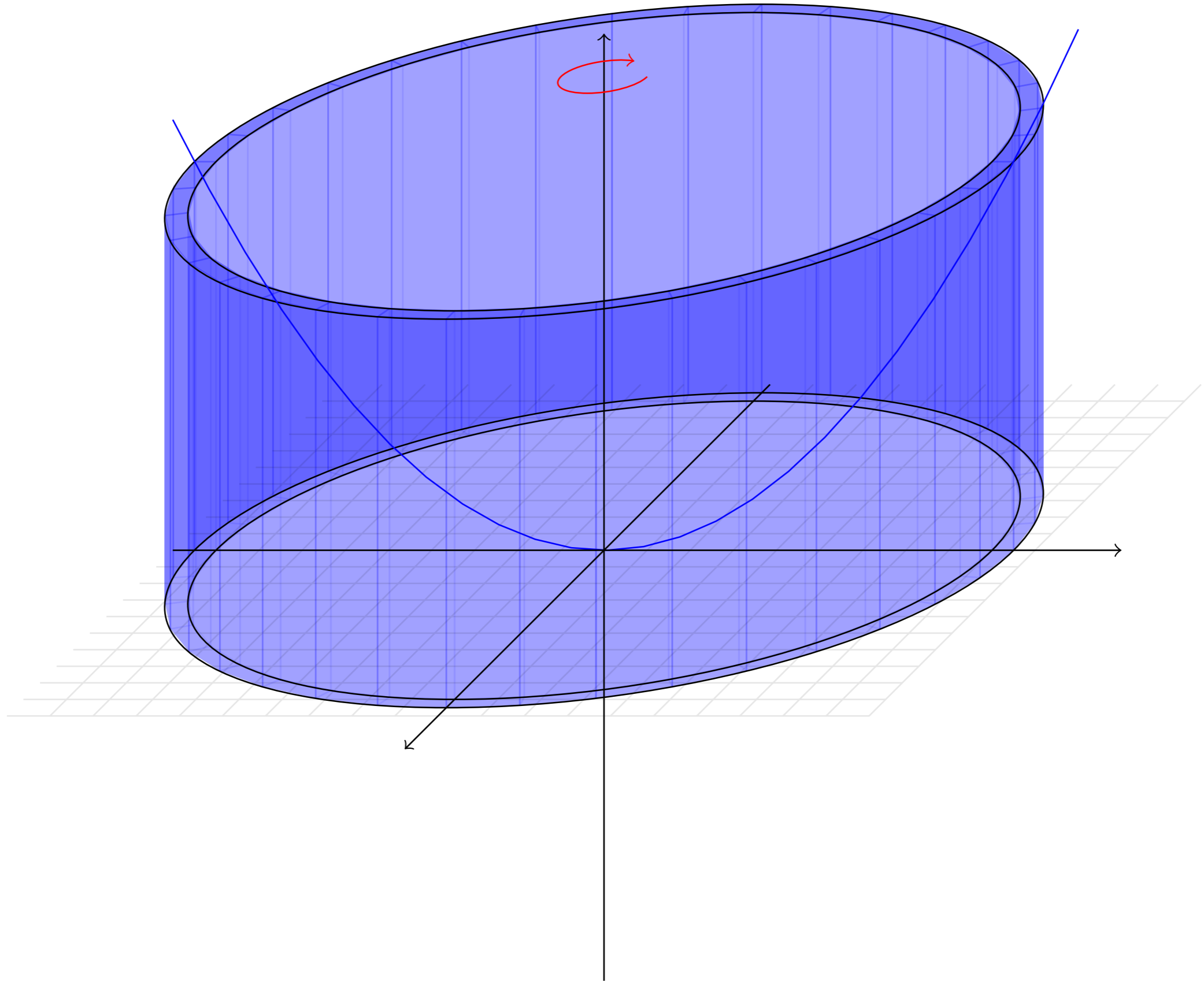


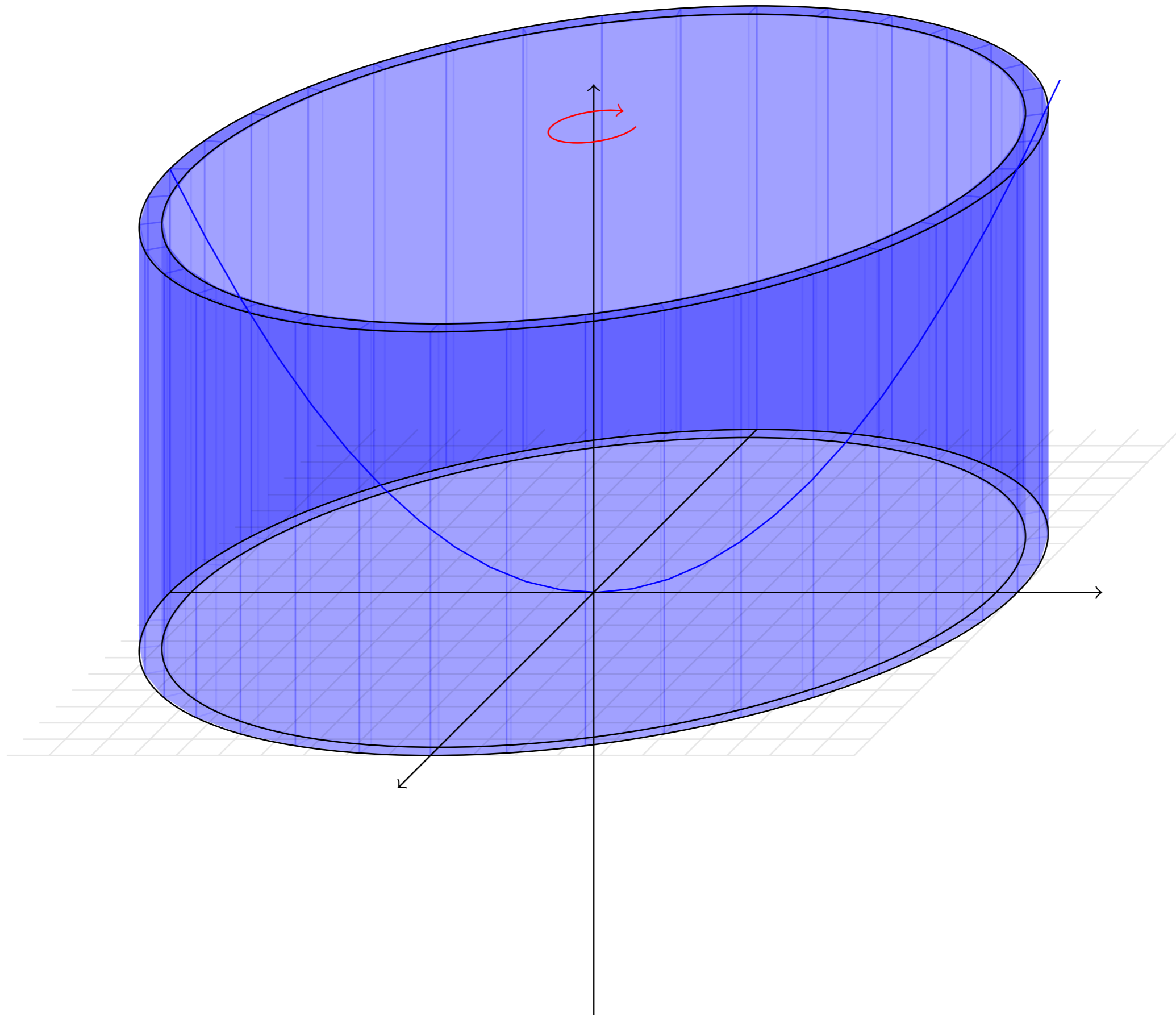


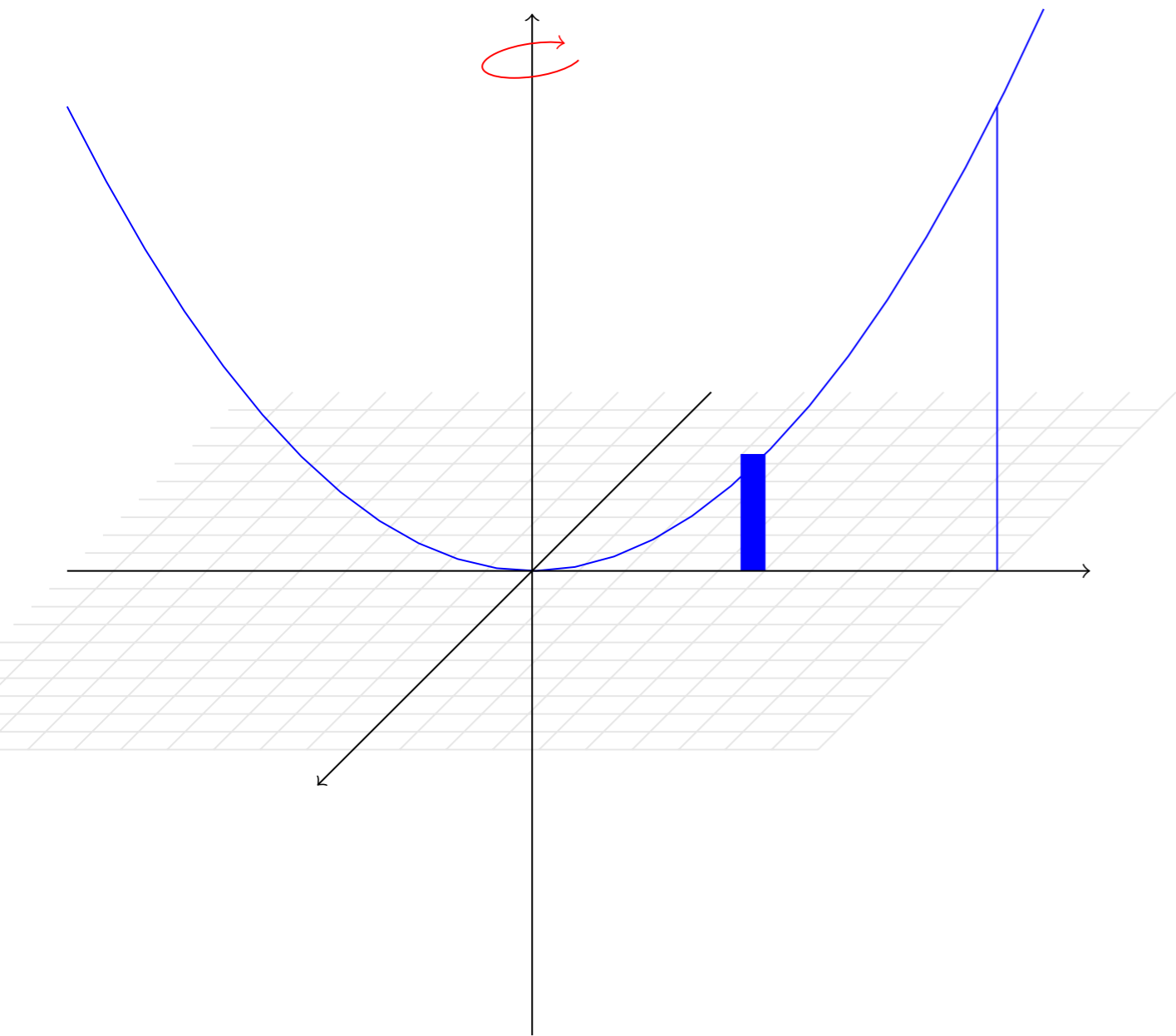


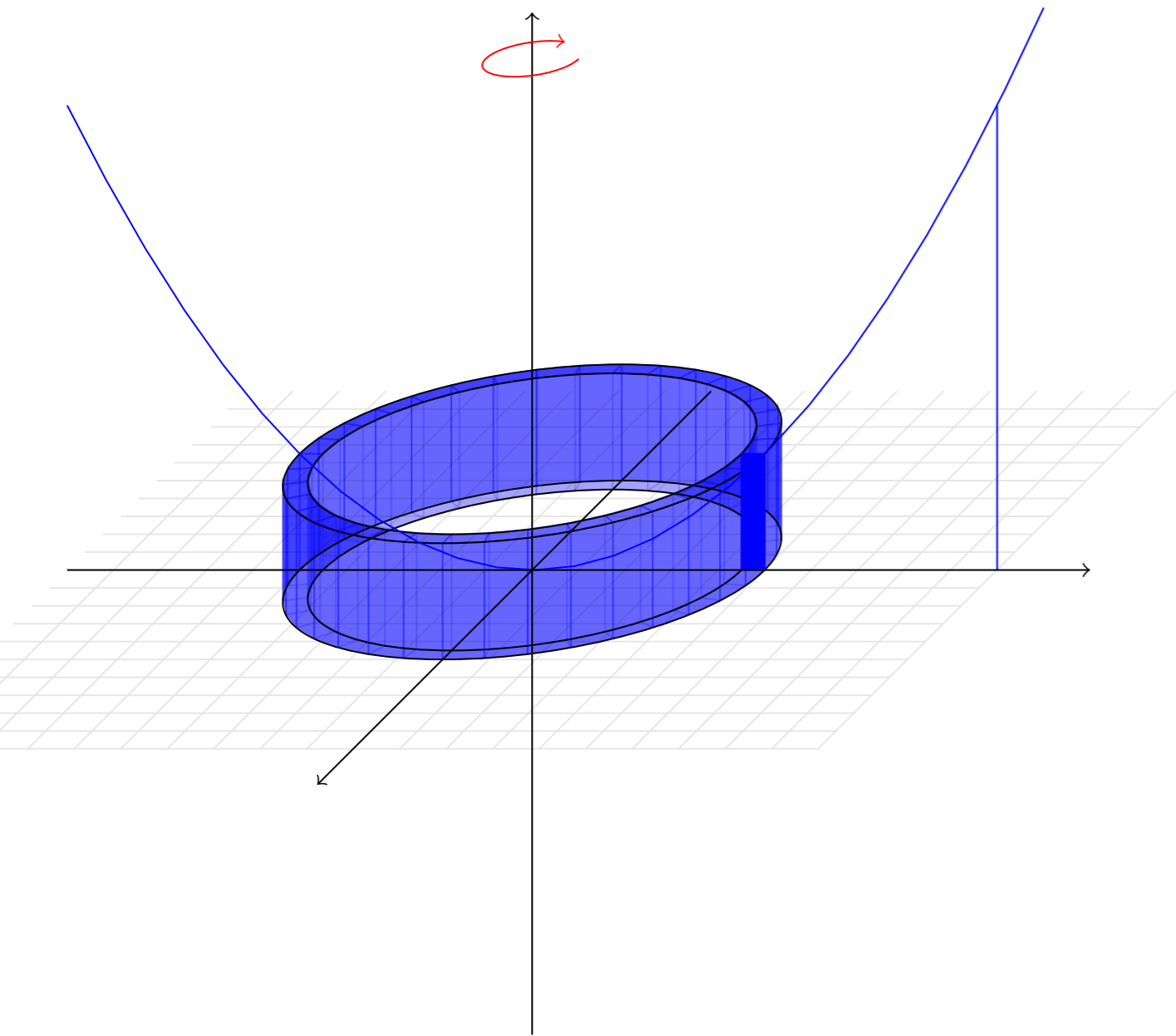


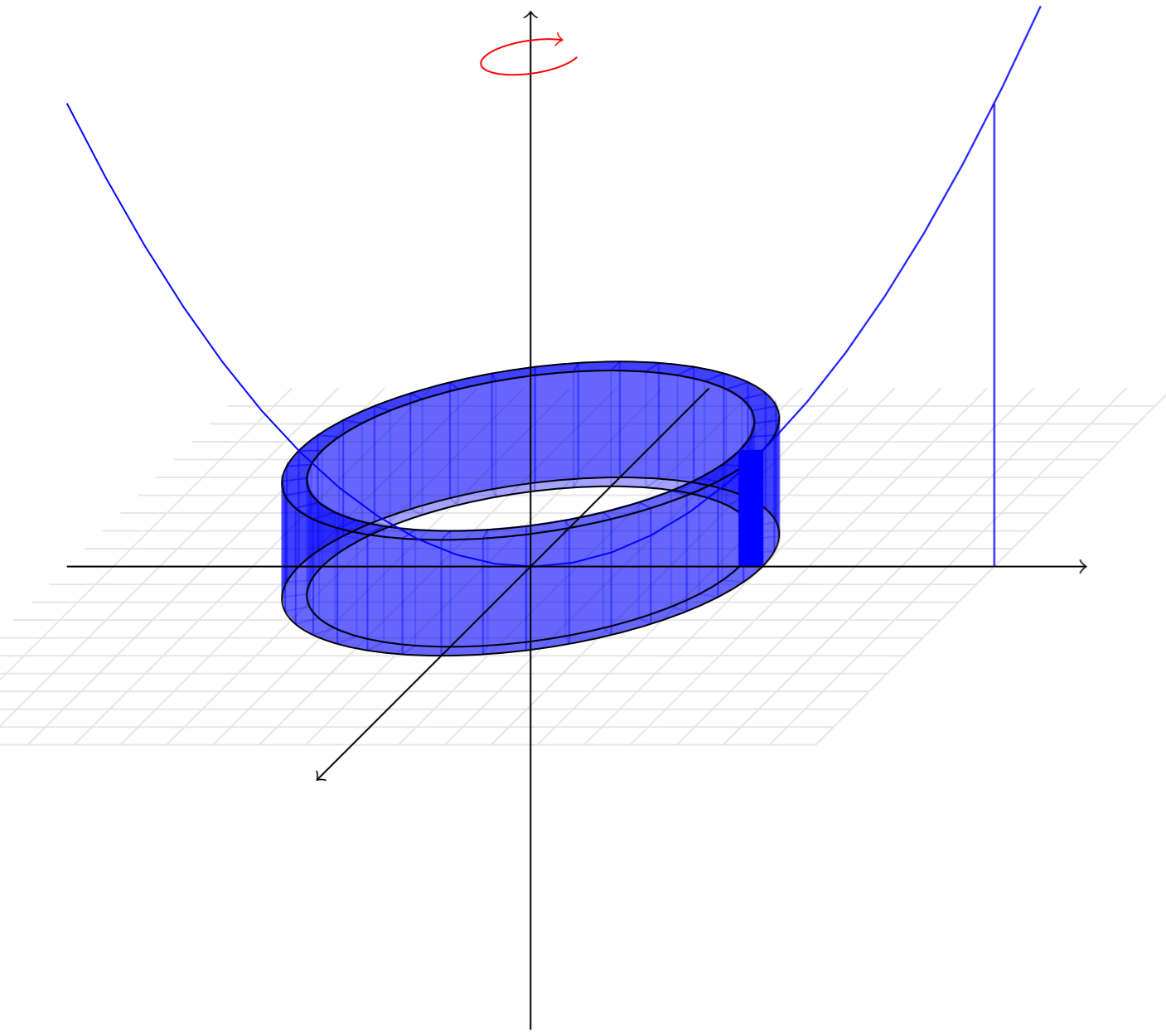




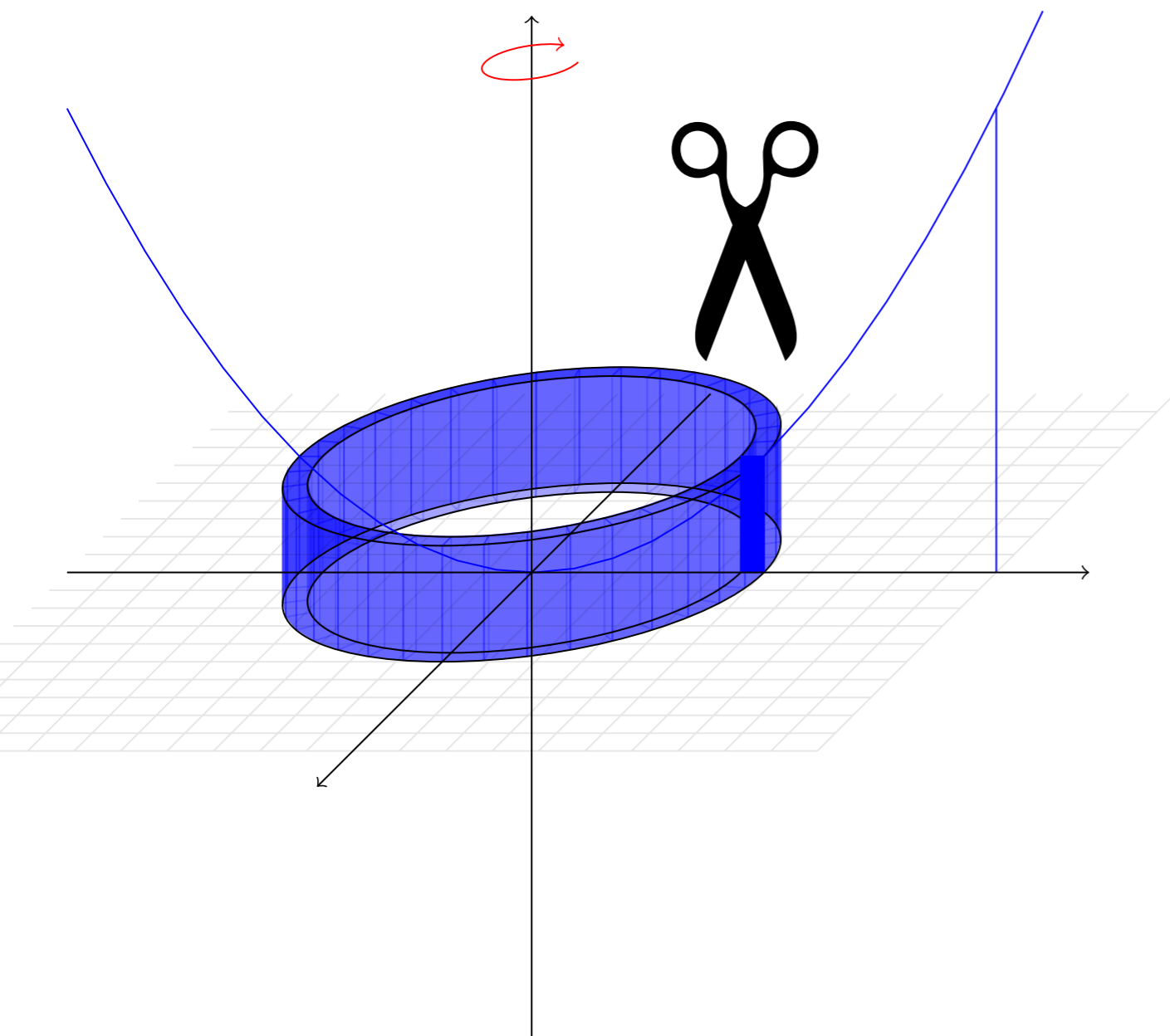




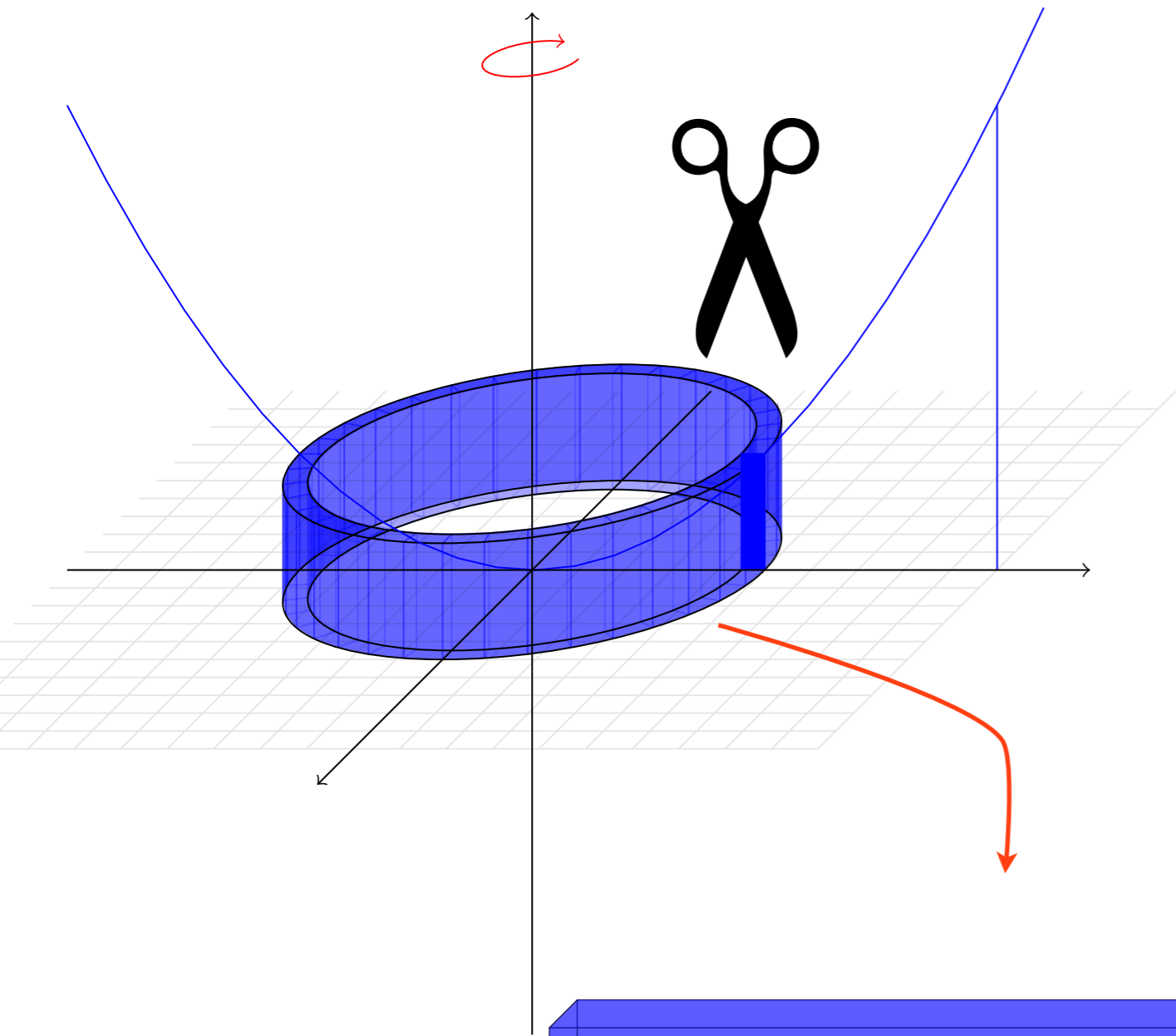




V_{tube}

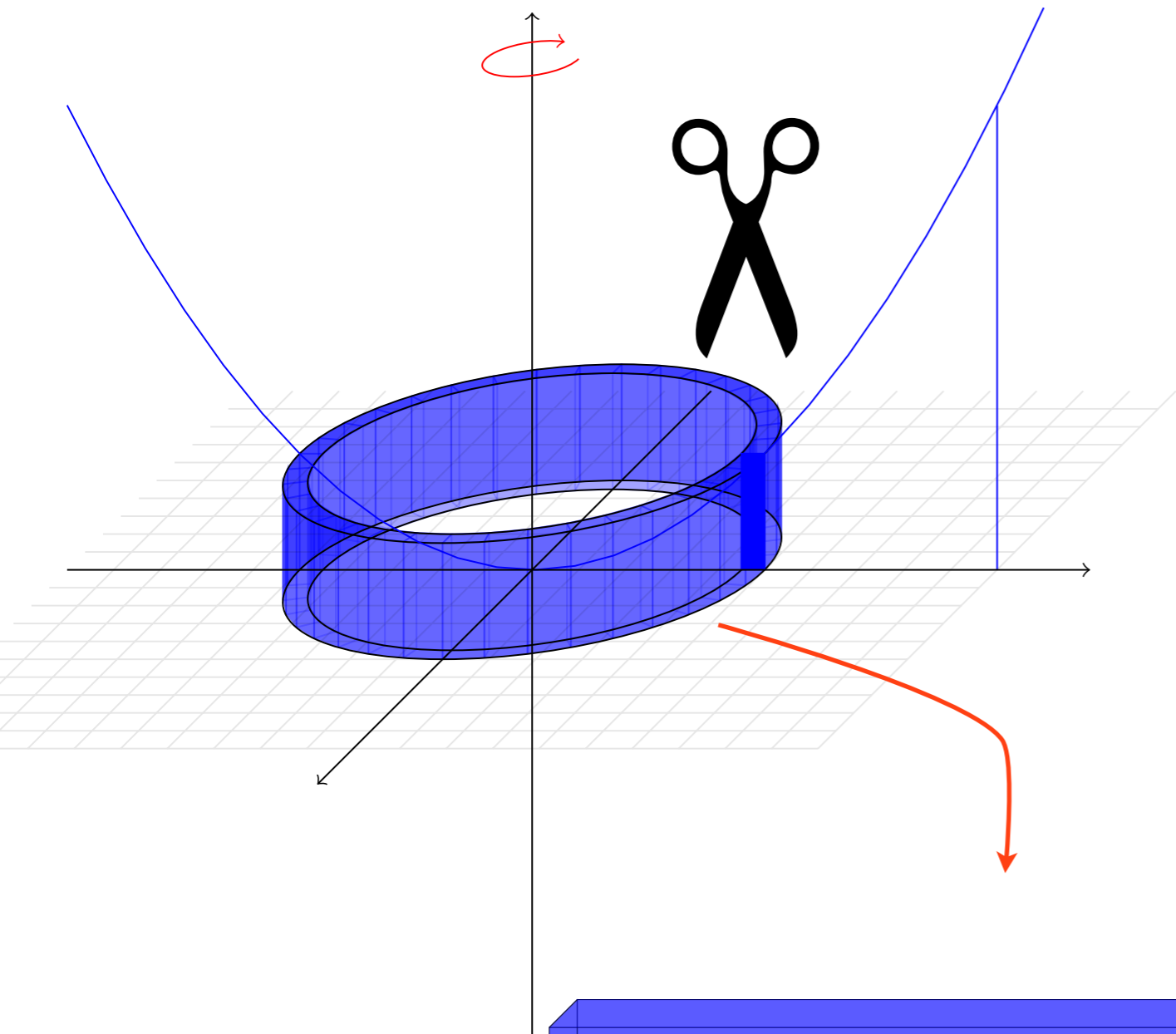


V_{tube}



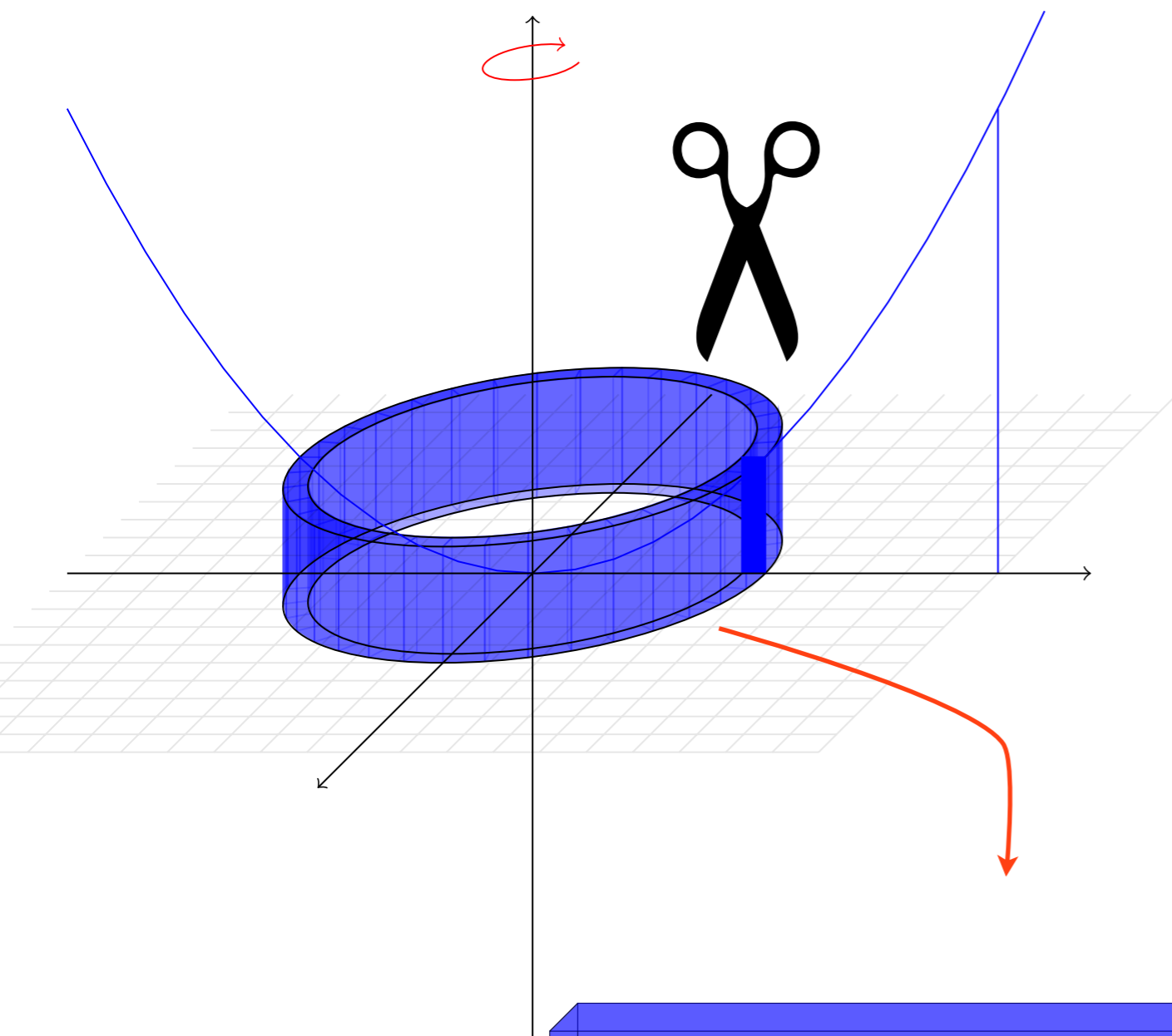
V_{tube}



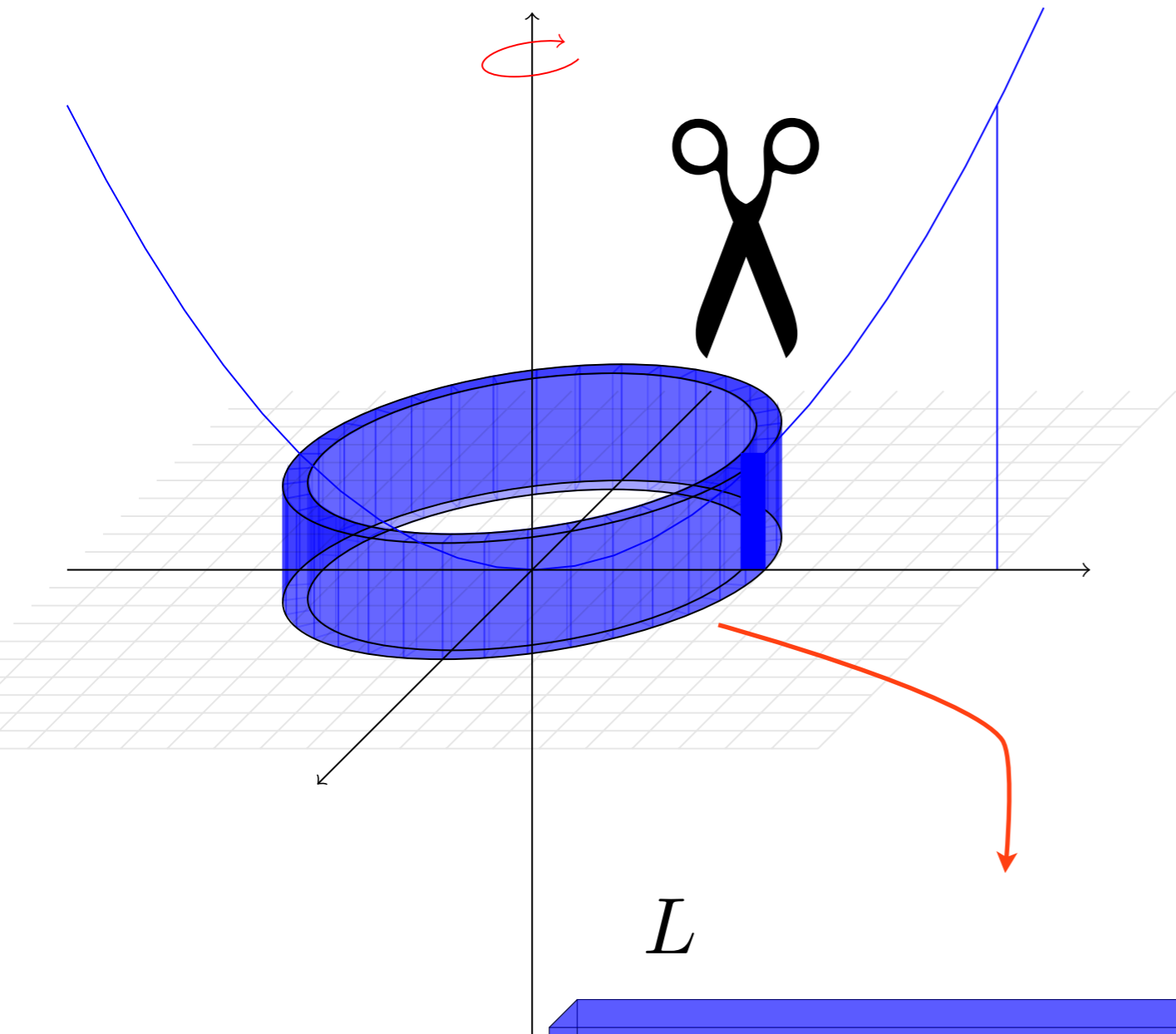


$$\text{Vol}_{\text{tube}} \approx \text{Vol}_{\text{boîte}}$$

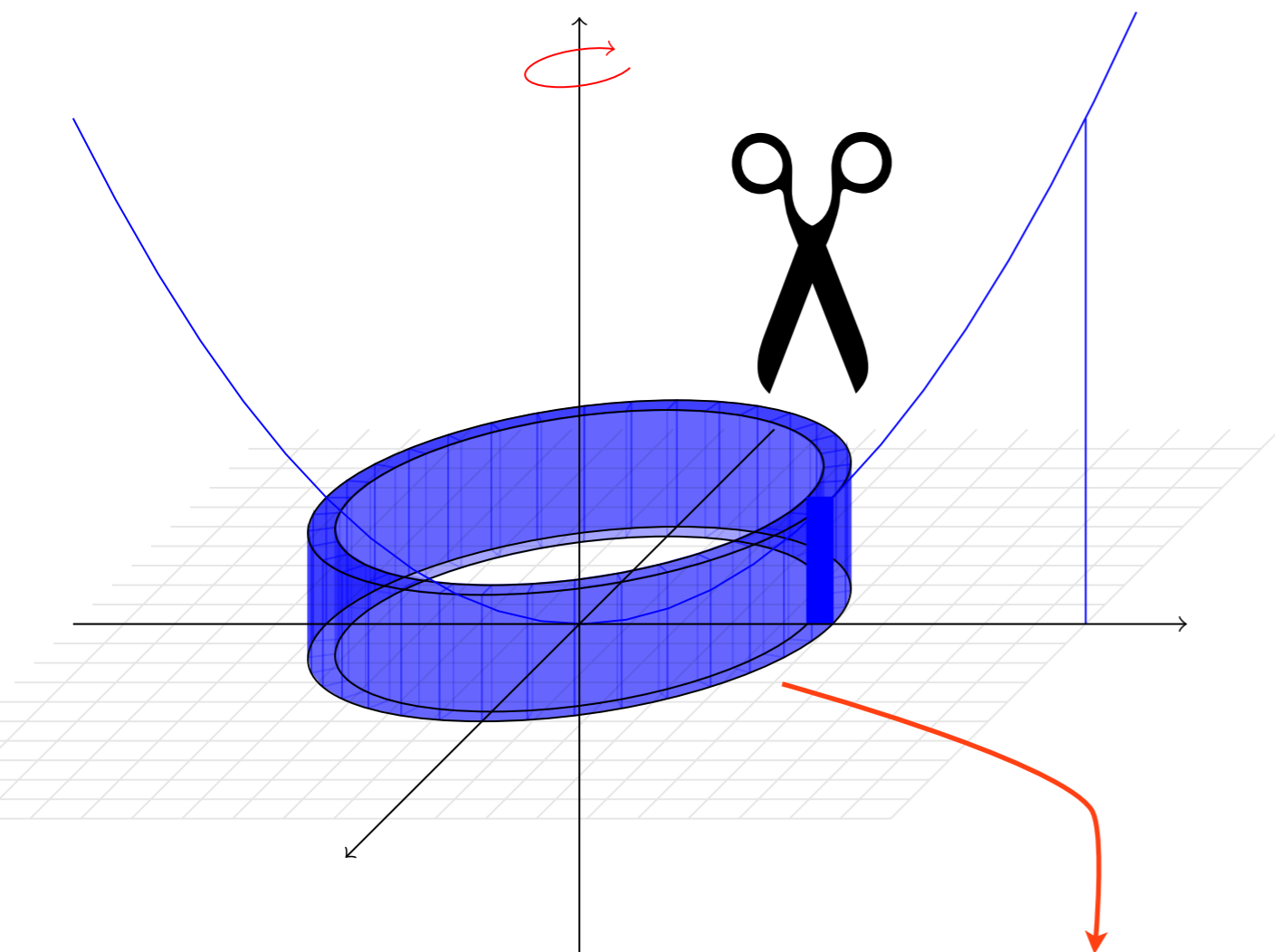




$$\text{Vol}_{\text{tube}} \approx \text{Vol}_{\text{boîte}} \\ = LhE$$



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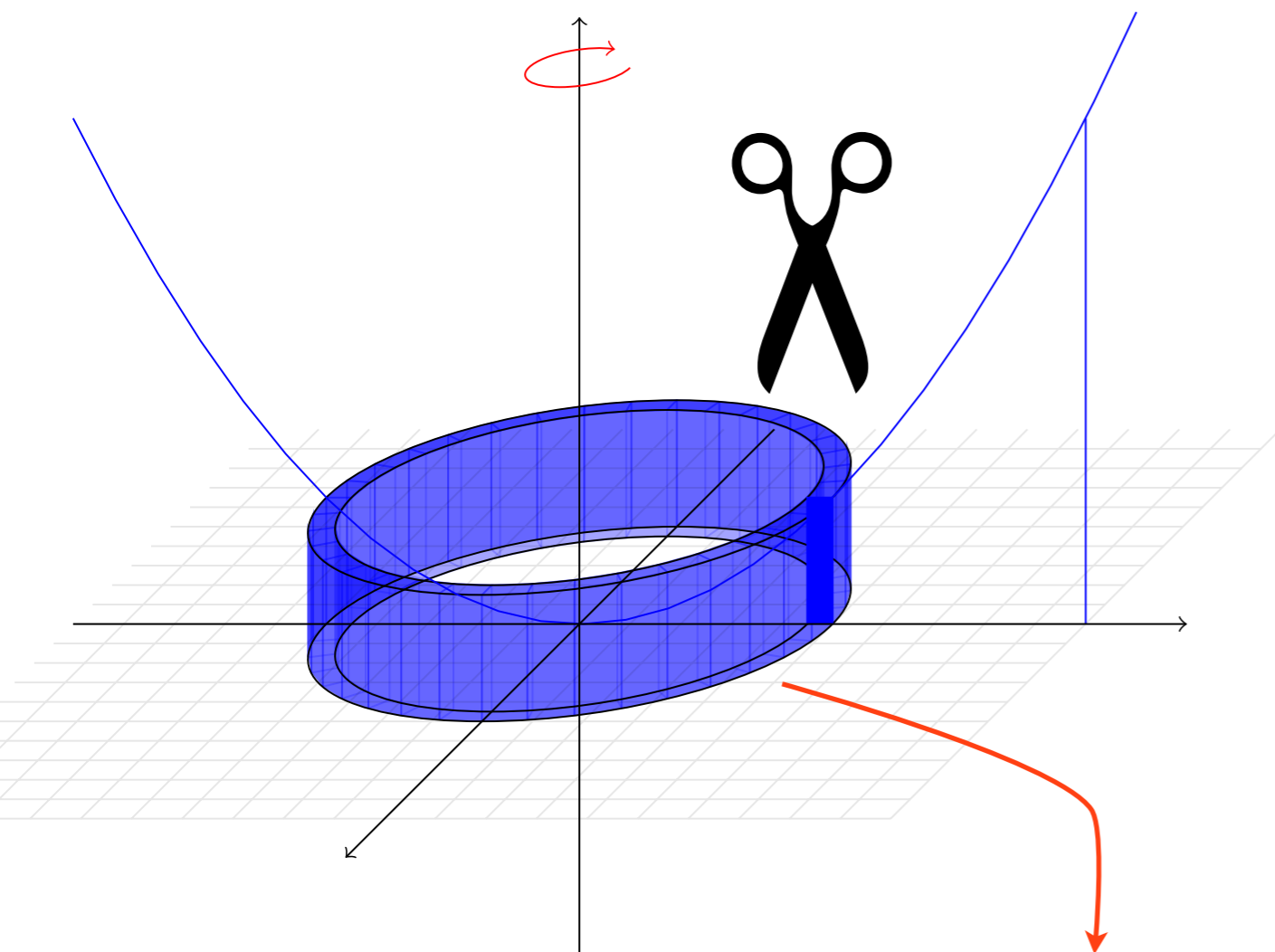


$$\text{Vol}_{\text{tube}} \approx \text{Vol}_{\text{boîte}} \\ = LhE$$

L

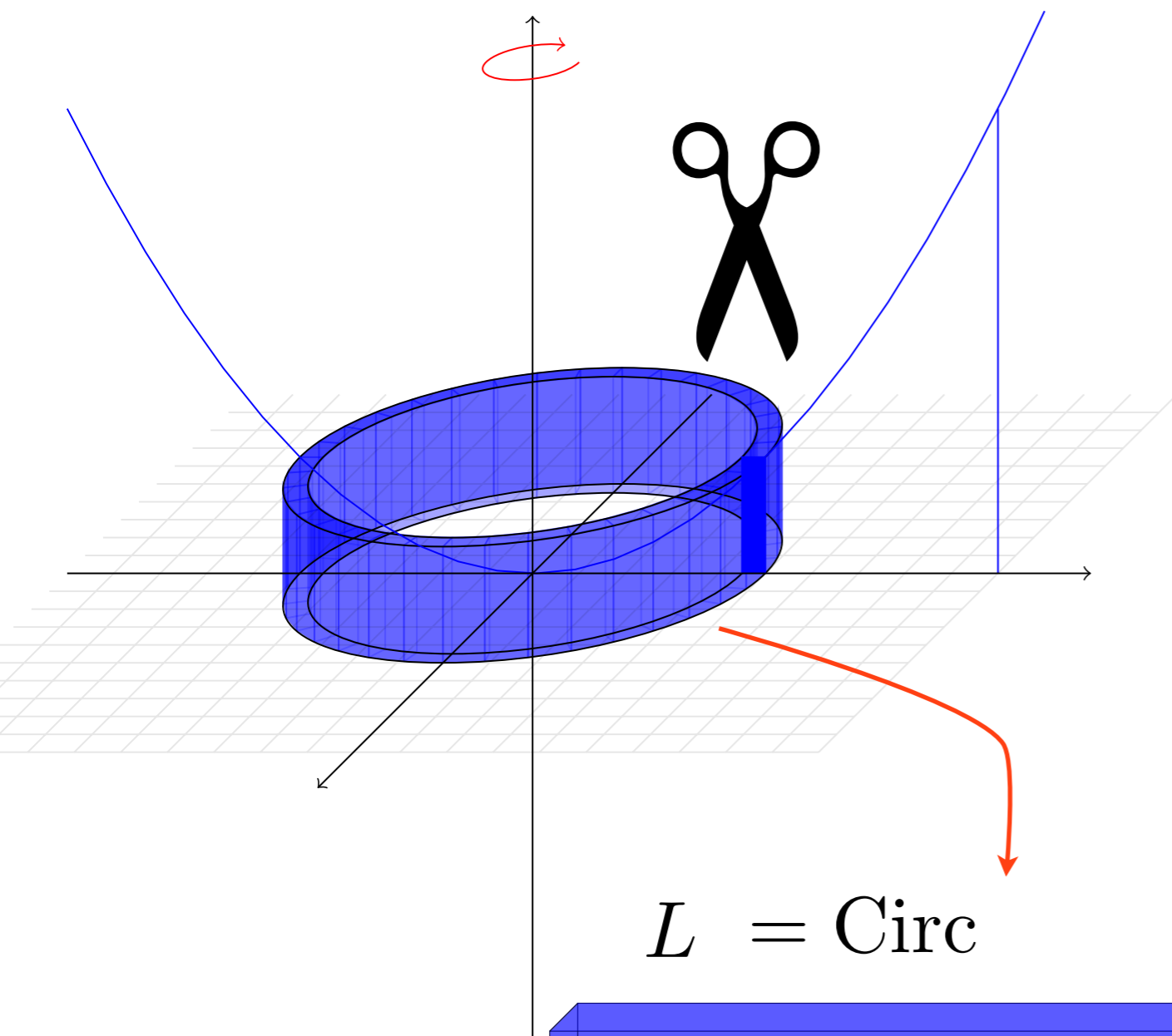


h



$$\text{Vol}_{\text{tube}} \approx \text{Vol}_{\text{boîte}} \\ = LhE$$



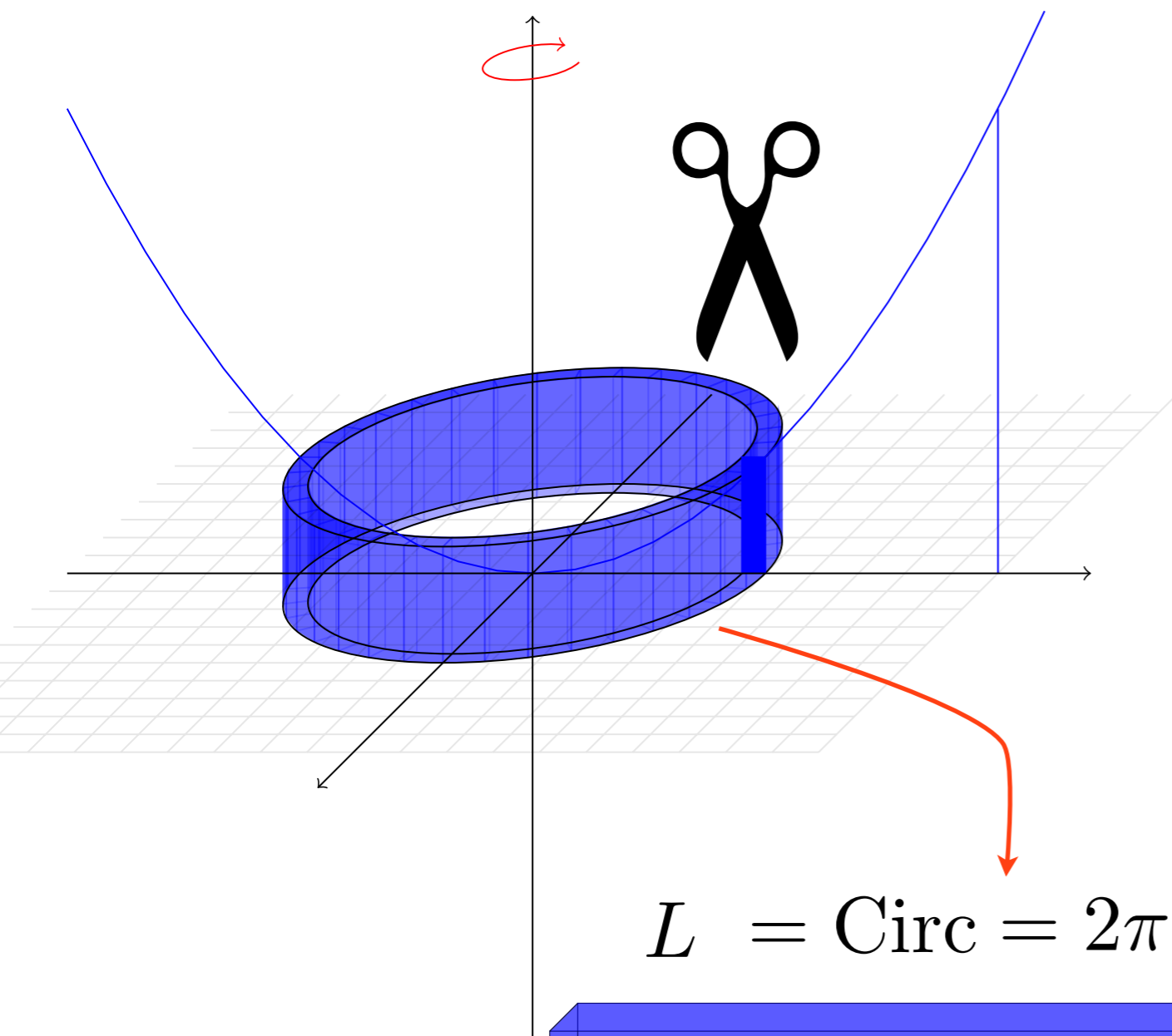


$$\text{Vol}_{\text{tube}} \approx \text{Vol}_{\text{boîte}} \\ = LhE$$



E

h



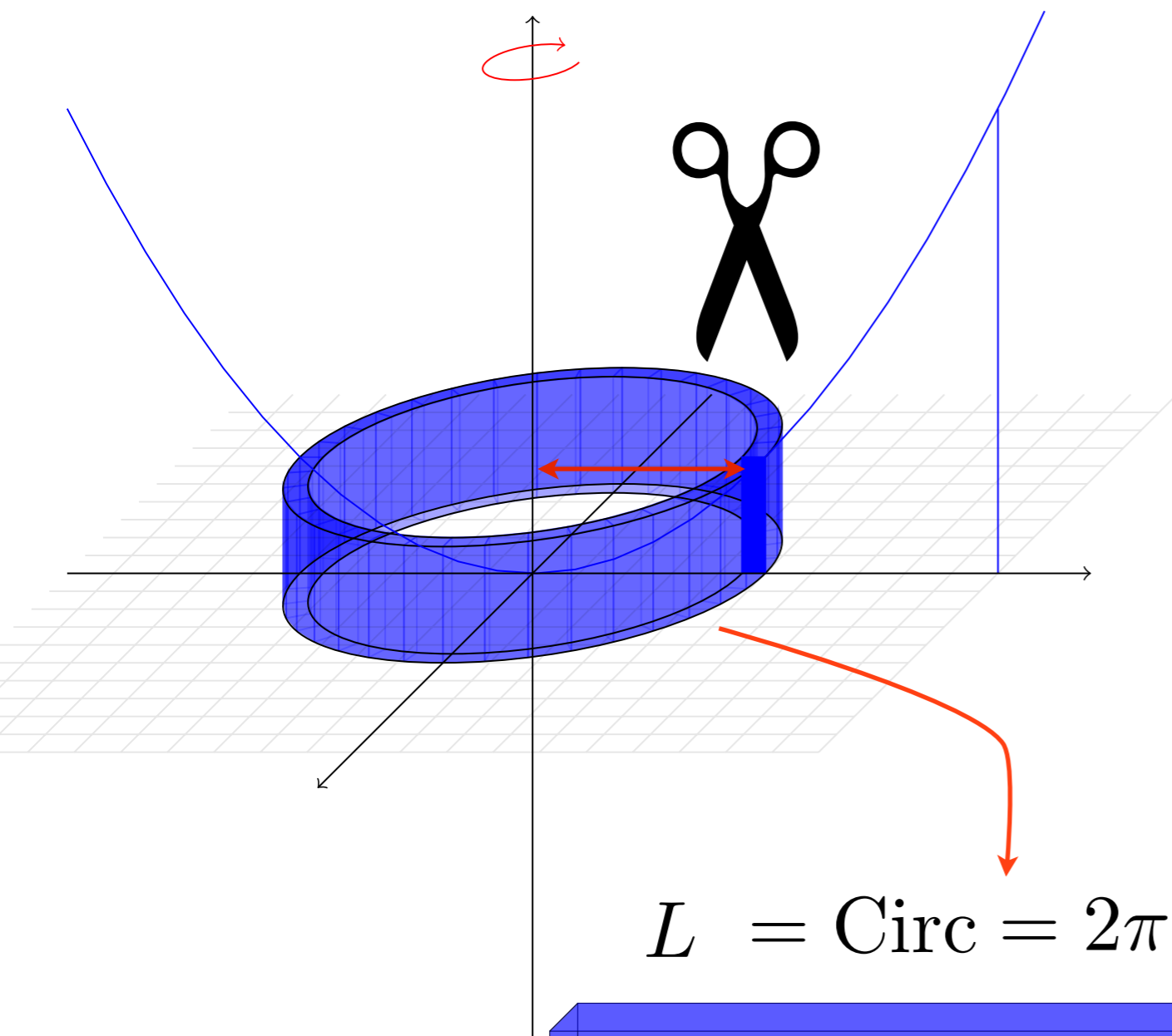
$$\text{Vol}_{\text{tube}} \approx \text{Vol}_{\text{boîte}} \\ = LhE$$

$$L = \text{Circ} = 2\pi R$$



E

h

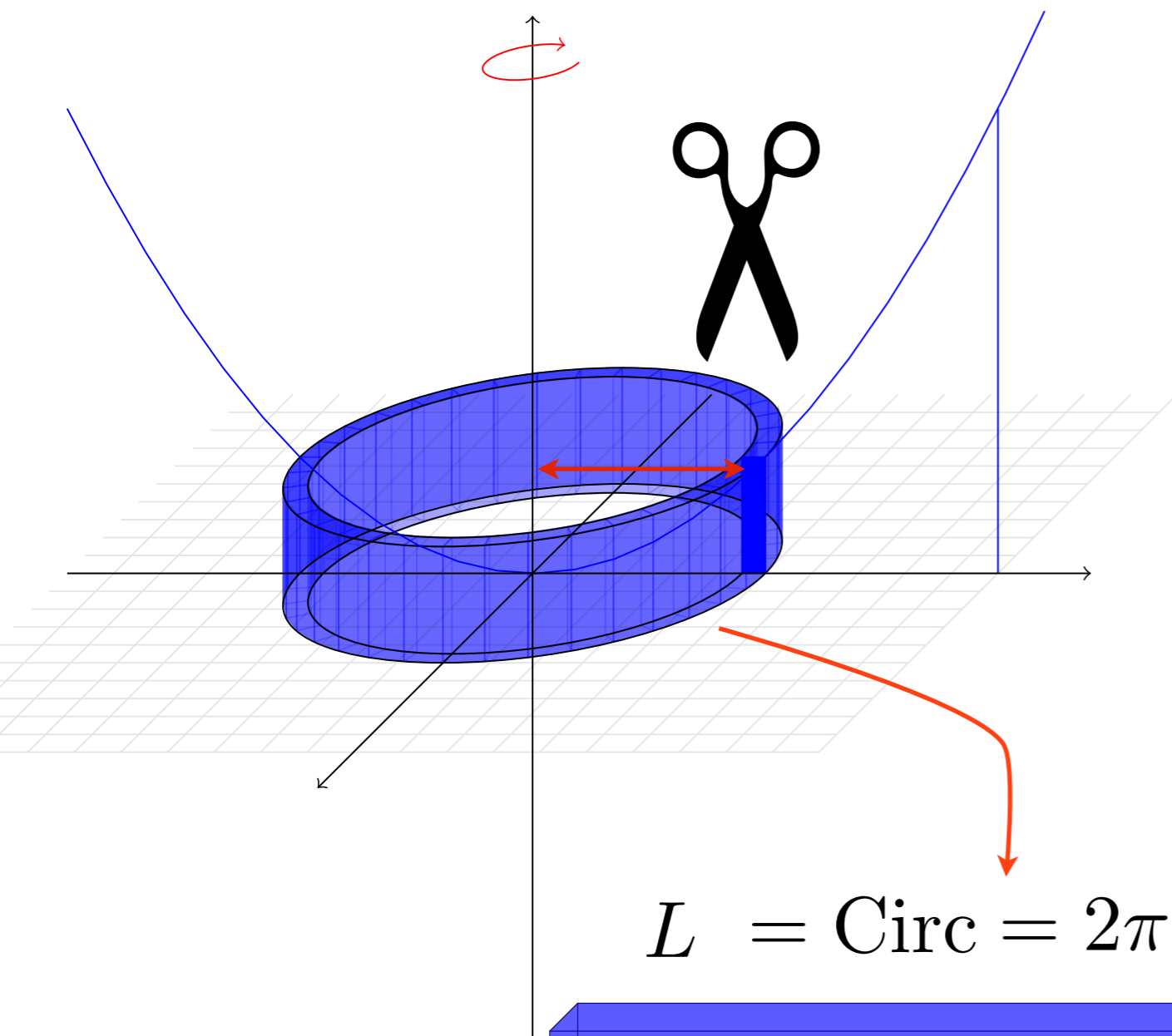


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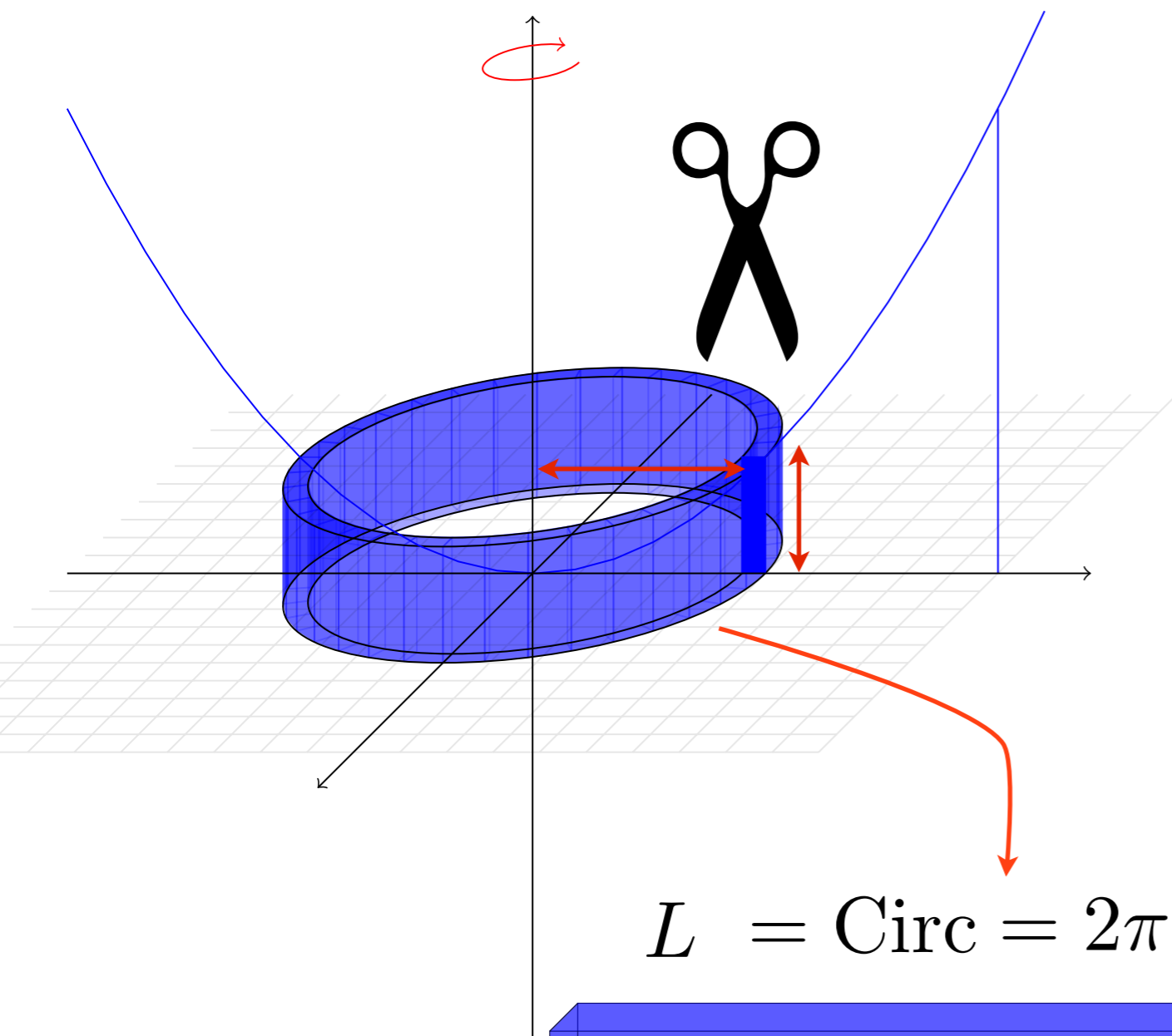


$$\begin{aligned} \text{Vol}_{\text{tube}} &\approx \text{Vol}_{\text{boîte}} \\ &= LhE \\ &= 2\pi RhE \end{aligned}$$

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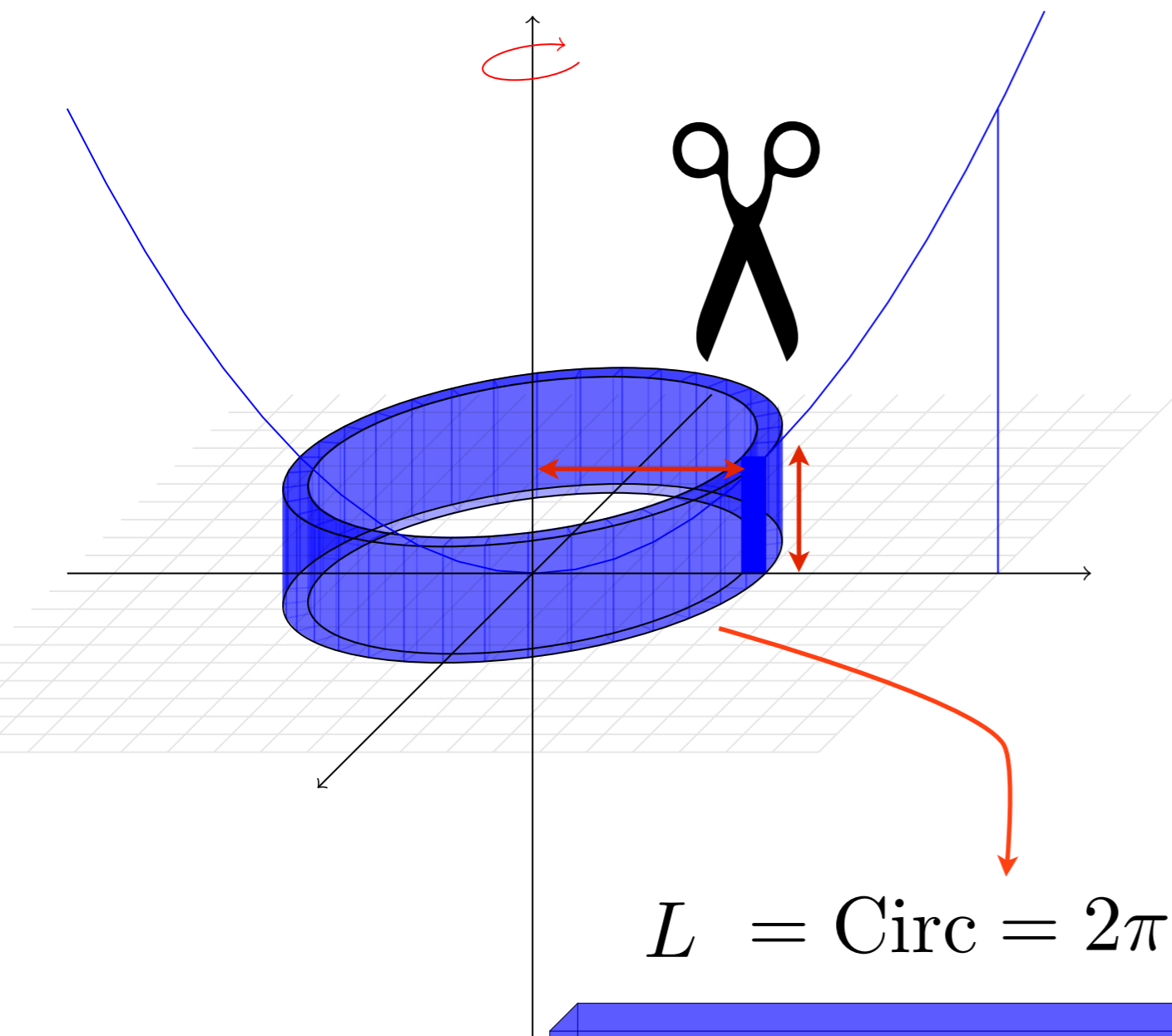


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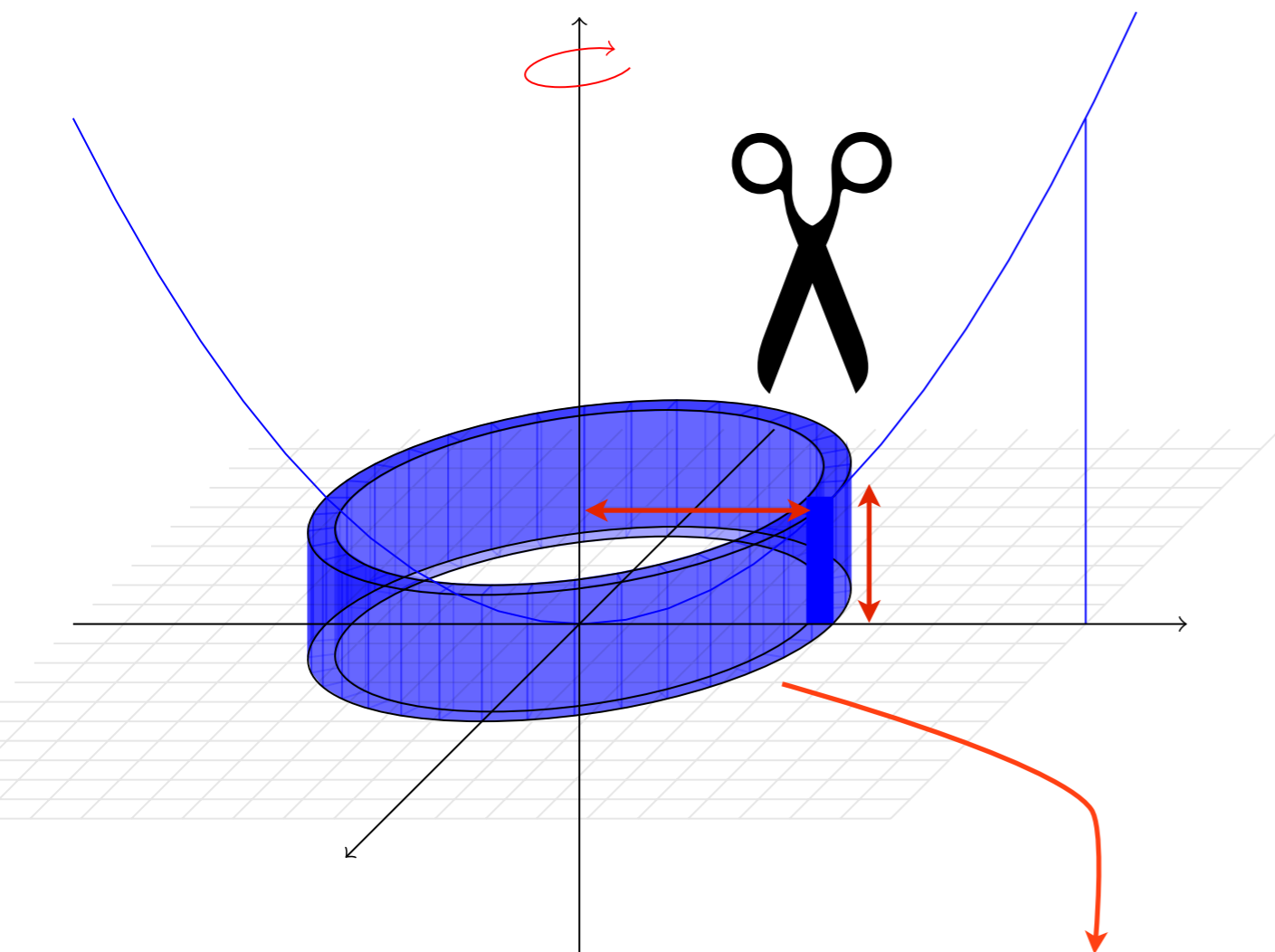
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$$h = f(x)$$

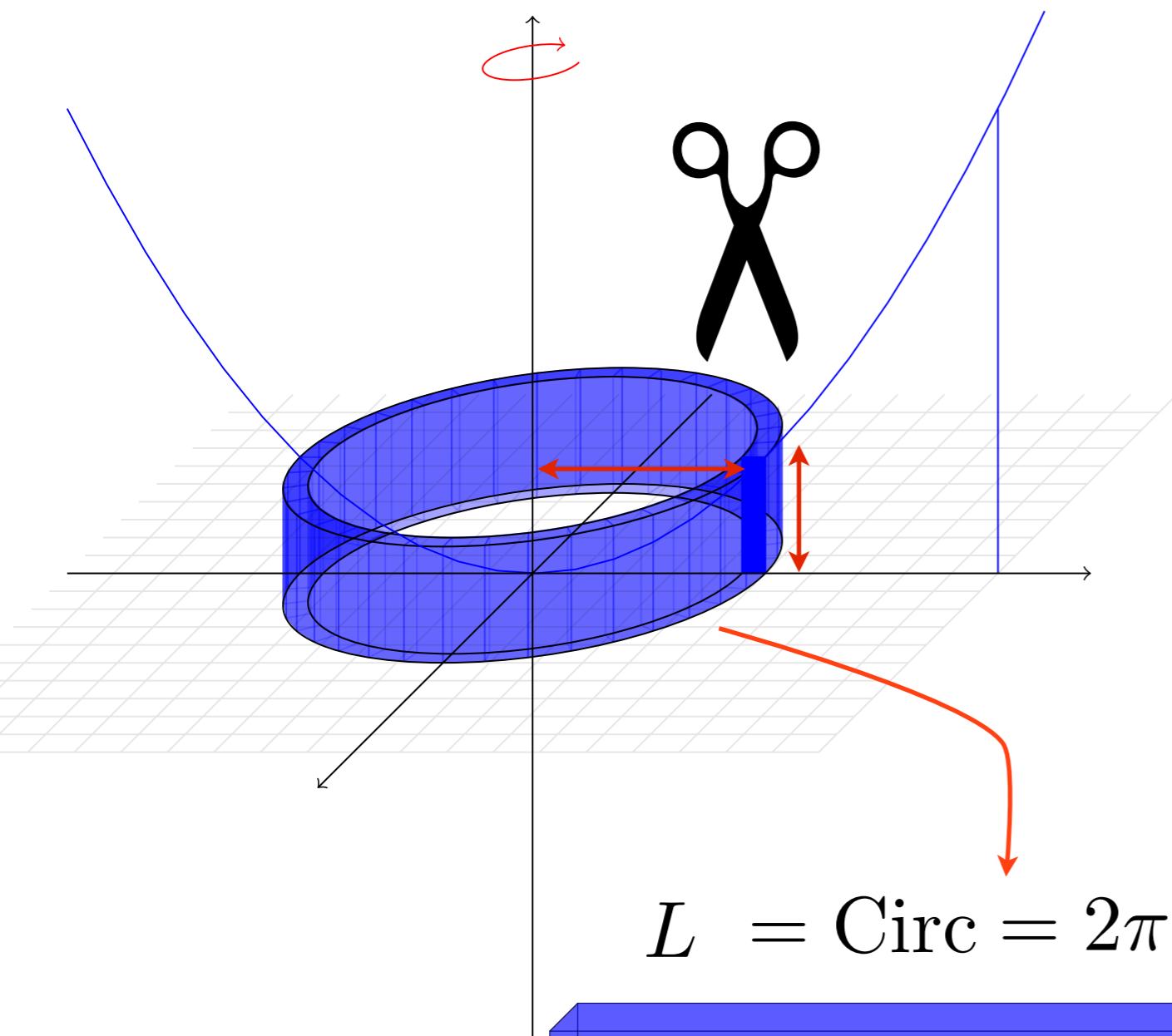


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$$L = \text{Circ} = 2\pi R = 2\pi x \quad E$$



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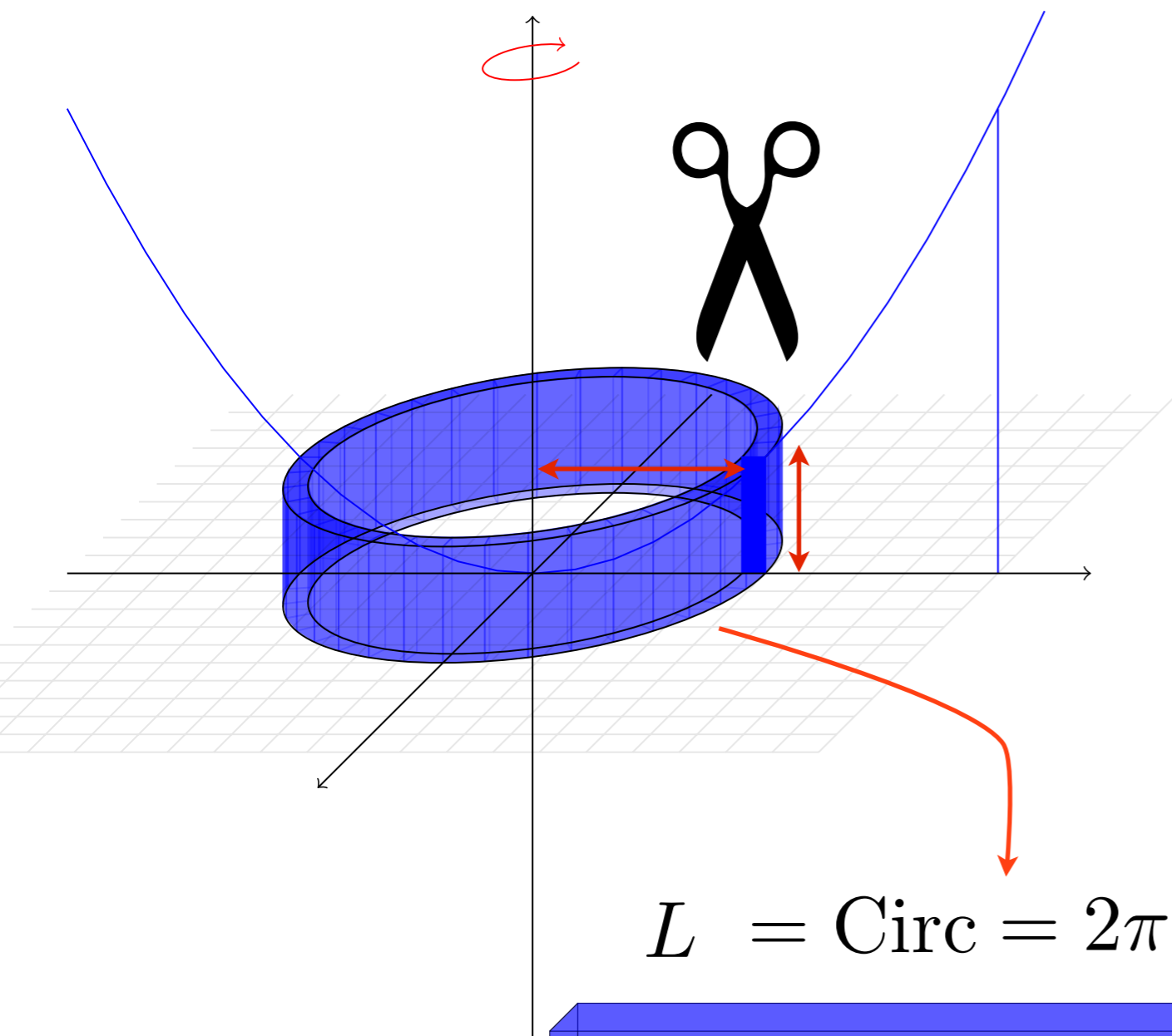


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$$L = \text{Circ} = 2\pi R = 2\pi x$$

$$E = \Delta x$$

$$h = f(x)$$



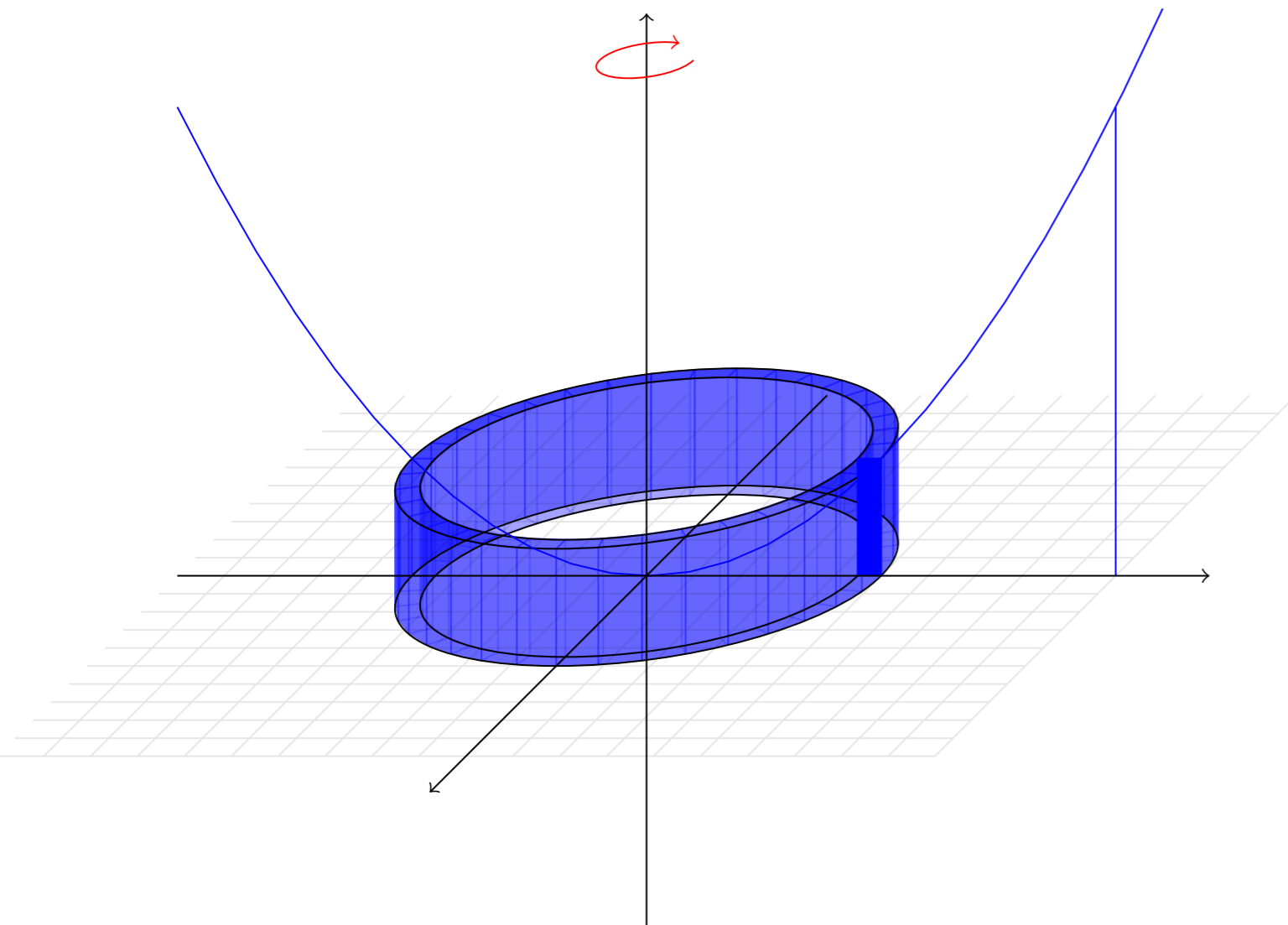
$$\begin{aligned}
 \text{Vol}_{\text{tube}} &\approx \text{Vol}_{\text{boîte}} \\
 &= LhE \\
 &= 2\pi RhE \\
 &= 2\pi x f(x) \Delta x
 \end{aligned}$$

$$L = \text{Circ} = 2\pi R = 2\pi x$$

$$E = \Delta x$$

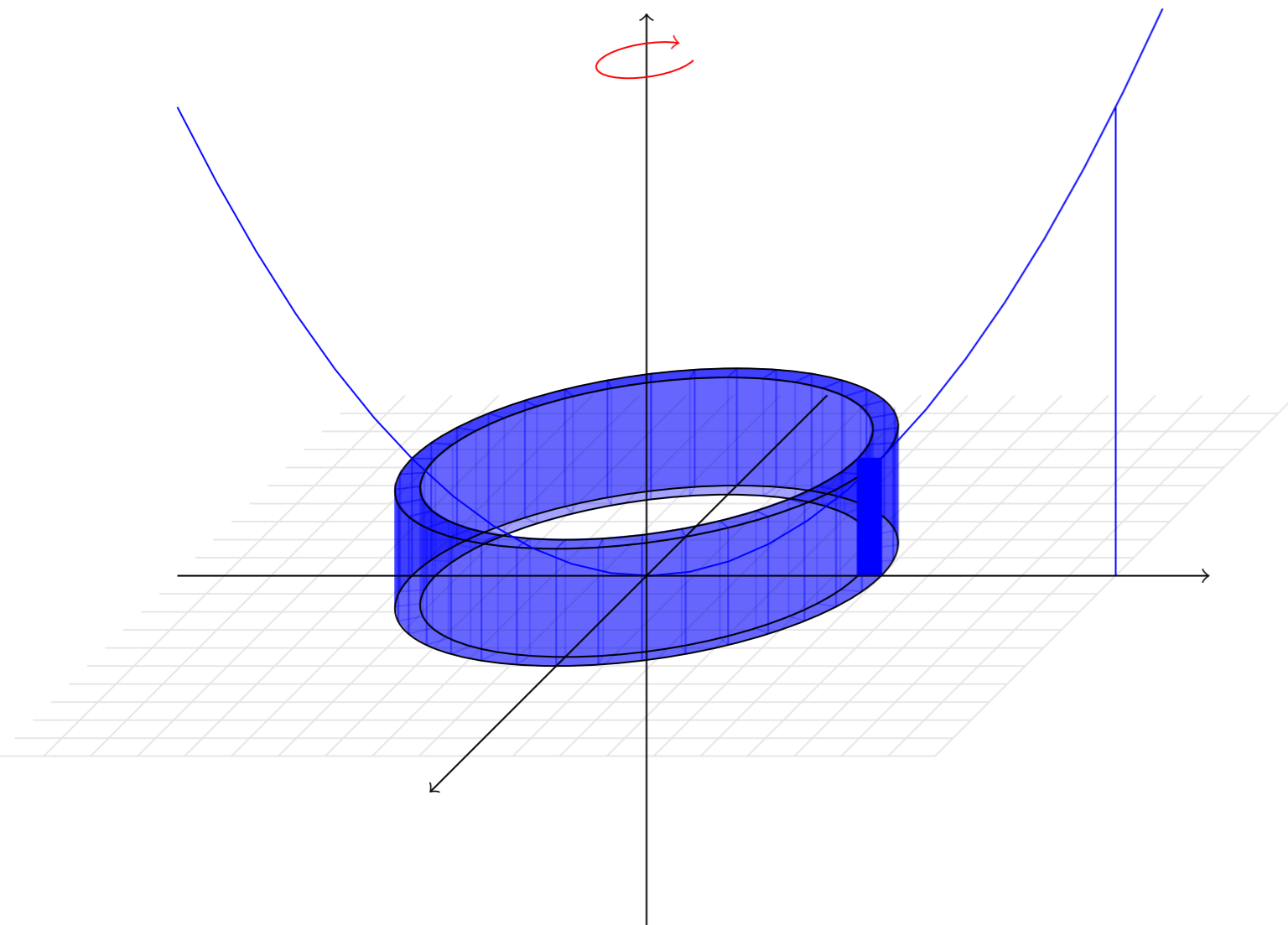
$$h = f(x)$$

$$\text{Vol}_{\text{tube}} = 2\pi x f(x) \Delta x$$



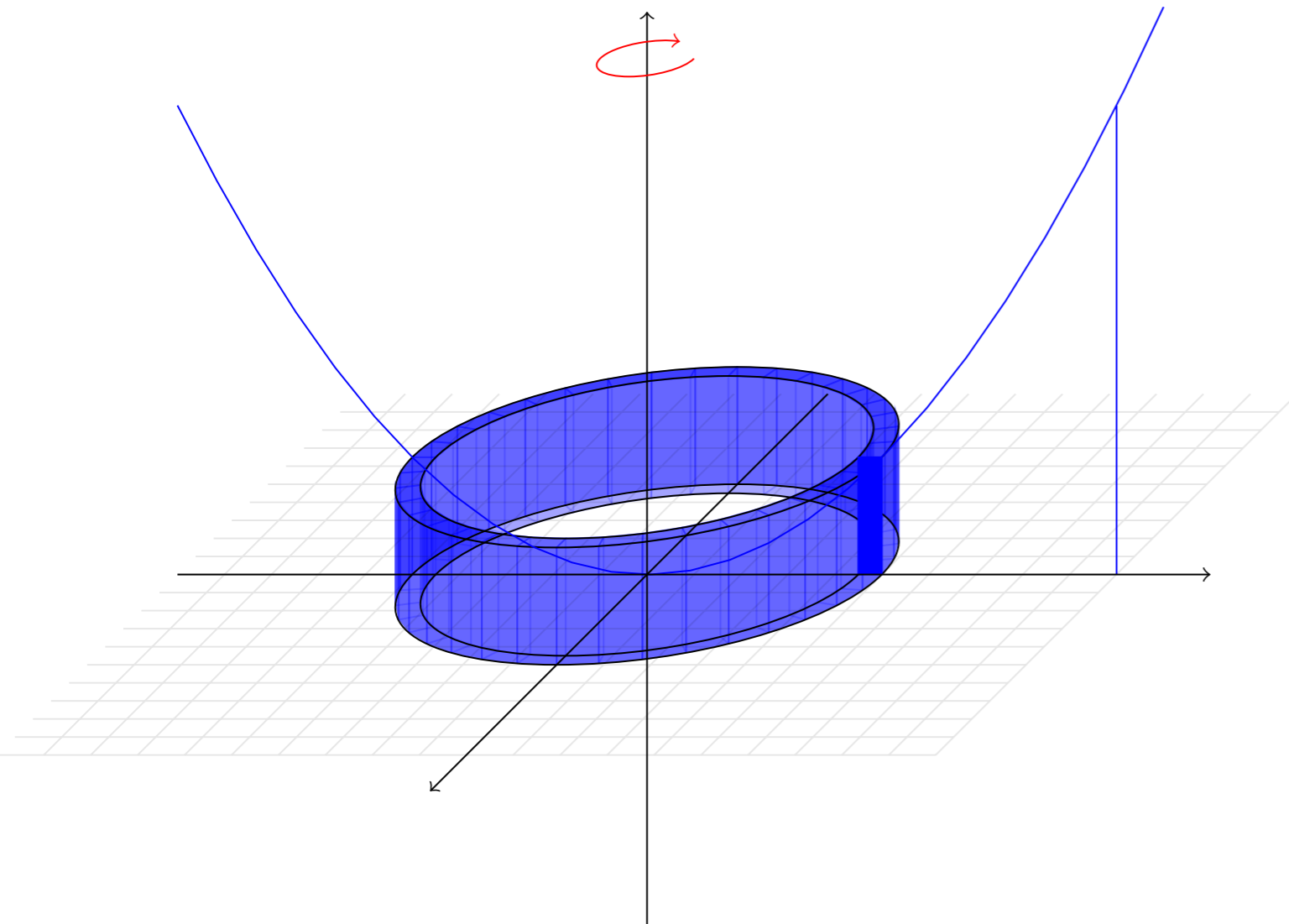
$$\text{Vol}_{\text{tube}} = 2\pi x f(x) \Delta x$$

$$\text{Vol}_{\text{tot}} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \text{Vol}_{\text{tube}_k}$$



$$\text{Vol}_{\text{tube}} = 2\pi x f(x) \Delta x$$

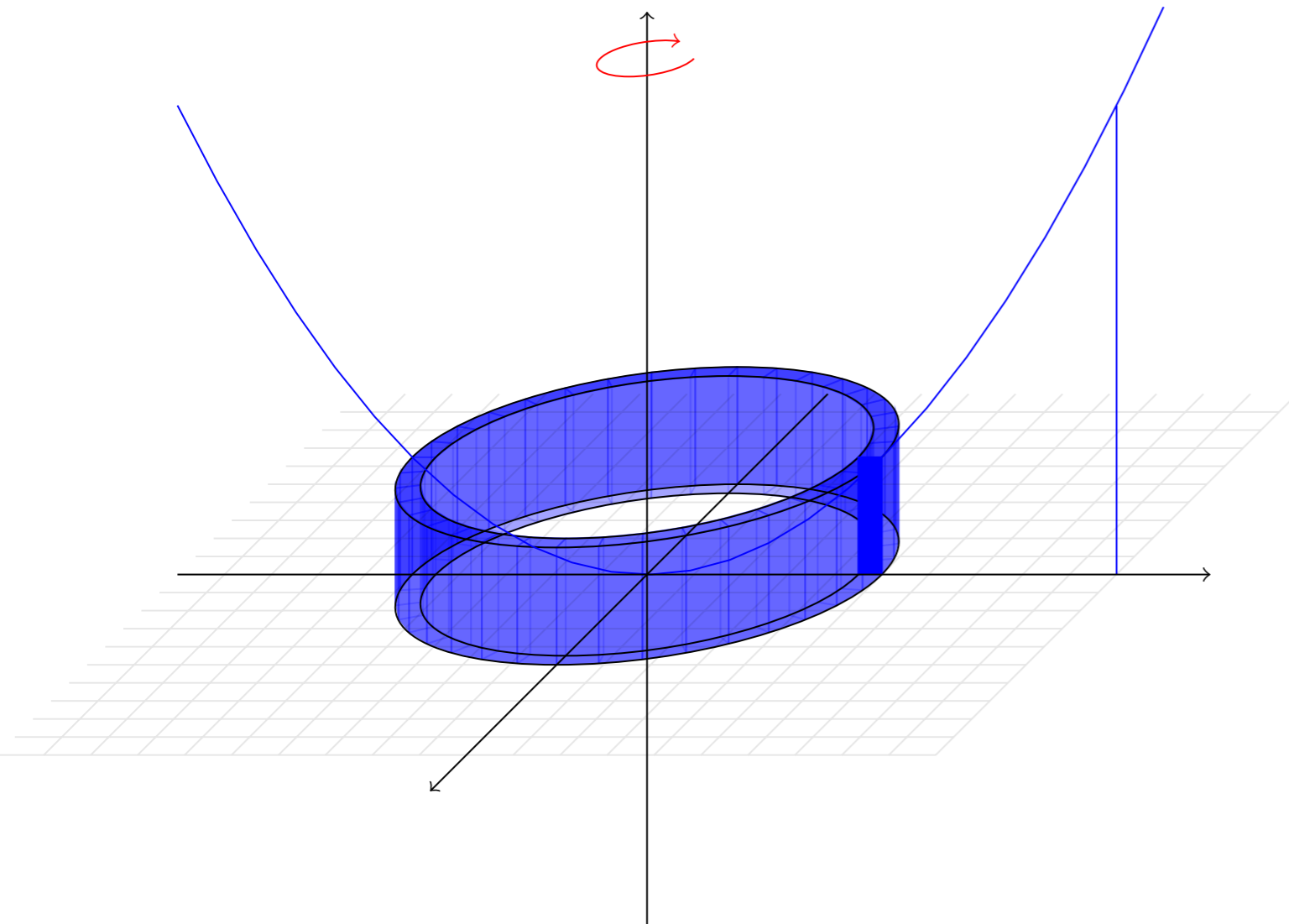
$$\text{Vol}_{\text{tot}} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \text{Vol}_{\text{tube}_k} = \lim_{n \rightarrow \infty} \sum_{k=1}^n 2\pi x f(x) \Delta x$$



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$$= \int_a^b 2\pi x f(x) dx$$

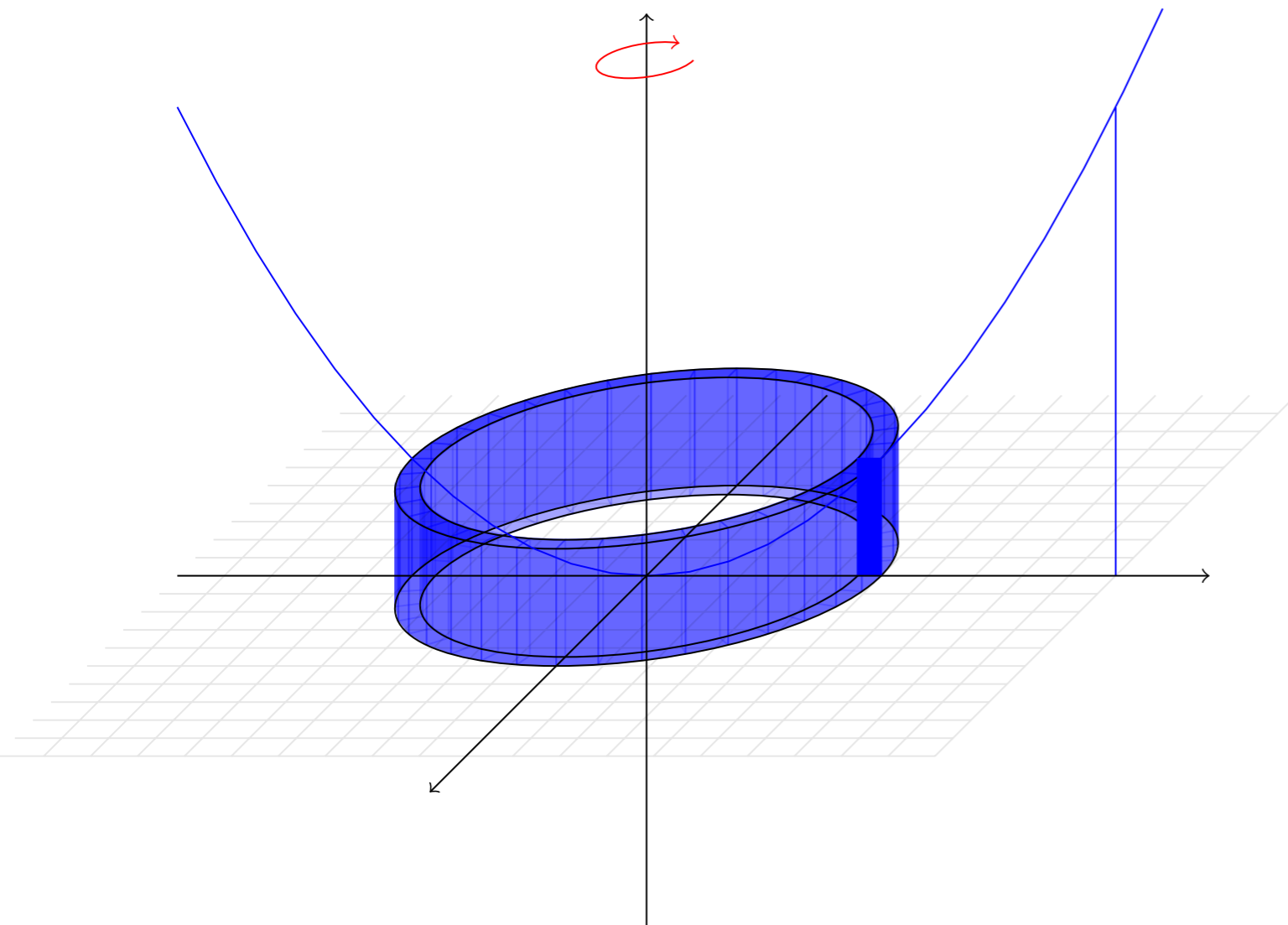


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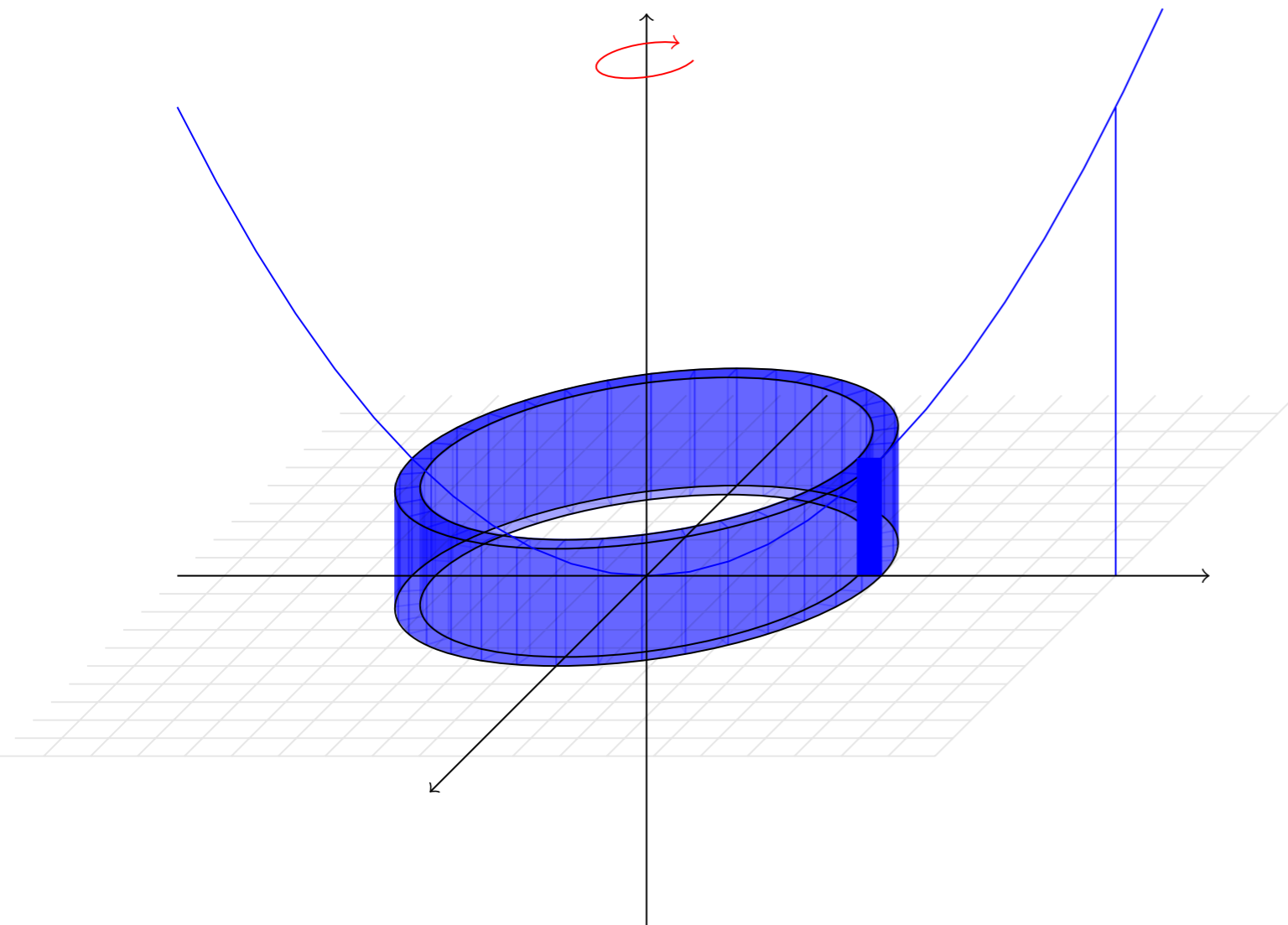
$$\text{Vol}_{\text{tot}} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \text{Vol}_{\text{tube}_k}$$

$$= \lim_{n \rightarrow \infty} \sum_{k=1}^n 2\pi x f(x) \Delta x$$

$$= \int_a^b 2\pi x f(x) dx$$

$$= 2\pi \int_a^b x f(x) dx$$

$$= 2\pi \int_a^b Rh dx$$

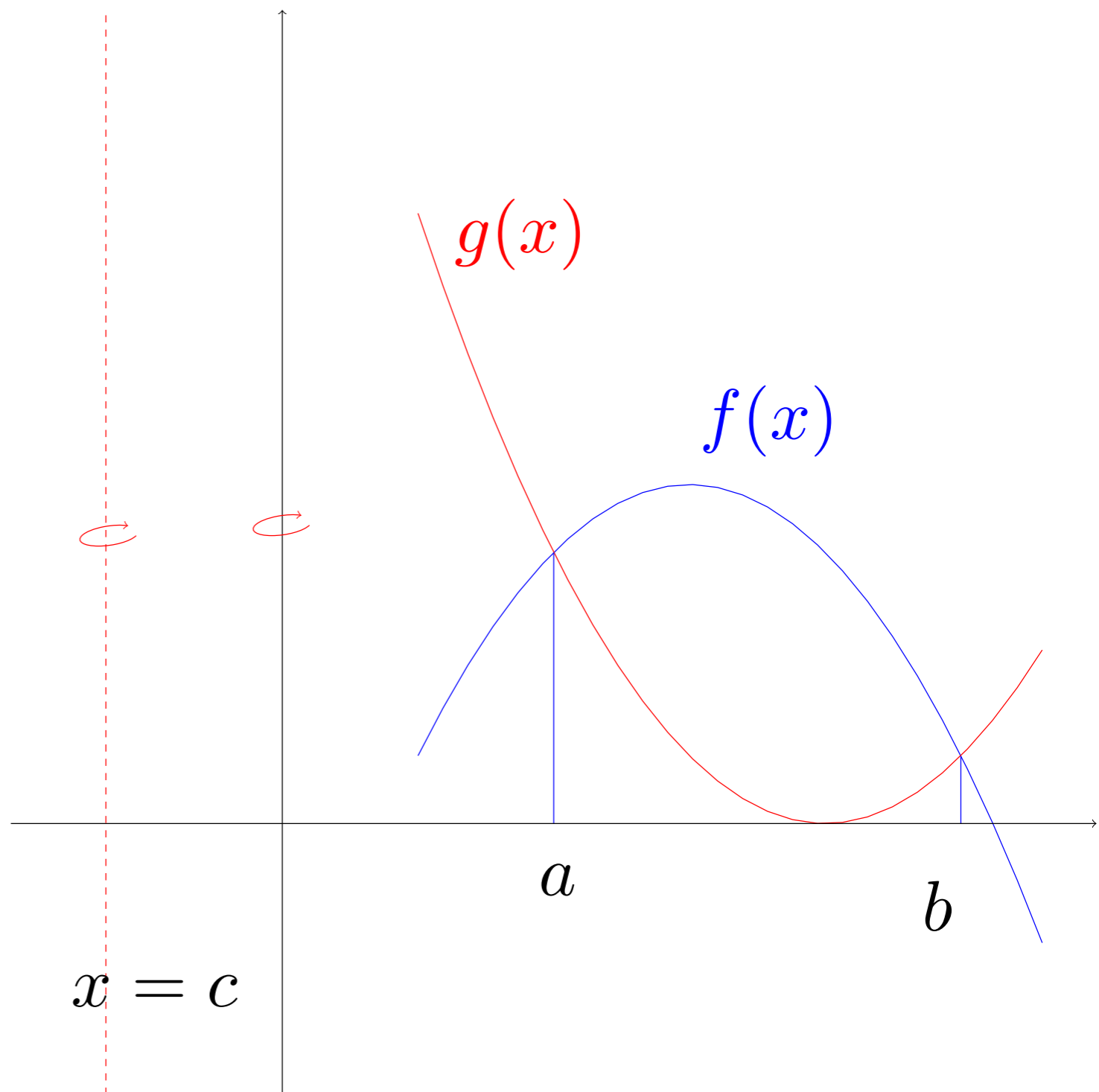


Faites les exercices suivants

Déterminer l'intégrale permettant de calculer le volume, à l'aide de la méthode des tubes, du solide obtenu en faisant

1) tourner la région entre les deux fonction autour de l'axe des y

2) tourner la région entre les deux fonction autour de l'axe $x = c$

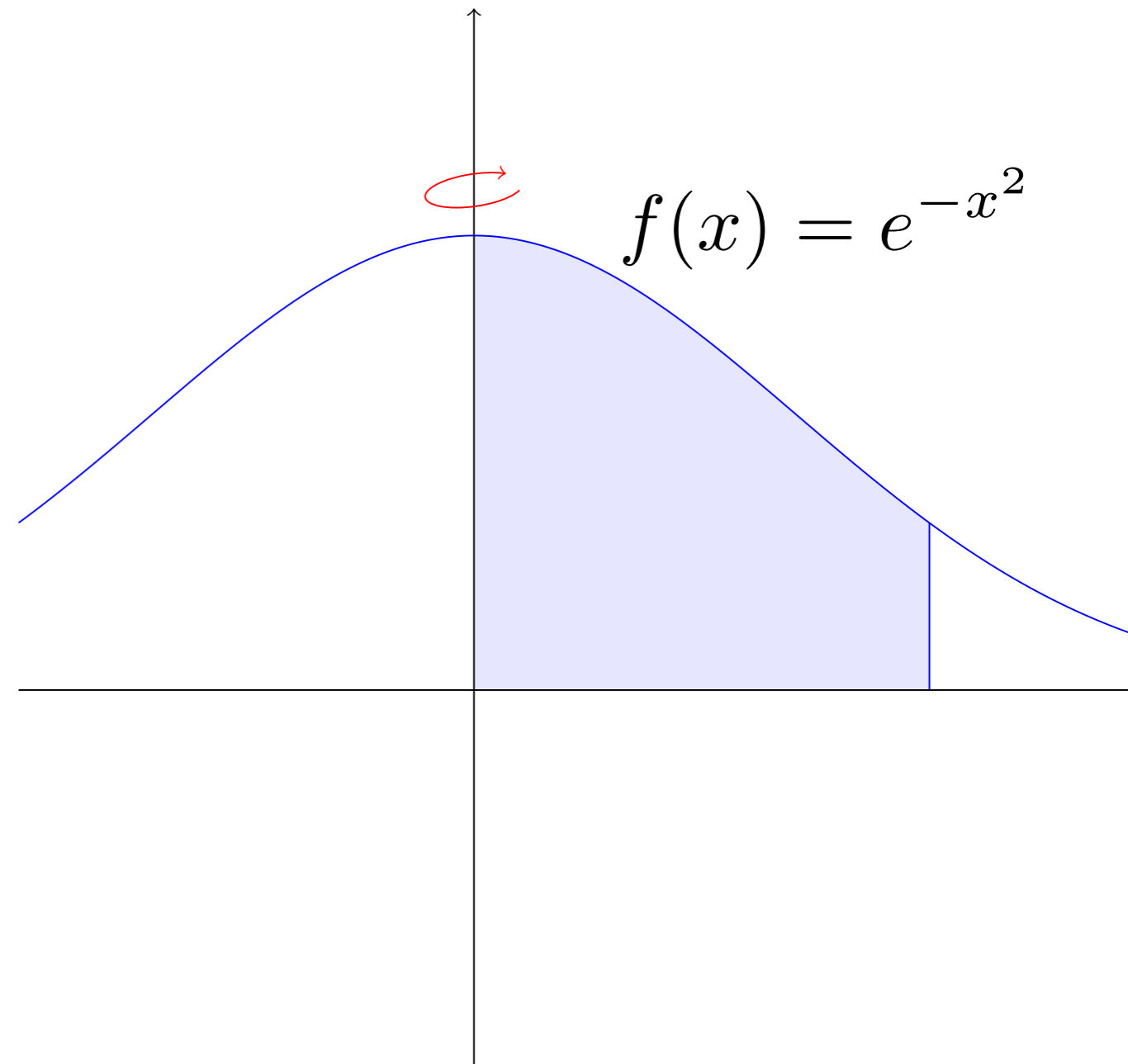


Exemple

Calculer le volume du solide obtenu en faisant tourner la fonction $f(x) = e^{-x^2}$ autour de l'axe des y .

Exemple

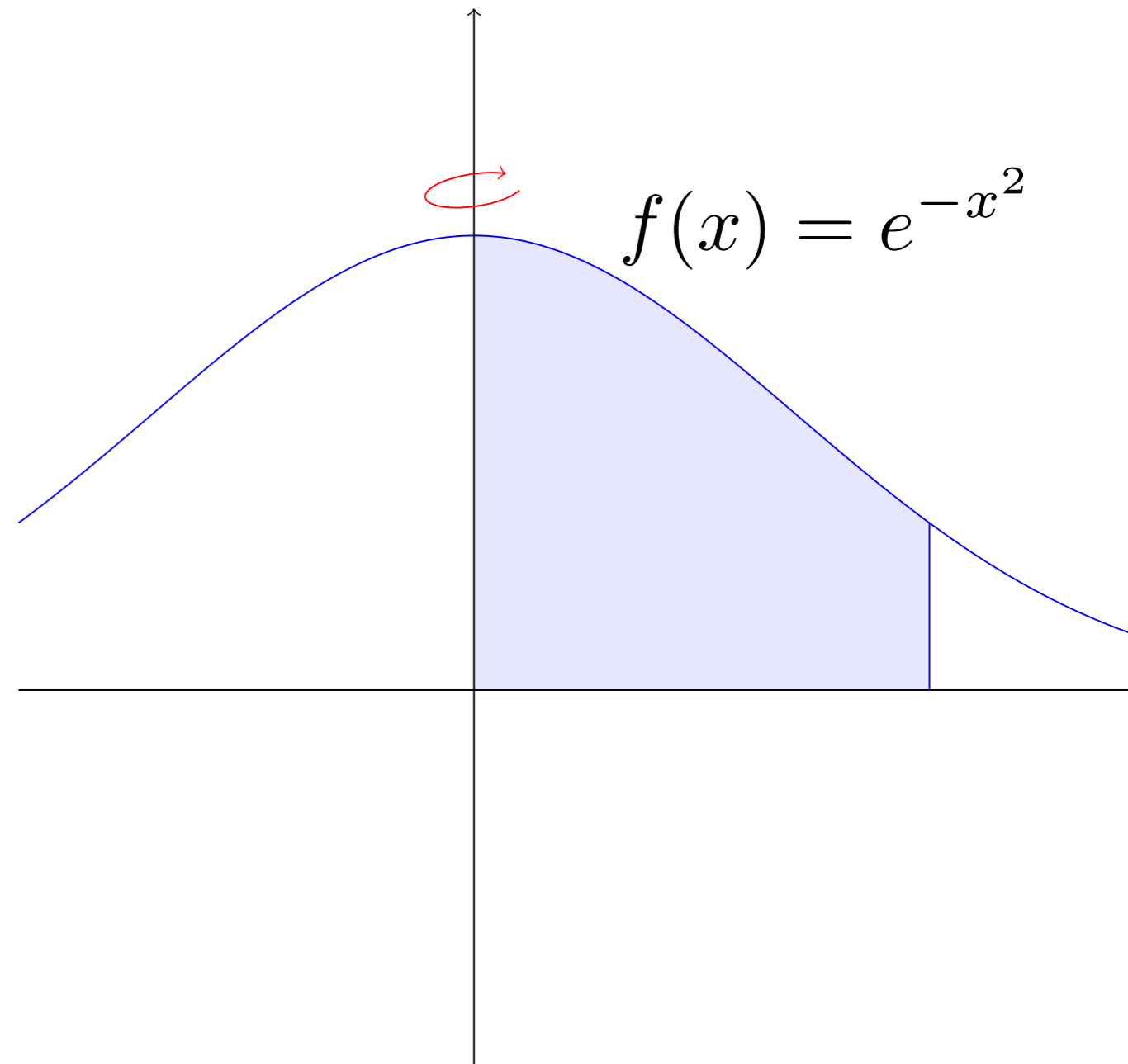
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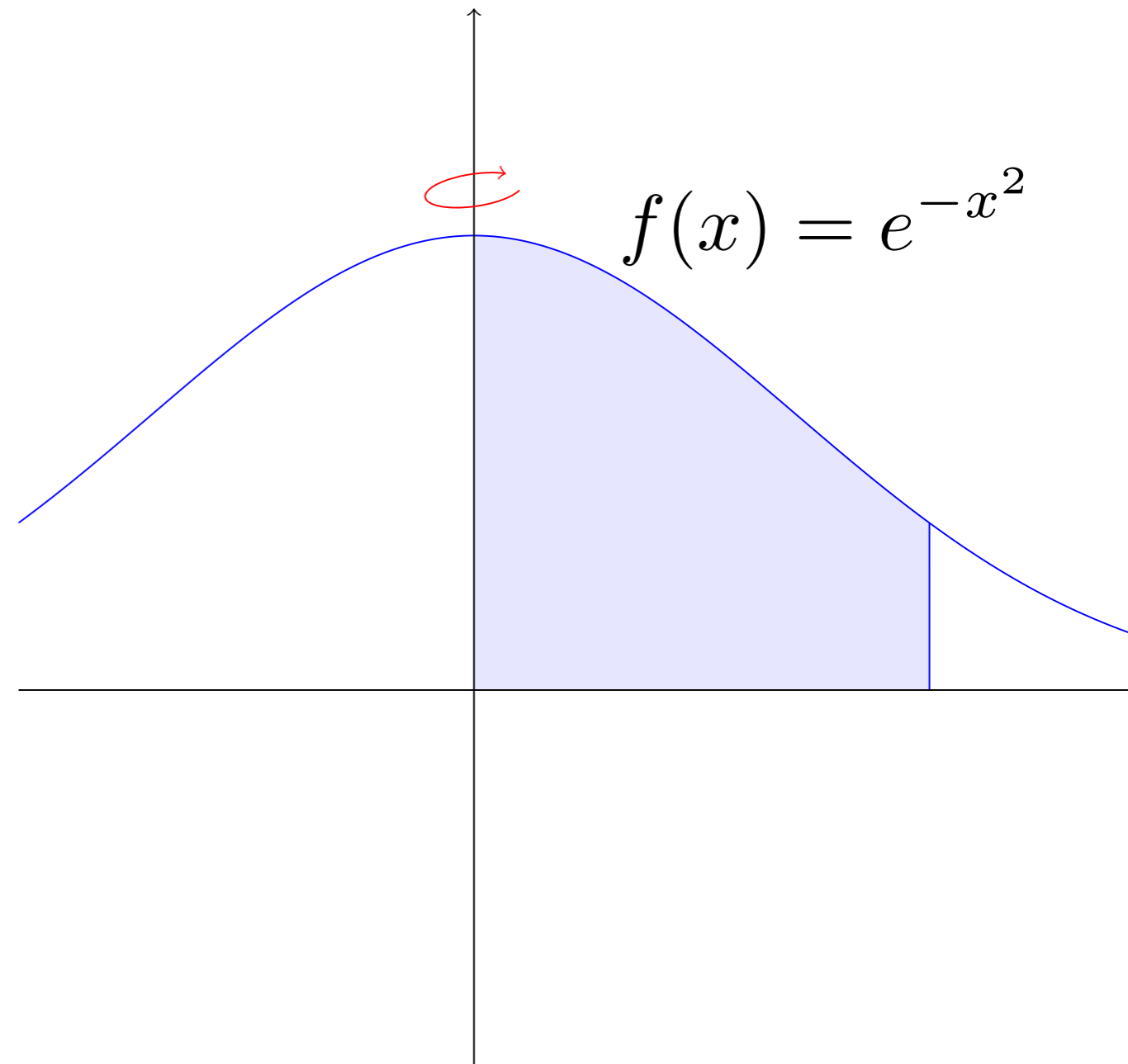
$$\int_0^1 2\pi R h \, dx$$



Exemple

Calculer le volume du solide obtenu en faisant tourner la fonction $f(x) = e^{-x^2}$ autour de l'axe des y .

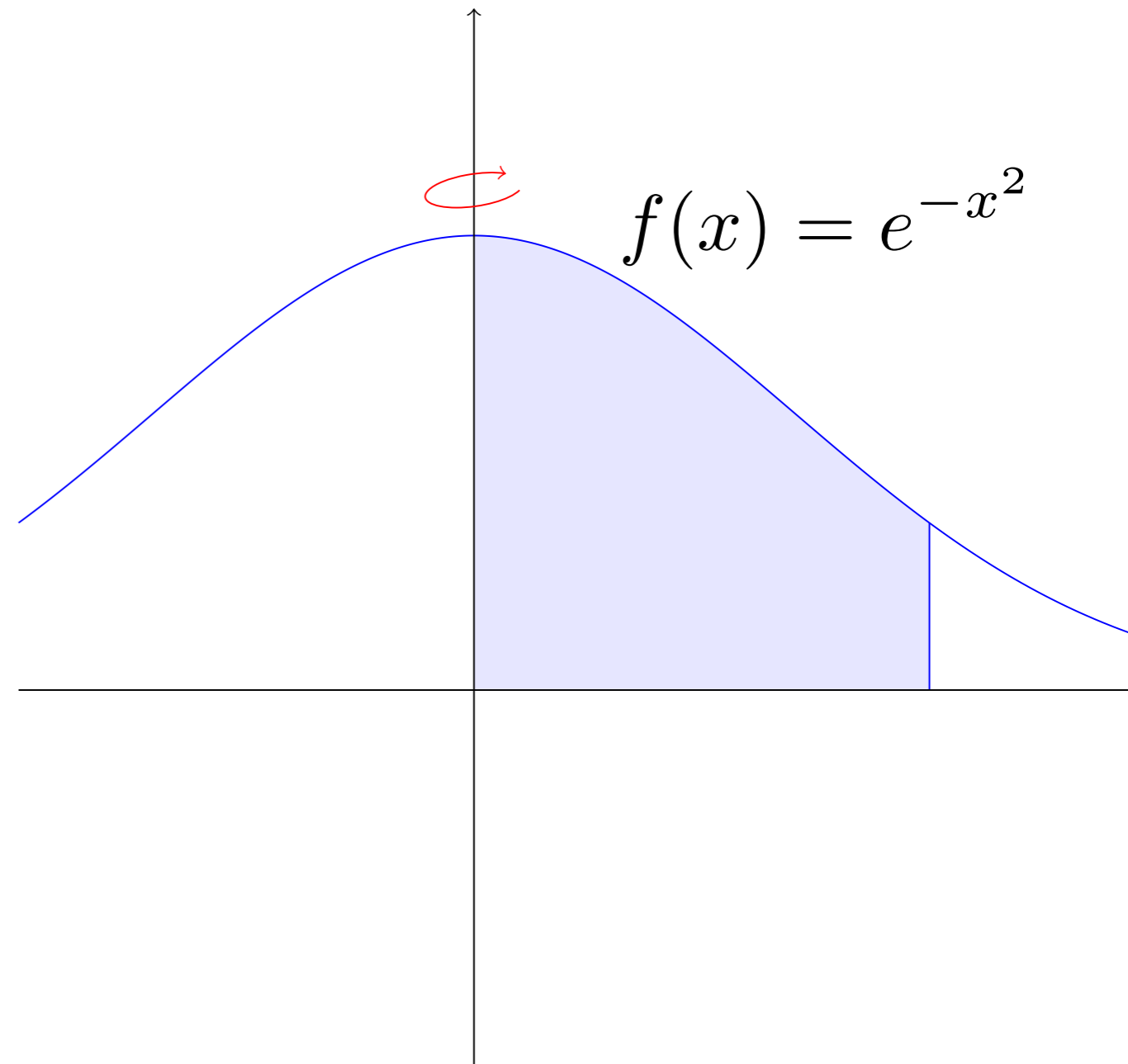
$$\int_0^1 2\pi R h \, dx = 2\pi \int_0^1 x e^{-x^2} \, dx$$



Exemple

Calculer le volume du solide obtenu en faisant tourner la fonction $f(x) = e^{-x^2}$ autour de l'axe des y .

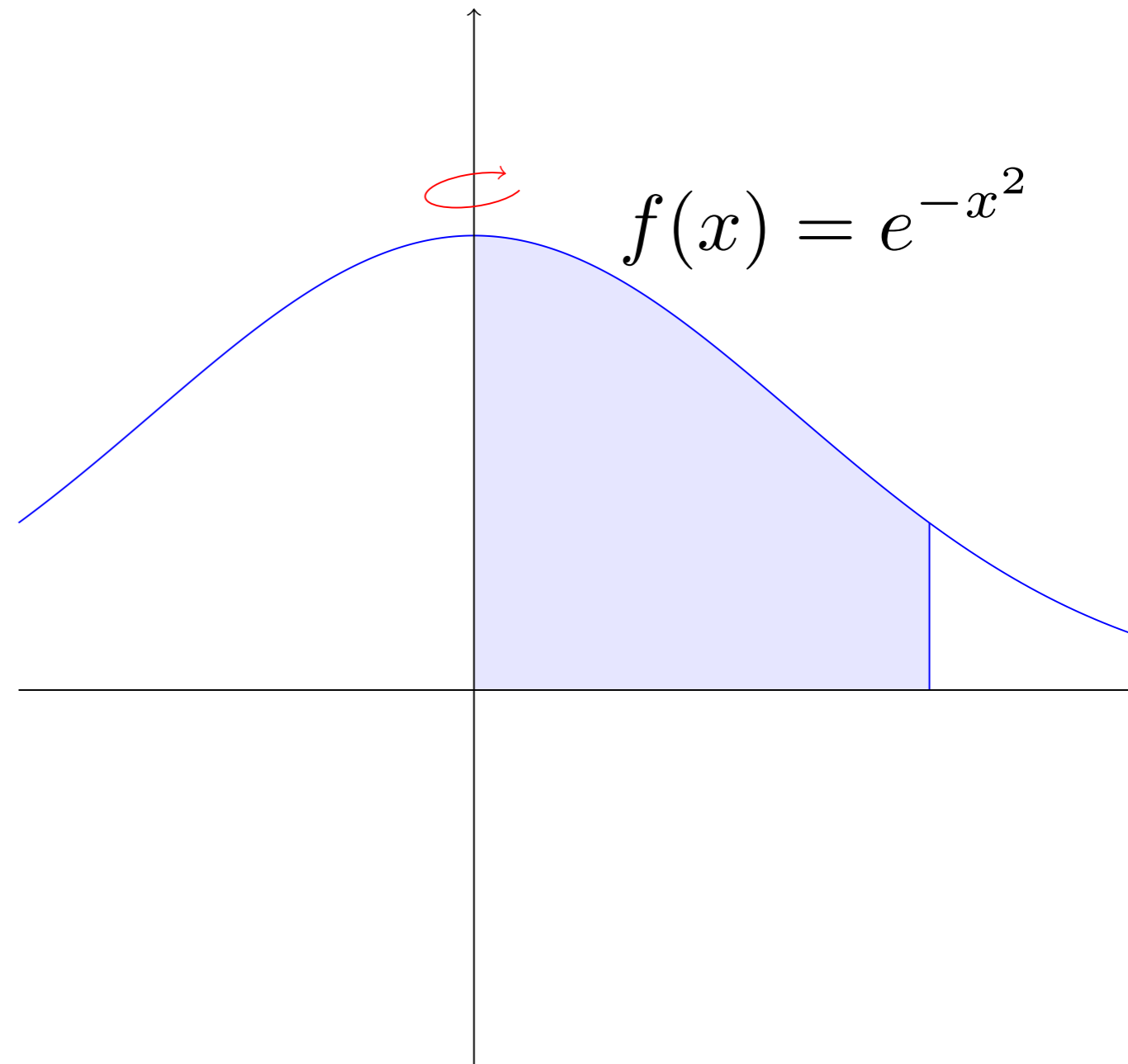
$$\int_0^1 2\pi R h \, dx = 2\pi \int_0^1 x e^{-x^2} \, dx \quad u = -x^2$$



Exemple

Calculer le volume du solide obtenu en faisant tourner la fonction $f(x) = e^{-x^2}$ autour de l'axe des y .

$$\int_0^1 2\pi R h \, dx = 2\pi \int_0^1 x e^{-x^2} \, dx \quad u = -x^2 \quad du = -2x \, dx$$

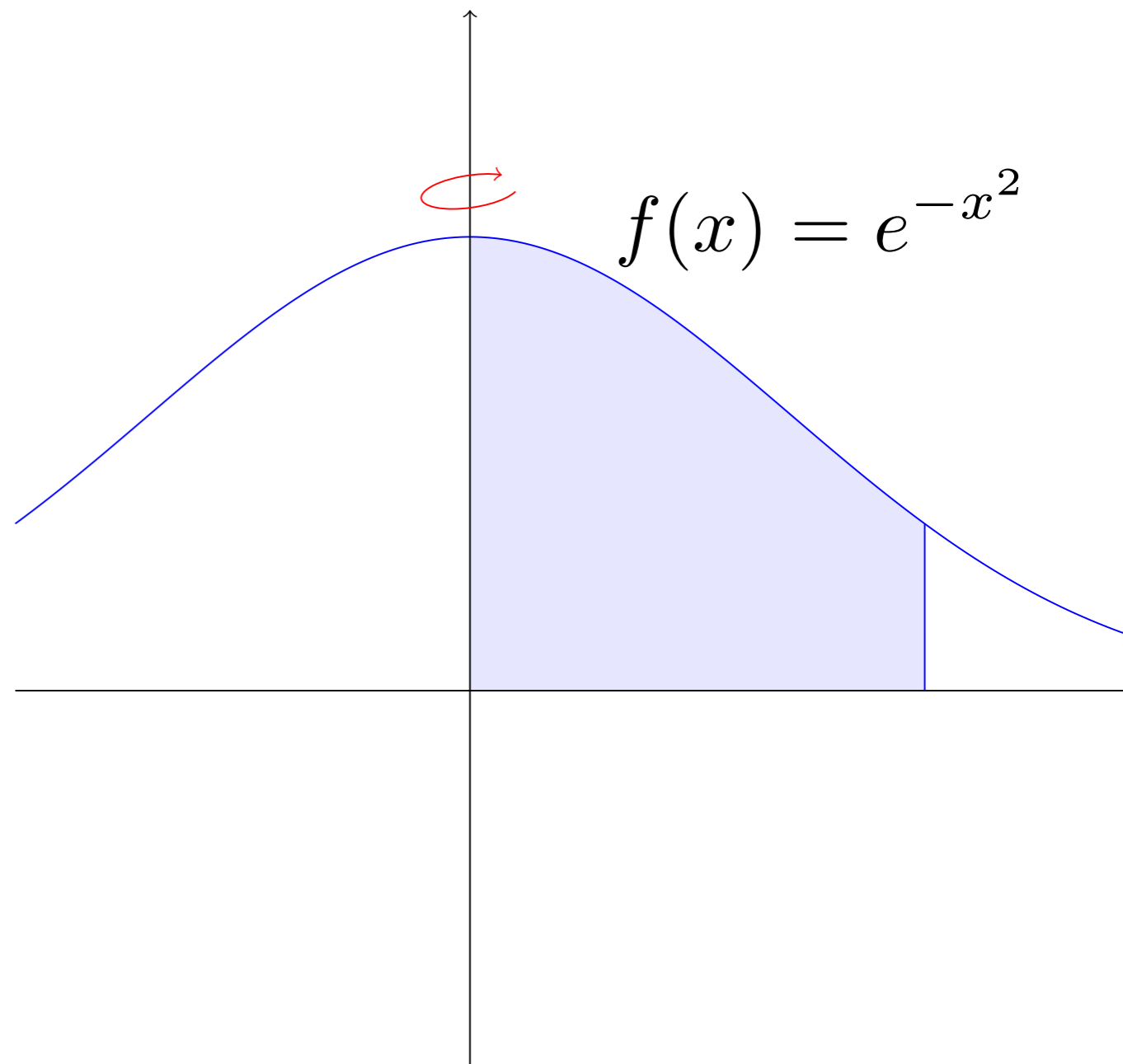


Exemple

Calculer le volume du solide obtenu en faisant tourner la fonction $f(x) = e^{-x^2}$ autour de l'axe des y.

$$\int_0^1 2\pi R h \, dx = 2\pi \int_0^1 x e^{-x^2} \, dx \quad u = -x^2 \quad du = -2x \, dx$$

$$= -\pi \int_{?}^{?} e^u \, du$$

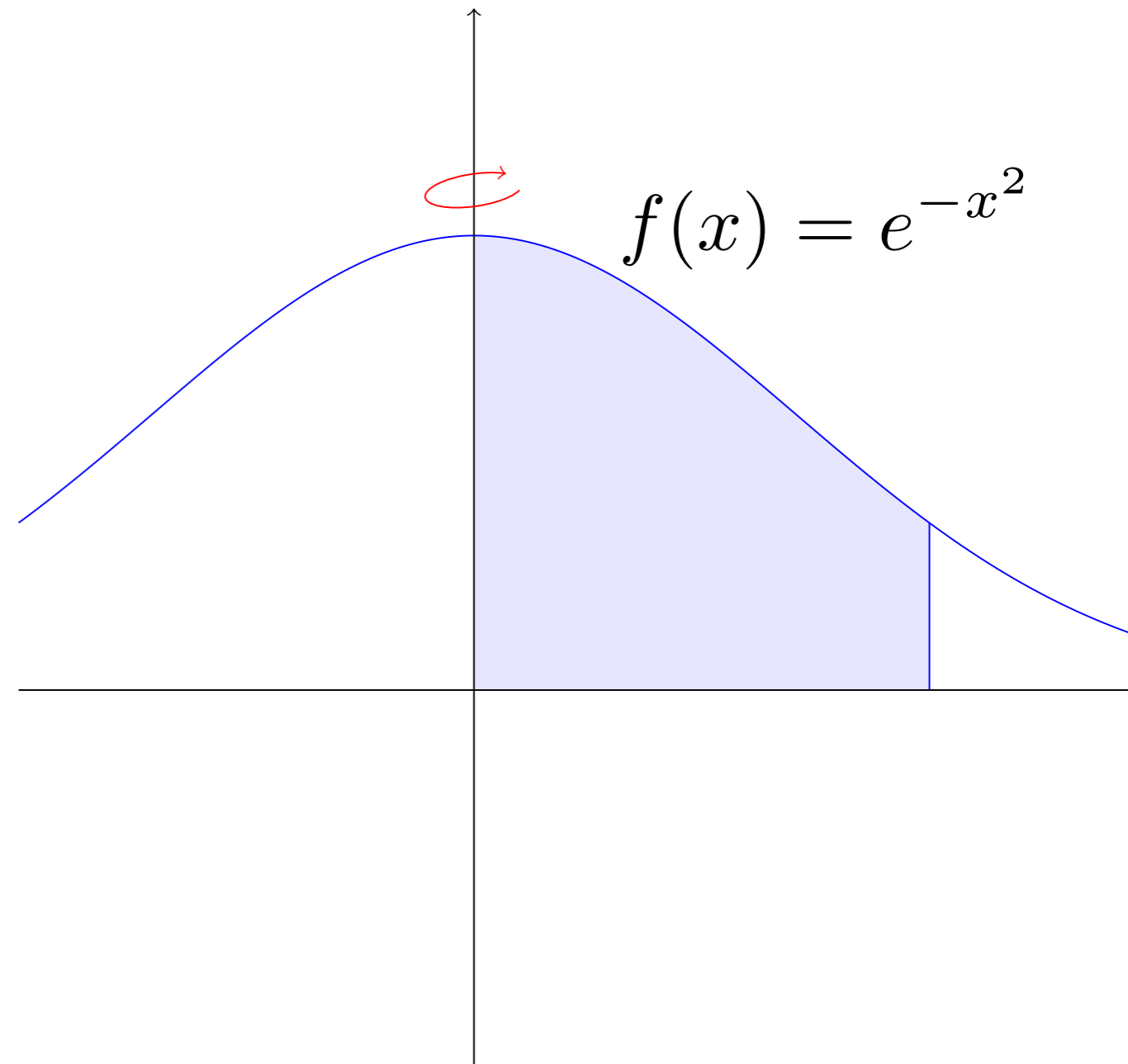


Exemple

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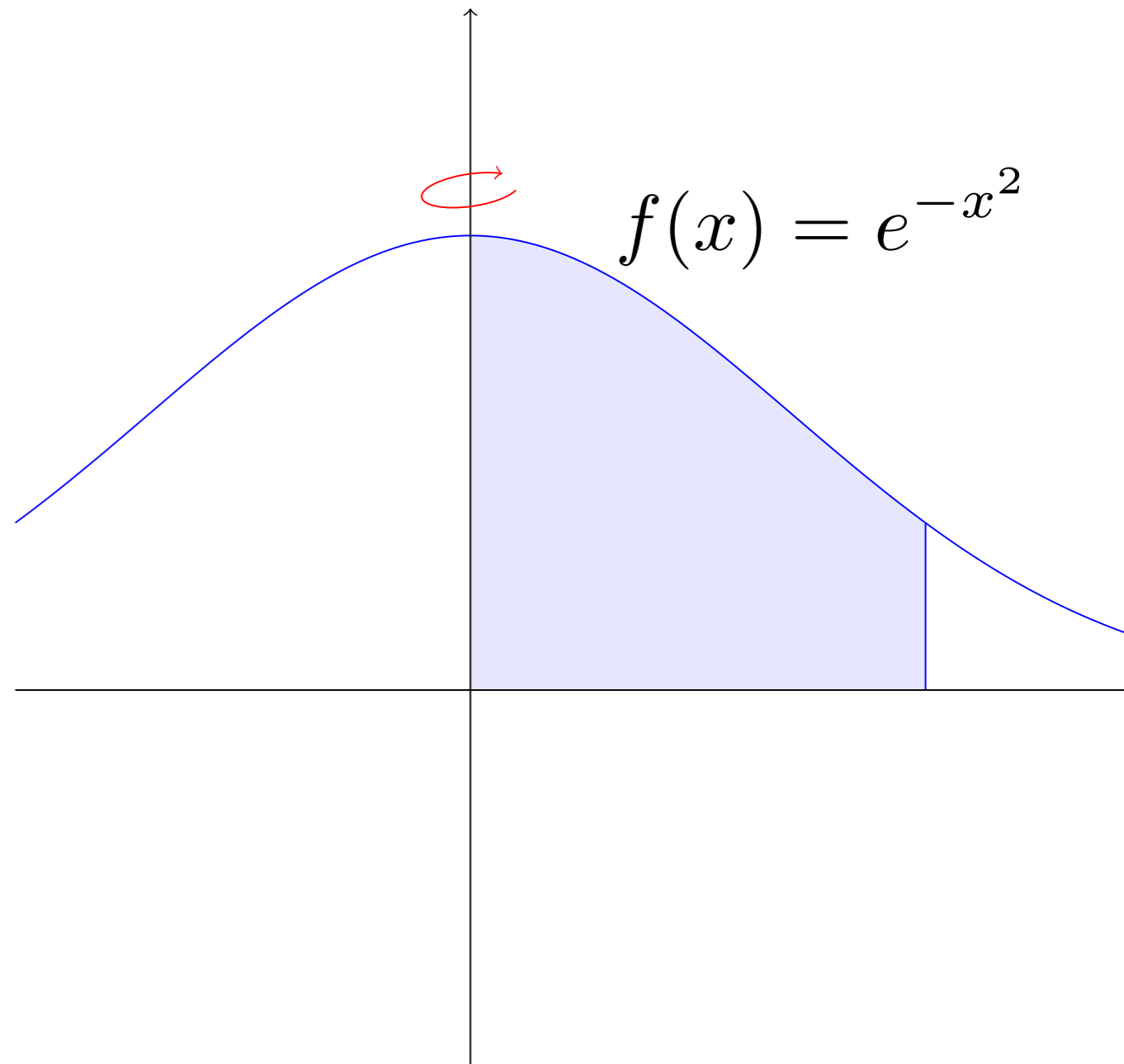
Exemple

Calculer le volume du solide obtenu en faisant tourner la fonction $f(x) = e^{-x^2}$ autour de l'axe des y.

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$$= -\pi e^{-1} + \pi e^0$$



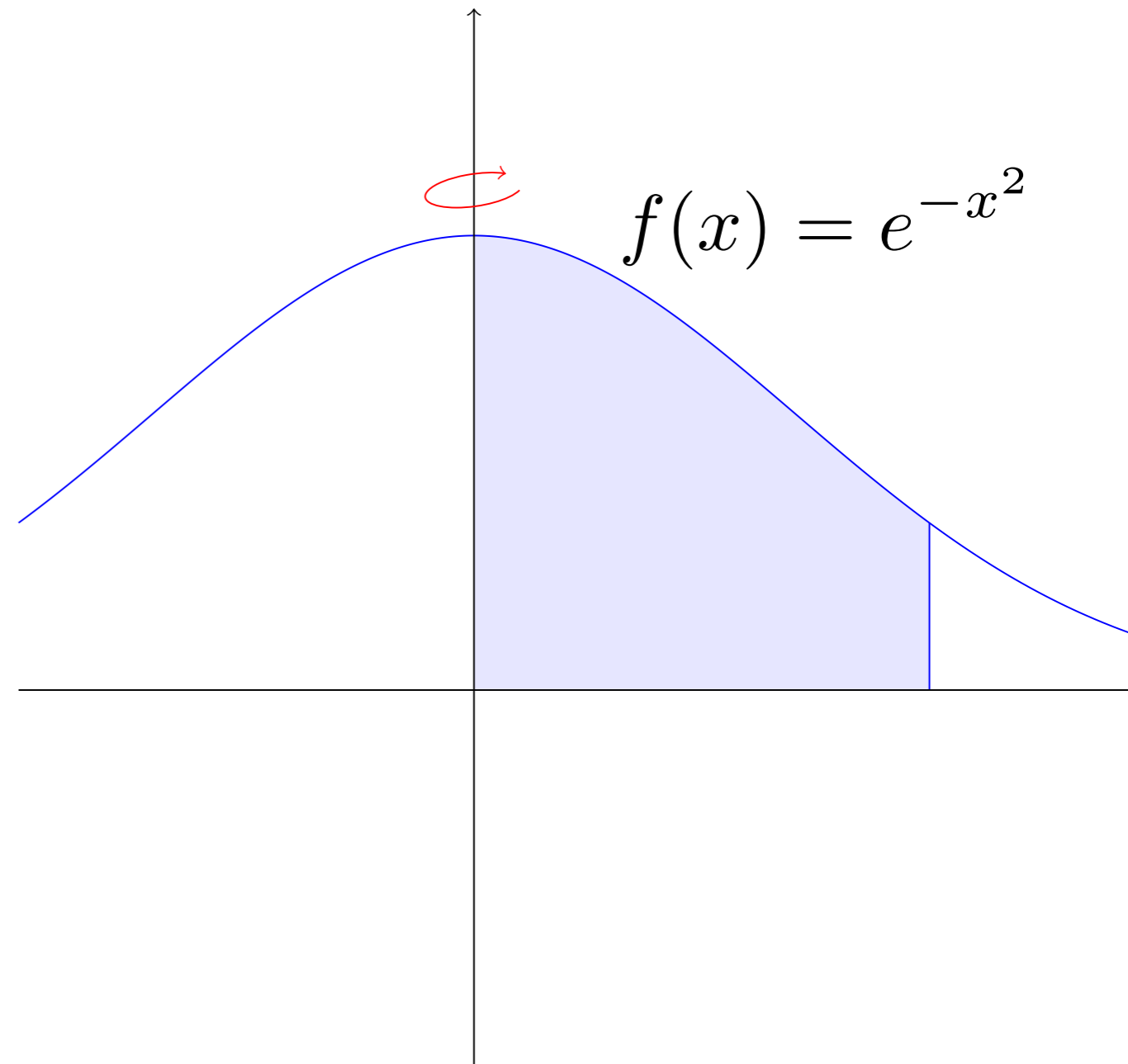
Exemple

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$$= -\pi \int_{?}^{?} e^u \, du = -\pi e^{-x^2} \Big|_0^1$$

$$= -\pi e^{-1} + \pi e^0 = \pi - \frac{\pi}{e}$$



Faites les exercices suivants

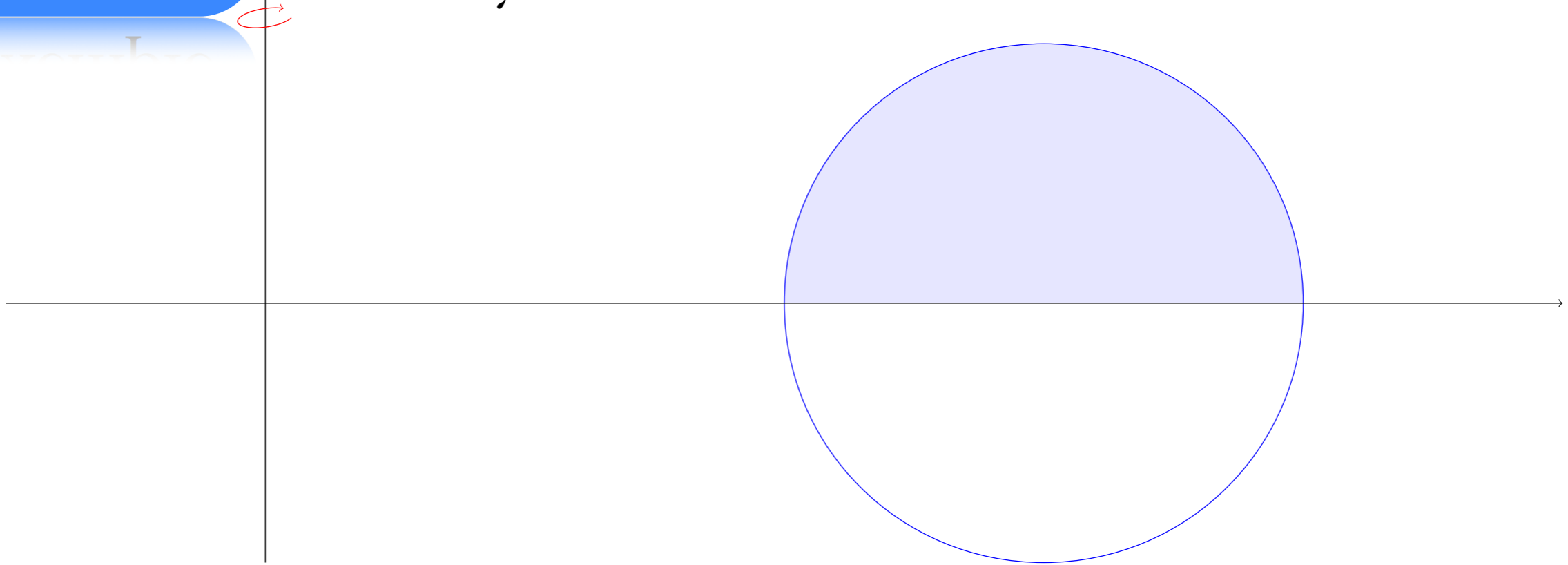
Section 3 #7

Exemple

Calculer le volume d'un beigne de rayon de pâte r et de rayon de «trou» R

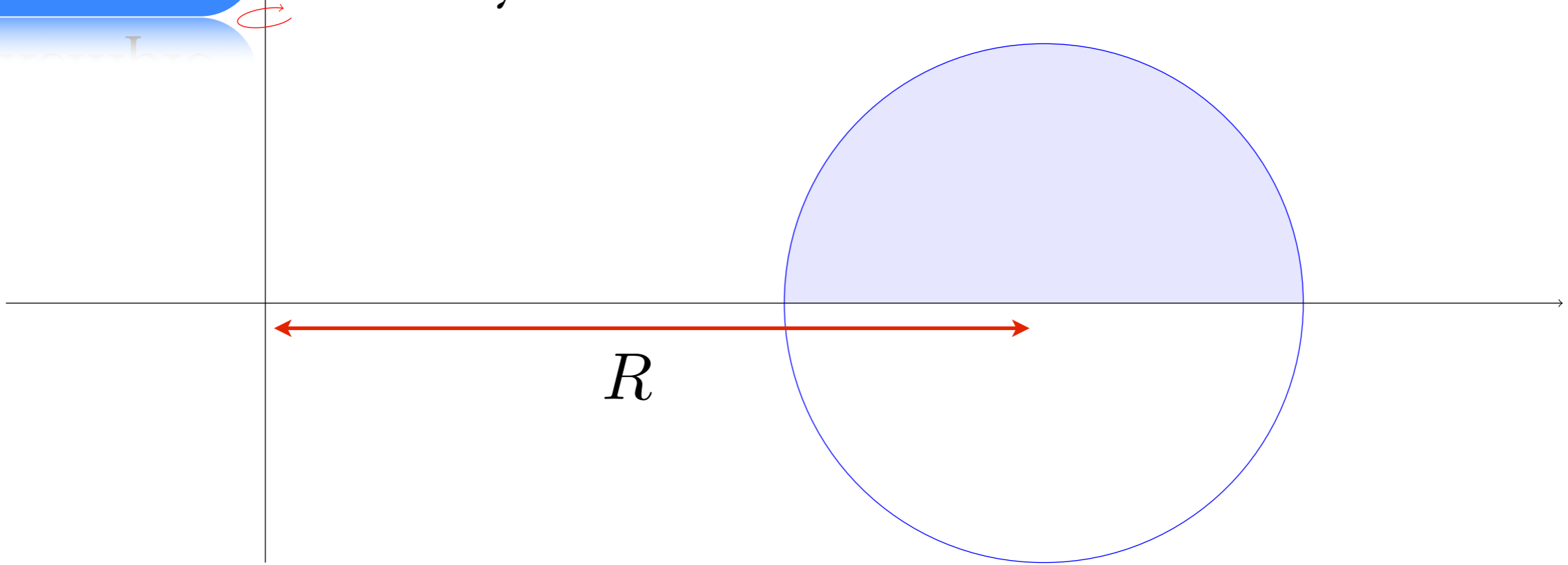
Exemple

Calculer le volume d'un beigne de rayon de pâte r
et de rayon de «trou» R



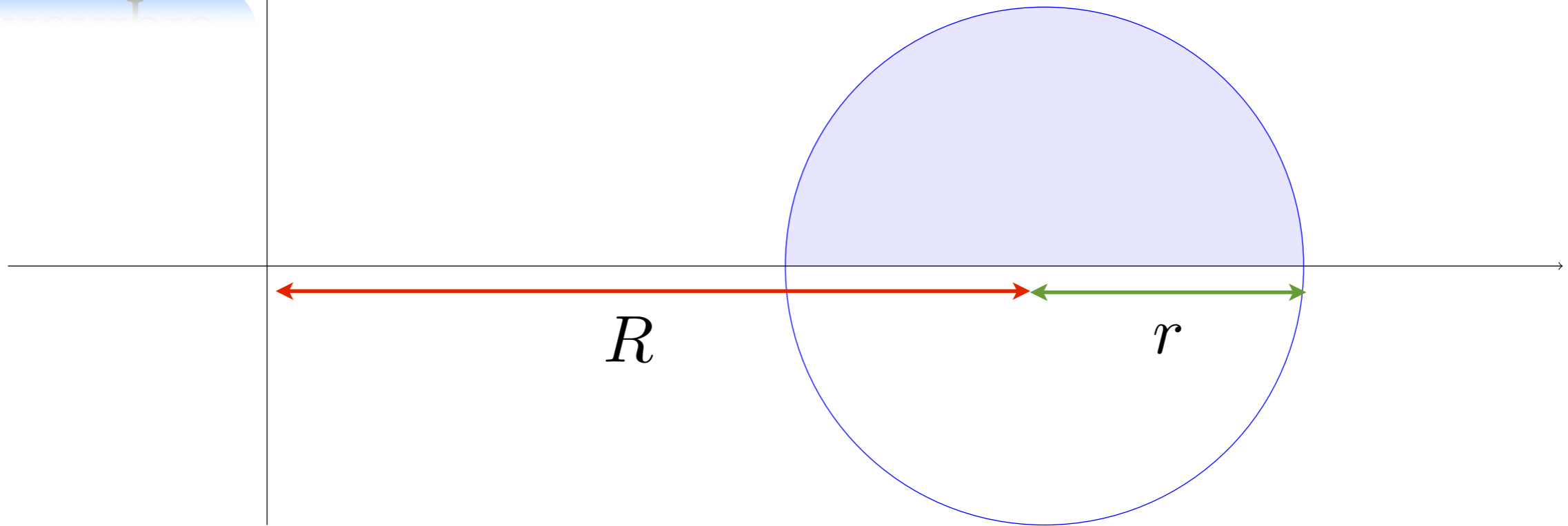
Exemple

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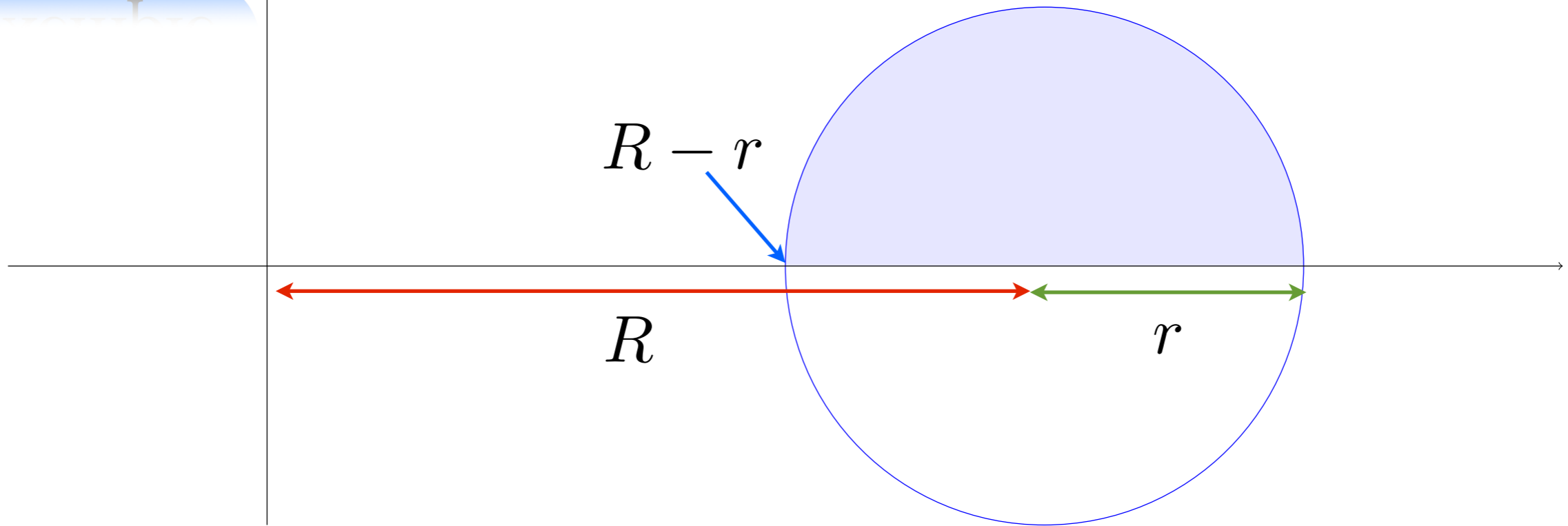
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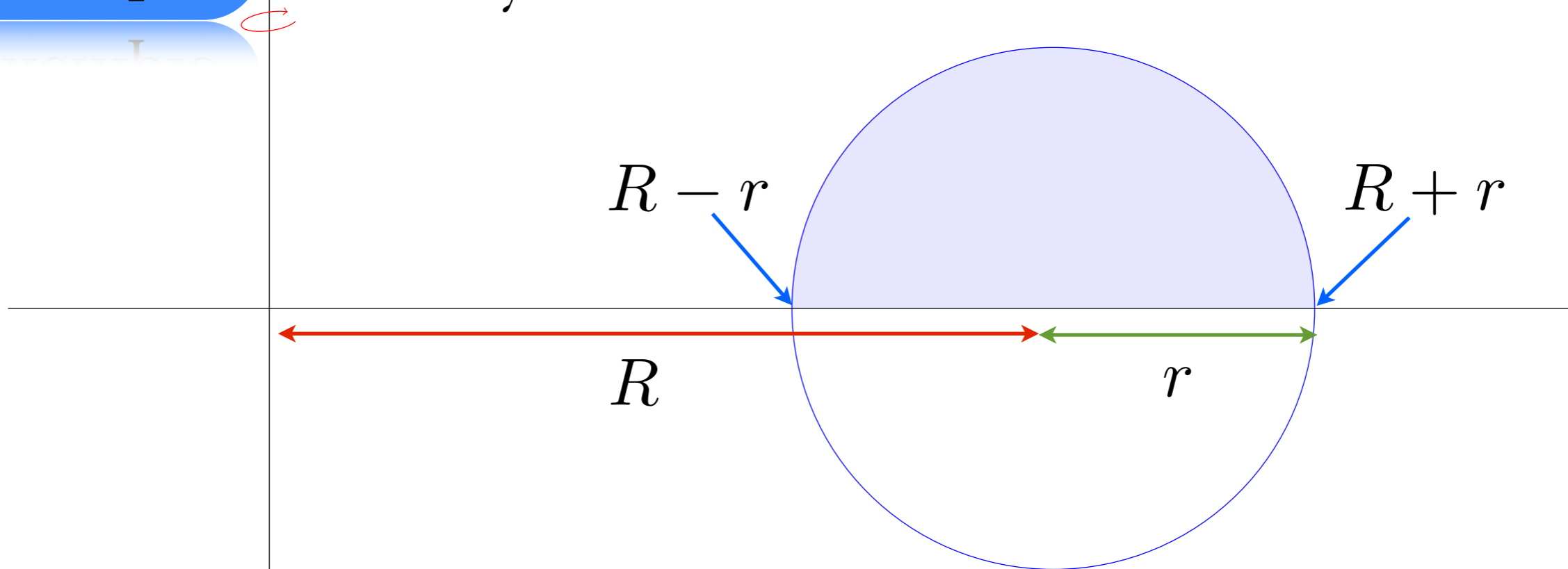
Exemple

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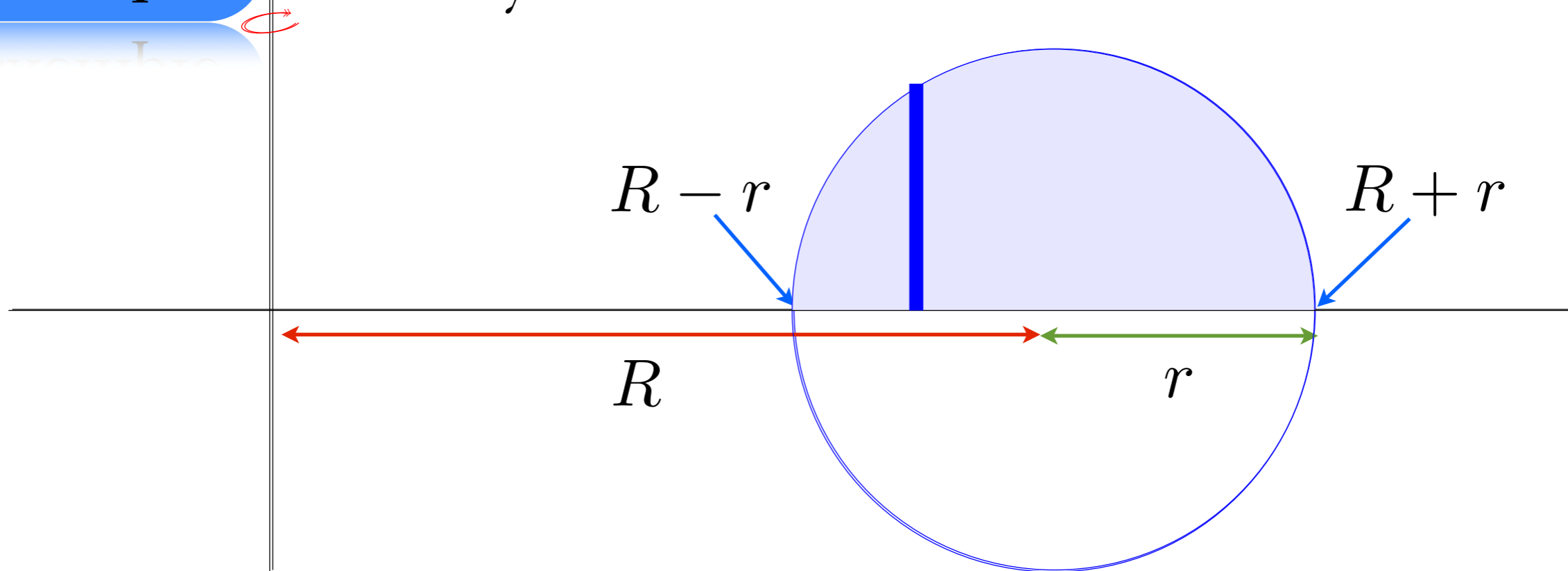
Exemple

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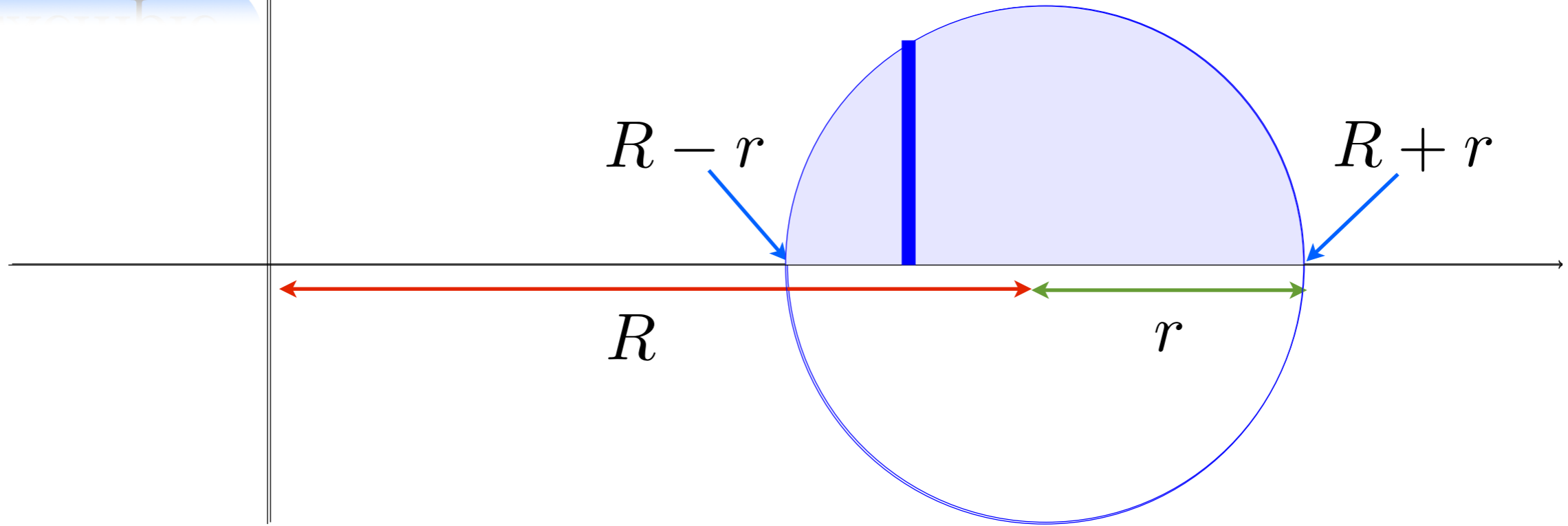
Exemple

Calculer le volume d'un beigne de rayon de pâte r et de rayon de «trou» R



Exemple

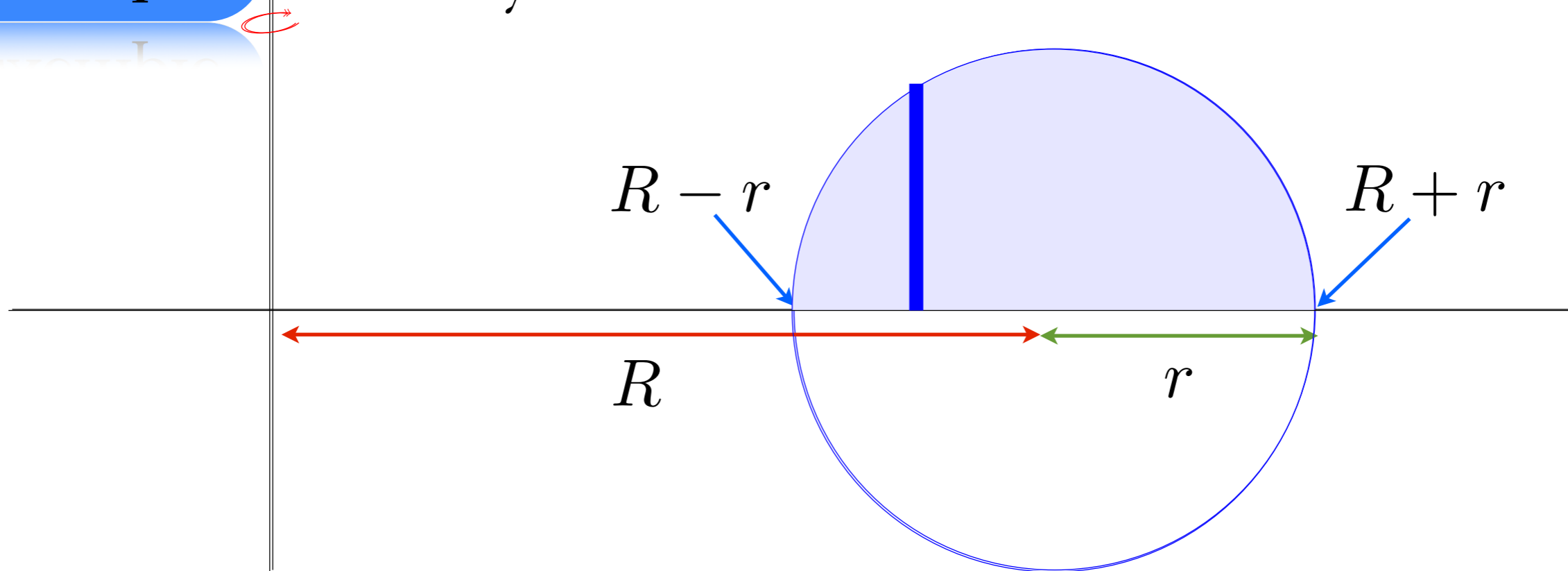
Calculer le volume d'un beigne de rayon de pâte r et de rayon de «trou» R



$\text{Vol}_{\frac{1}{2} \text{ beigne}}$

Exemple

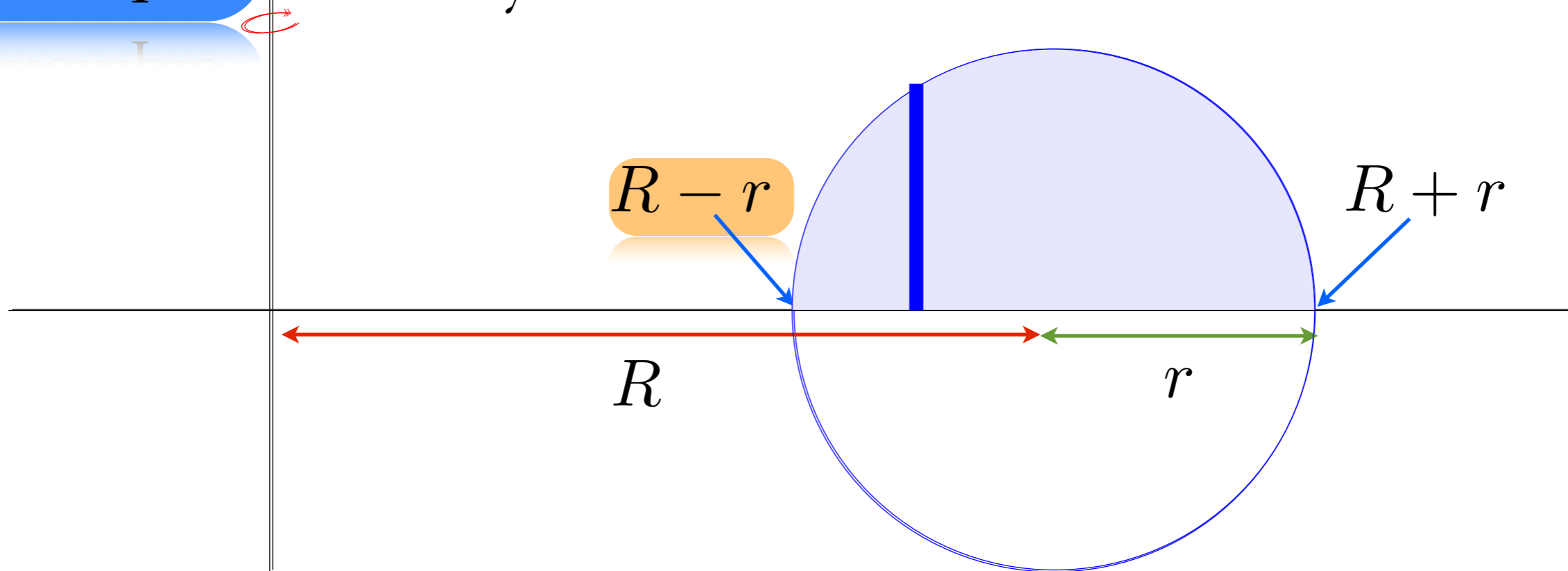
Calculer le volume d'un beigne de rayon de pâte r et de rayon de «trou» R



$$\text{Vol}_{\frac{1}{2} \text{ beigne}} = 2\pi \int_{R-r}^{R+r} x f(x) dx$$

Exemple

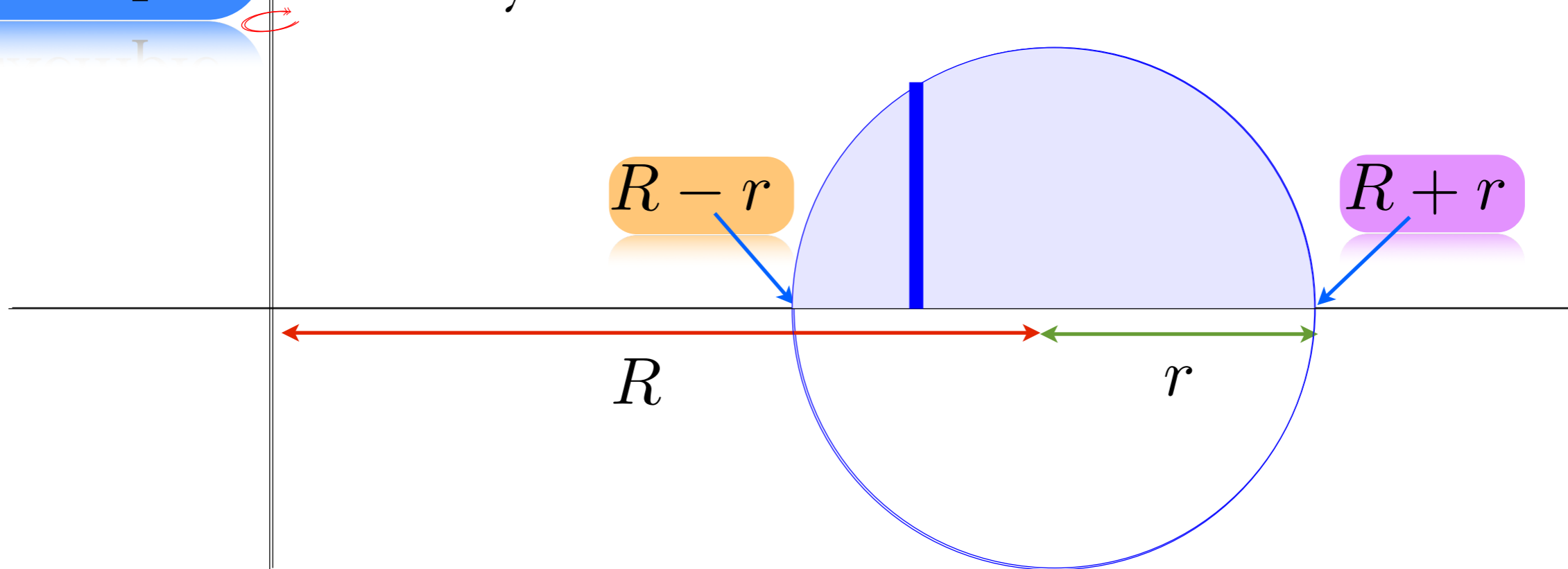
Calculer le volume d'un beigne de rayon de pâte r et de rayon de «trou» R



$$\text{Vol}_{\frac{1}{2} \text{ beigne}} = 2\pi \int_{R-r}^{R+r} x f(x) dx$$

Exemple

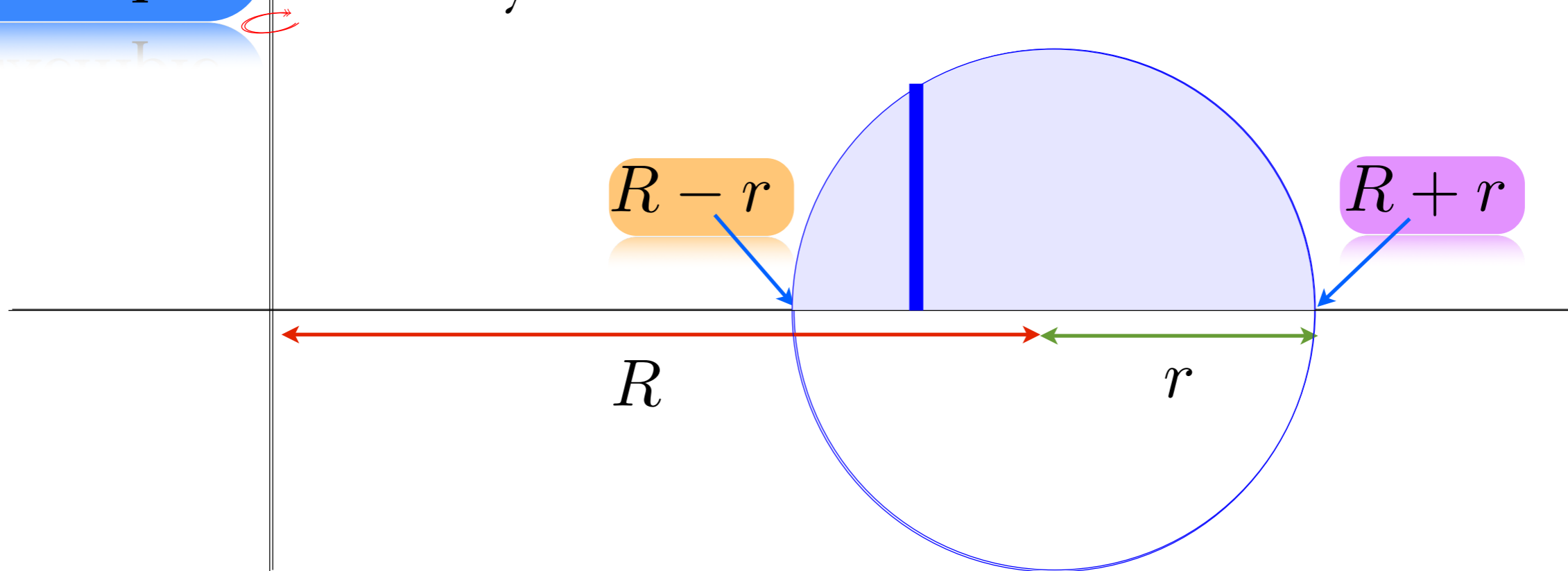
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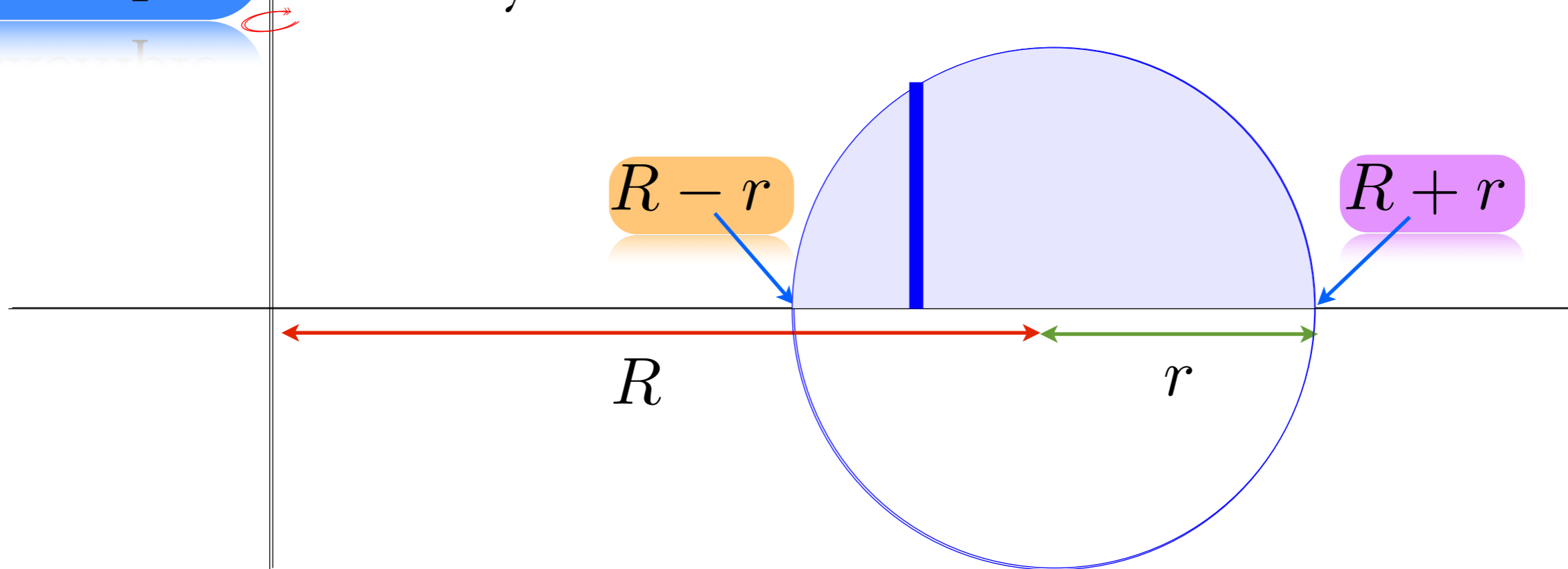


$$(x - R)^2 + y^2 = r^2$$

$$\text{Vol}_{\frac{1}{2} \text{ beigne}} = 2\pi \int_{R-r}^{R+r} x f(x) dx$$

Exemple

Calculer le volume d'un beigne de rayon de pâte r et de rayon de «trou» R

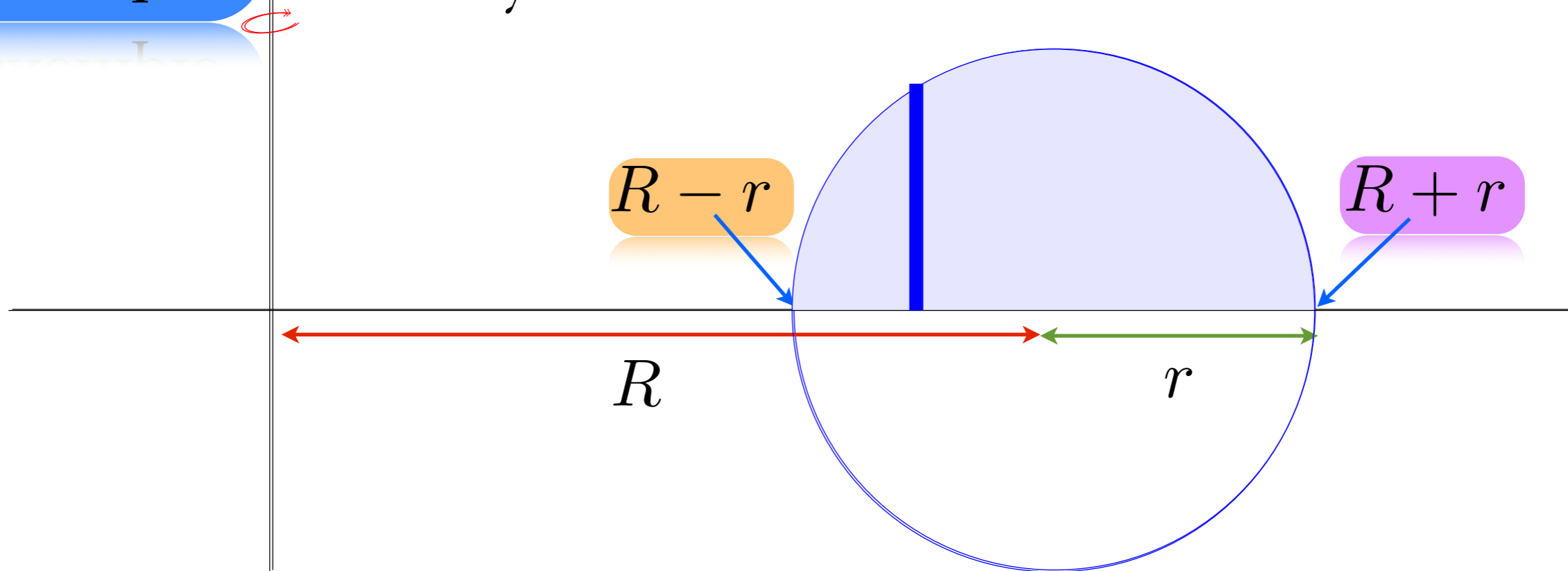


$$(x - R)^2 + y^2 = r^2 \quad \Longrightarrow \quad y = \sqrt{r^2 - (x - R)^2}$$

$$\text{Vol}_{\frac{1}{2} \text{ beigne}} = 2\pi \int_{R-r}^{R+r} x f(x) dx$$

Exemple

Calculer le volume d'un beigne de rayon de pâte r et de rayon de «trou» R

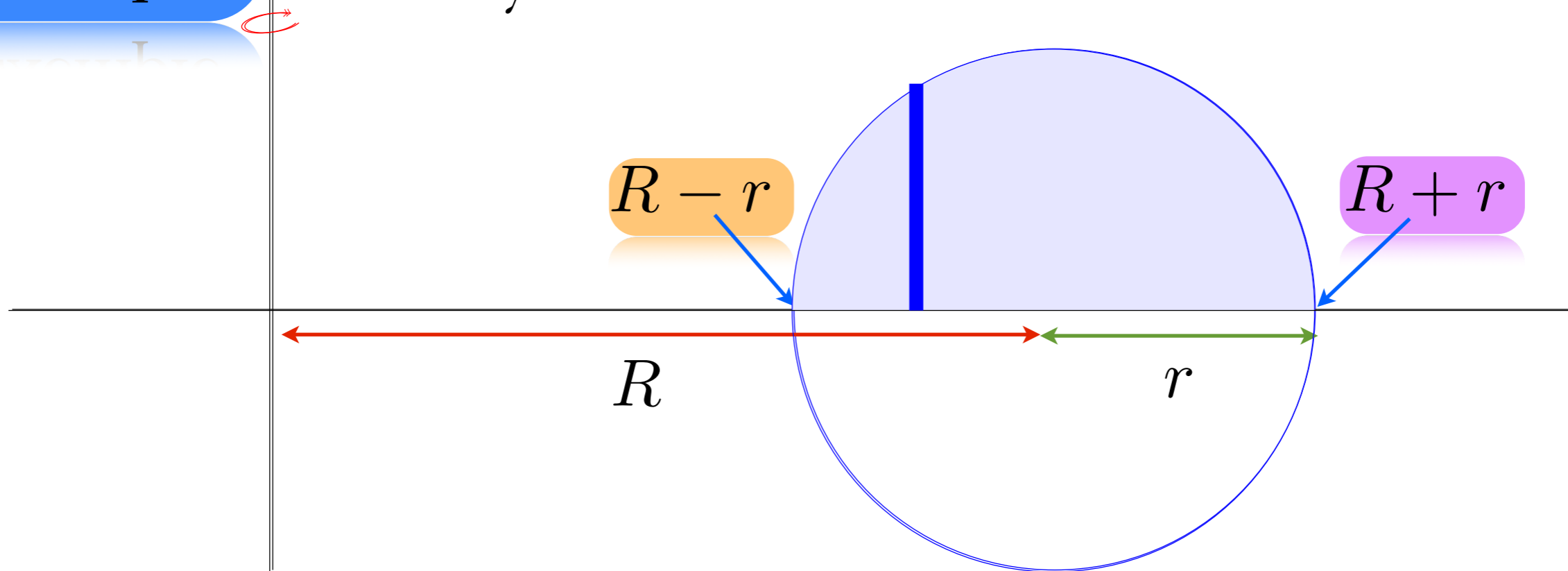


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Exemple

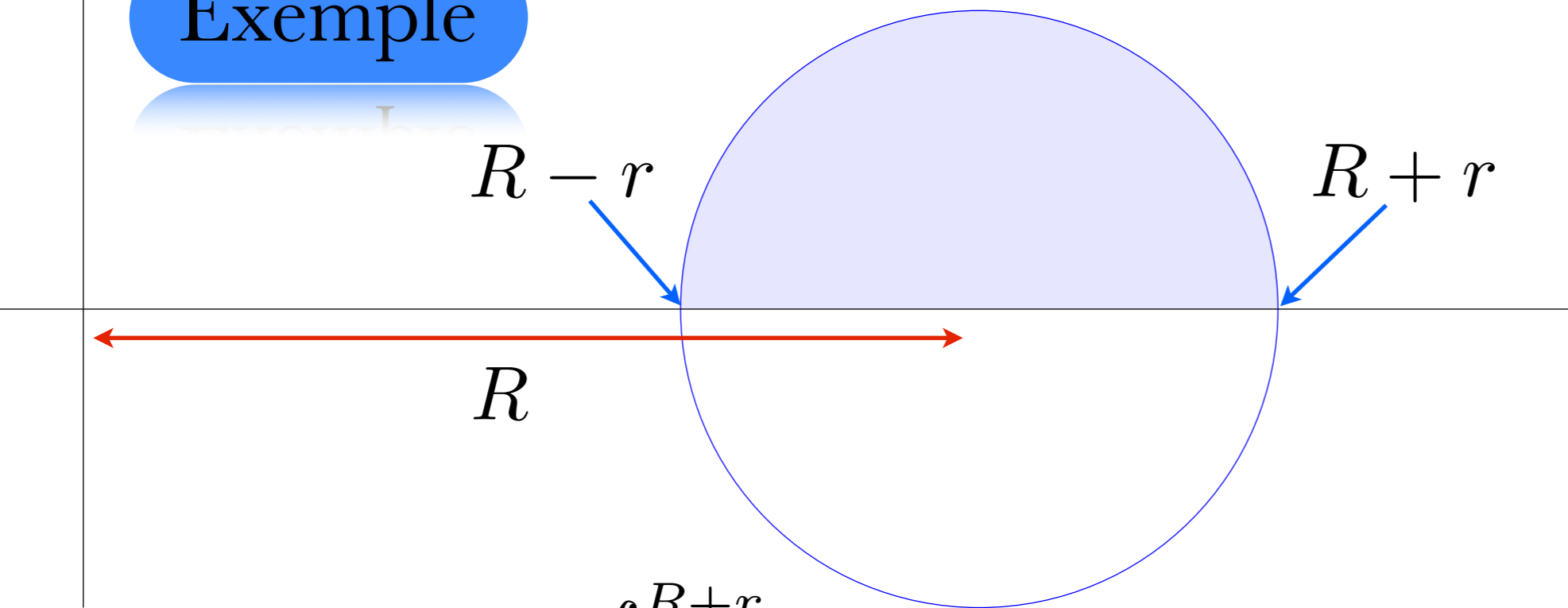
Calculer le volume d'un beigne de rayon de pâte r et de rayon de «trou» R



$$(x - R)^2 + y^2 = r^2 \implies y = \sqrt{r^2 - (x - R)^2}$$

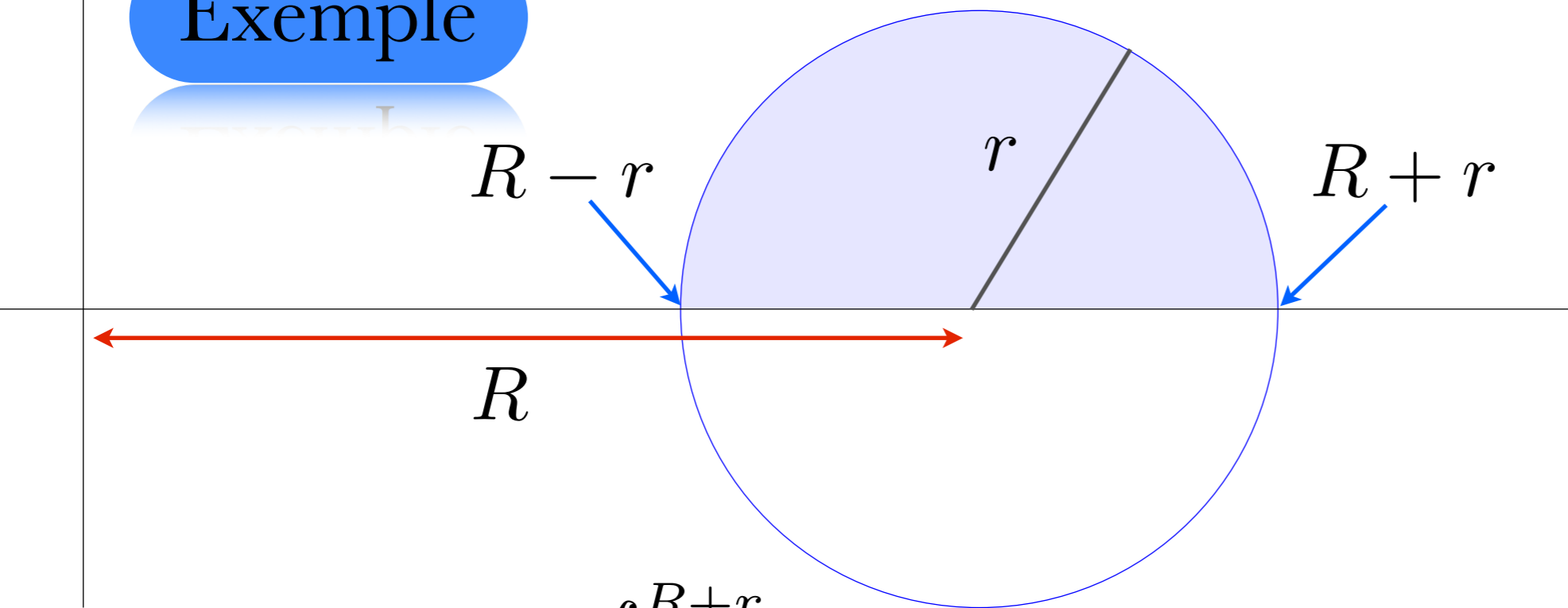
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Exemple



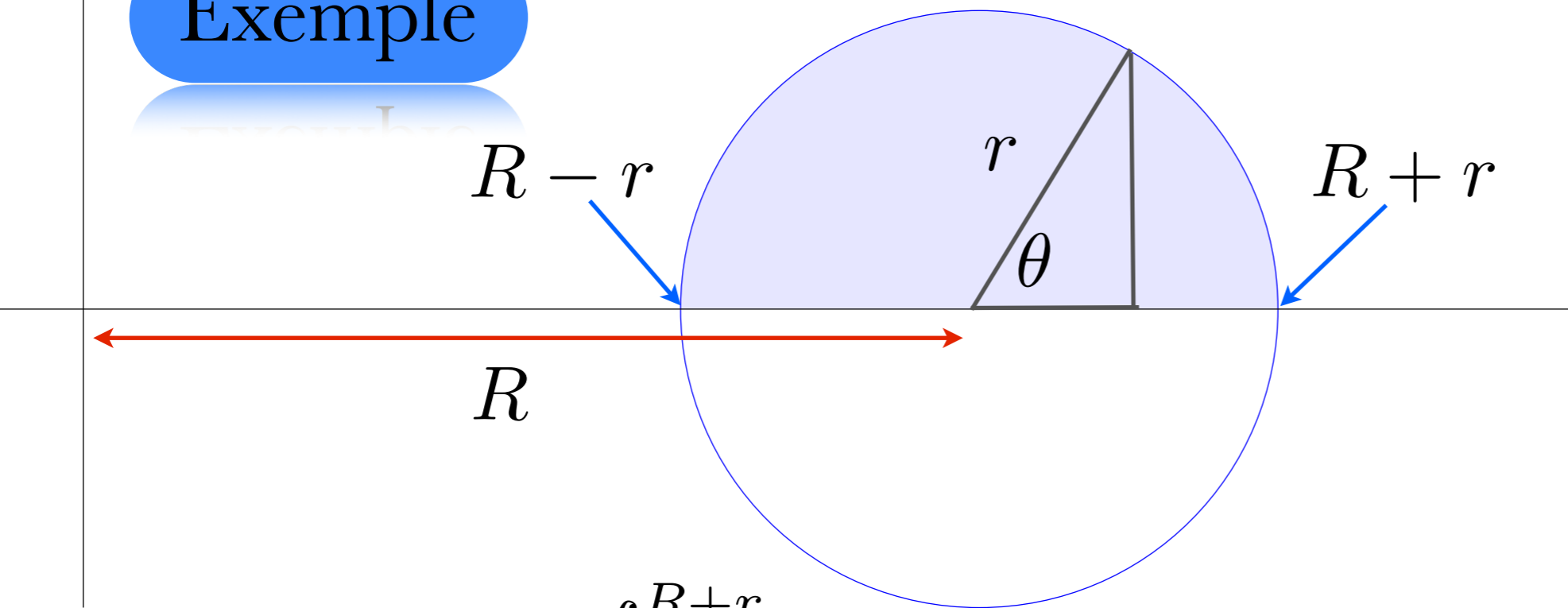
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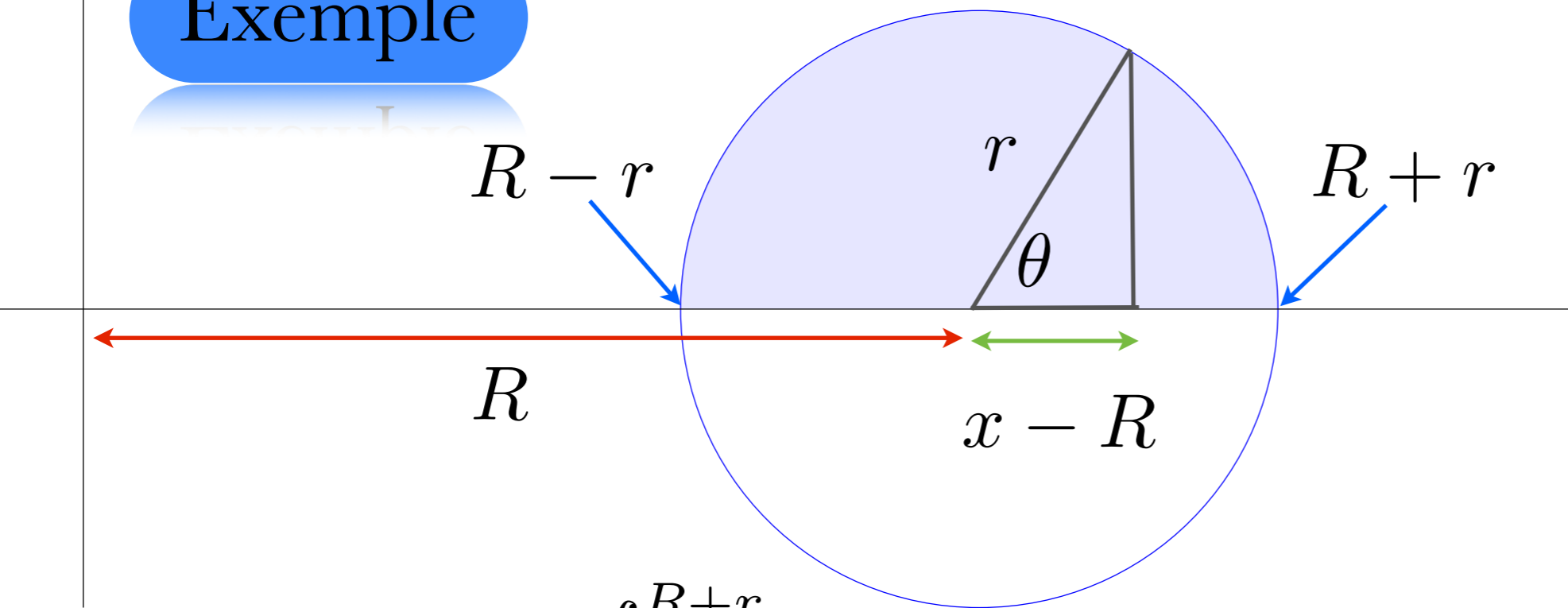
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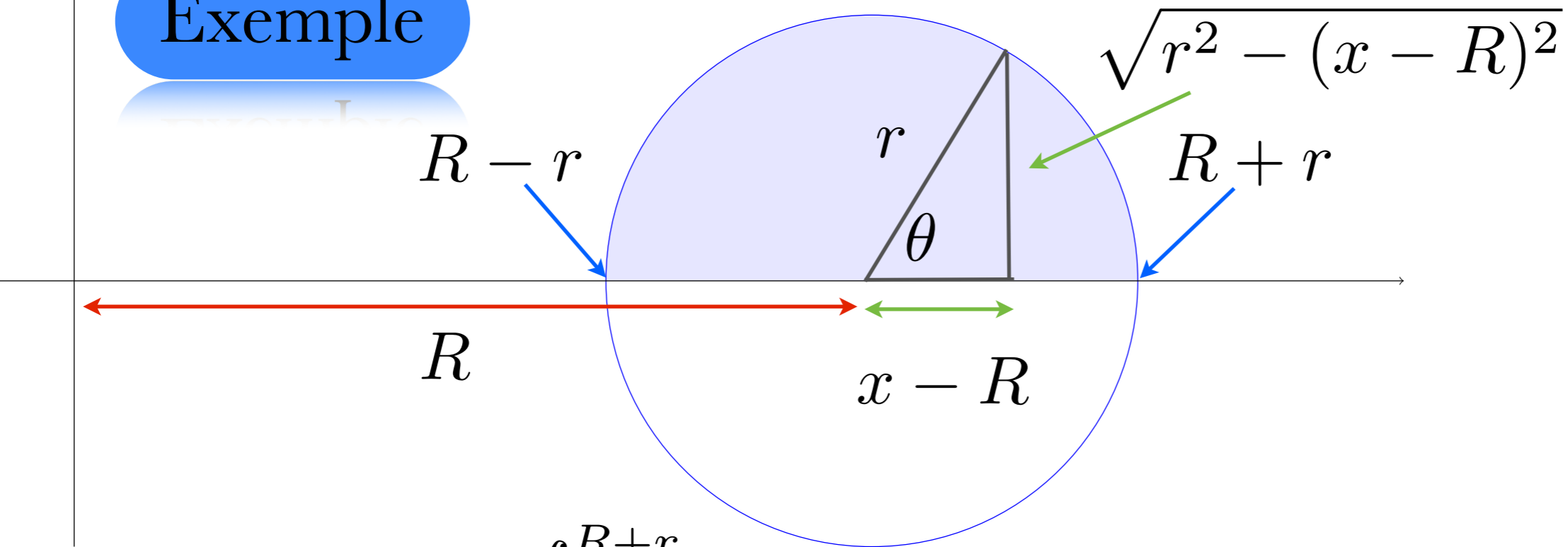
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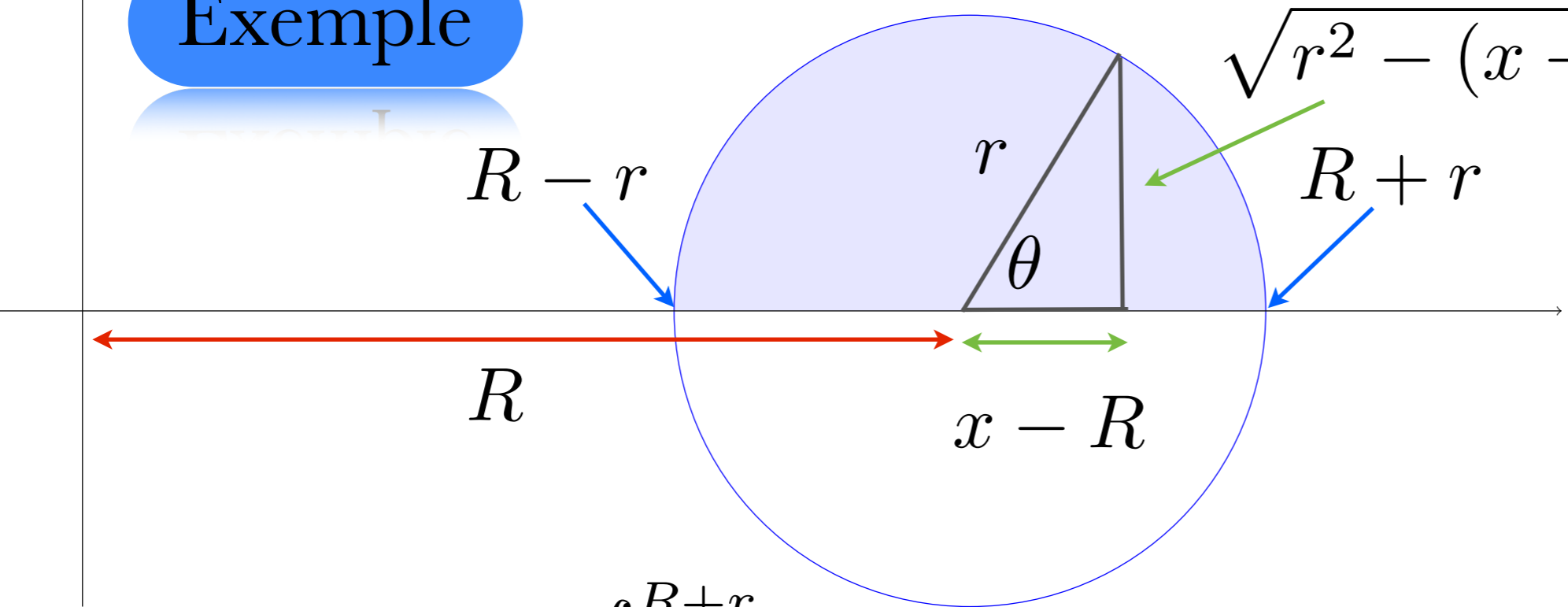
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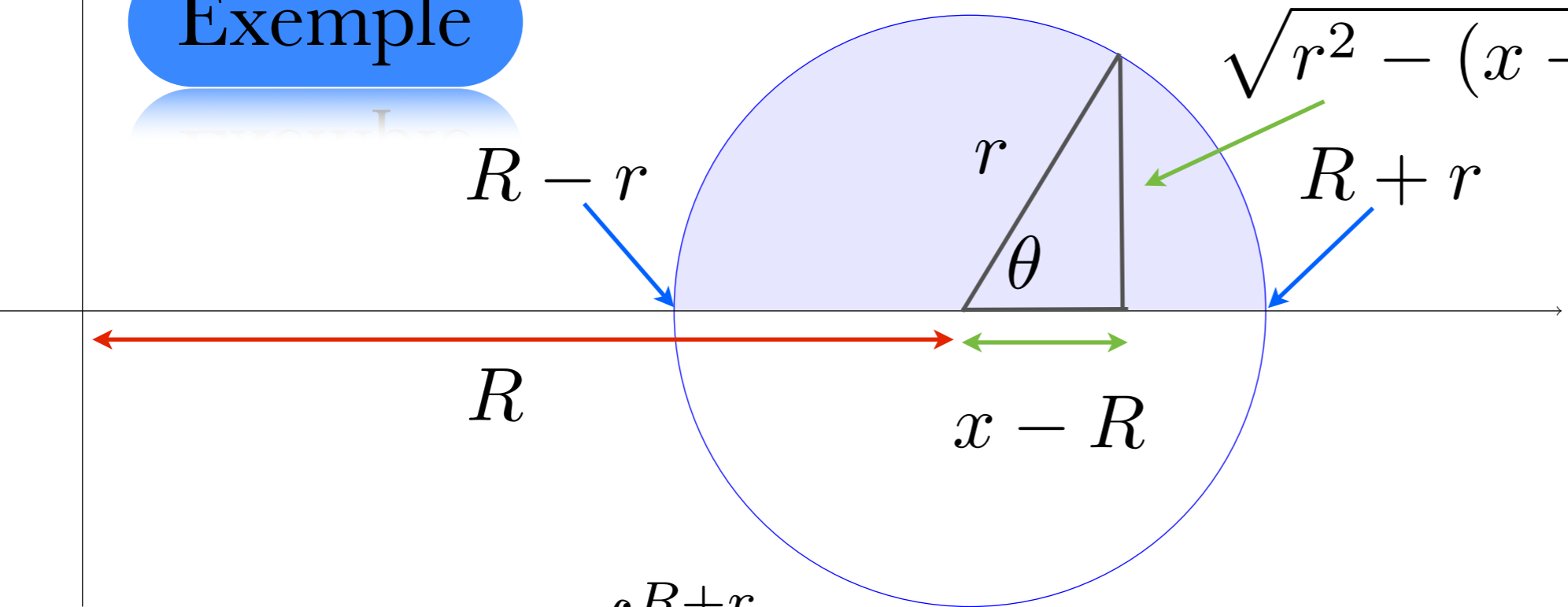
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$$\text{Vol}_{\frac{1}{2}\text{beigne}} = 2\pi \int_{R-r}^{R+r} x \sqrt{r^2 - (x - R)^2} dx$$

$$\frac{x - R}{r} = \cos \theta$$

Exemple

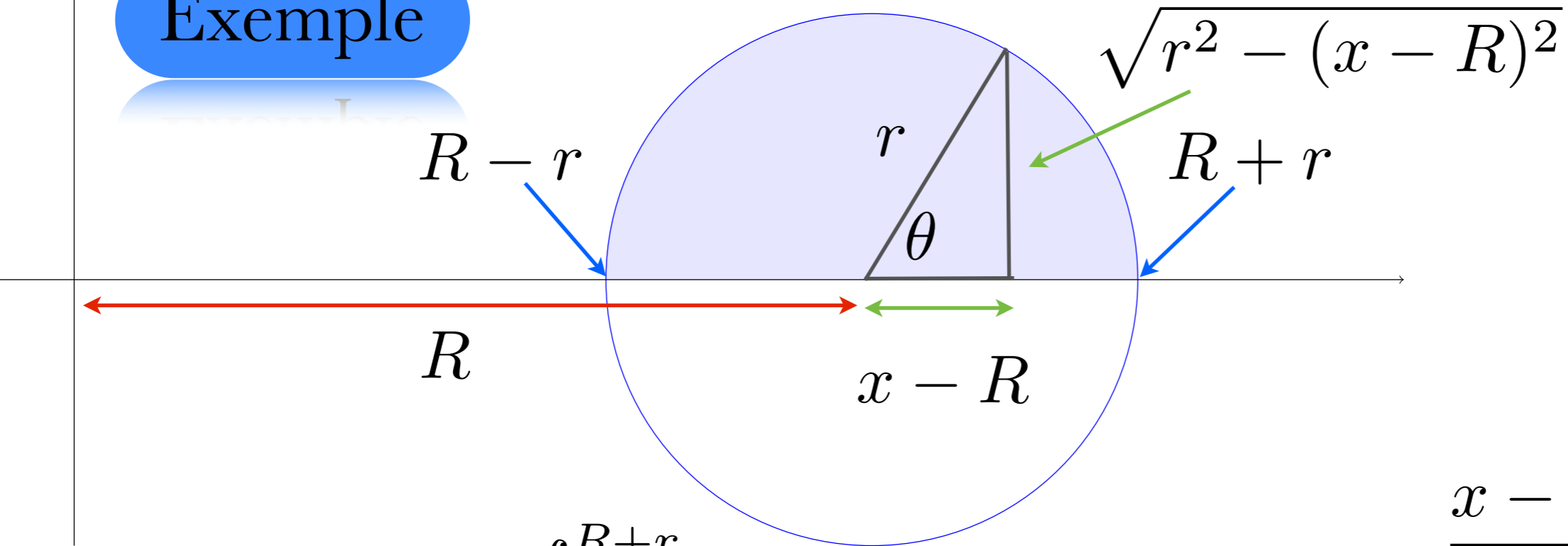


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$$x = r \cos \theta + R$$

Exemple



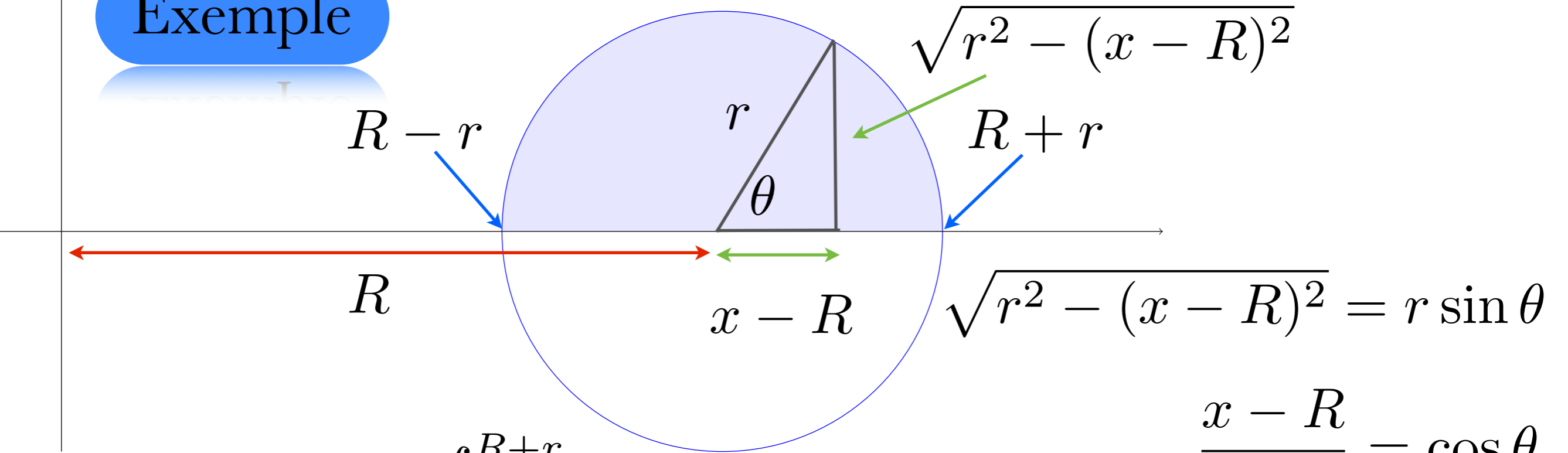
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Exemple



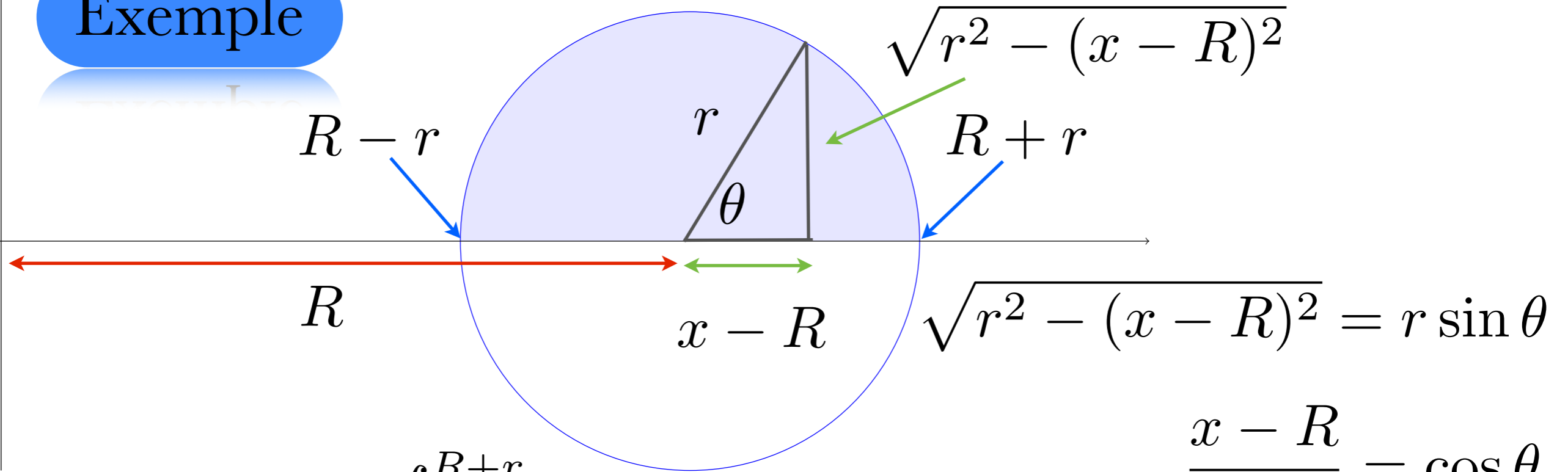
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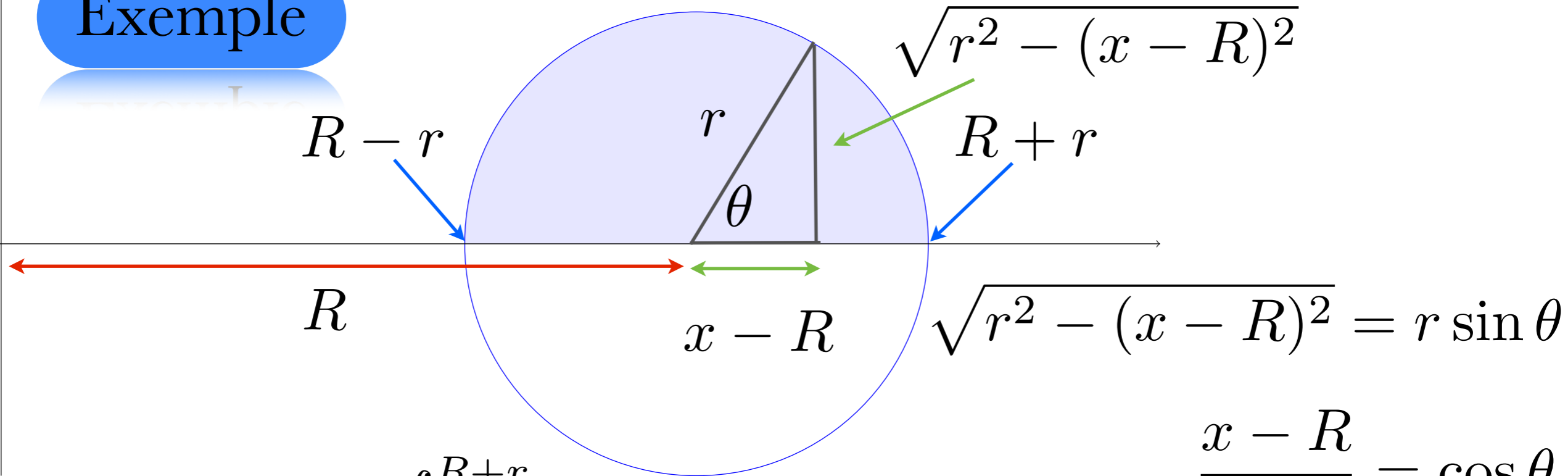
$$\frac{x - R}{r} = \cos \theta$$

$$x = r \cos \theta + R$$

$$= -2\pi \int_{?}^{?} (r \cos \theta + R) r^2 \sin^2 \theta d\theta$$

$$dx = -r \sin \theta d\theta$$

Exemple



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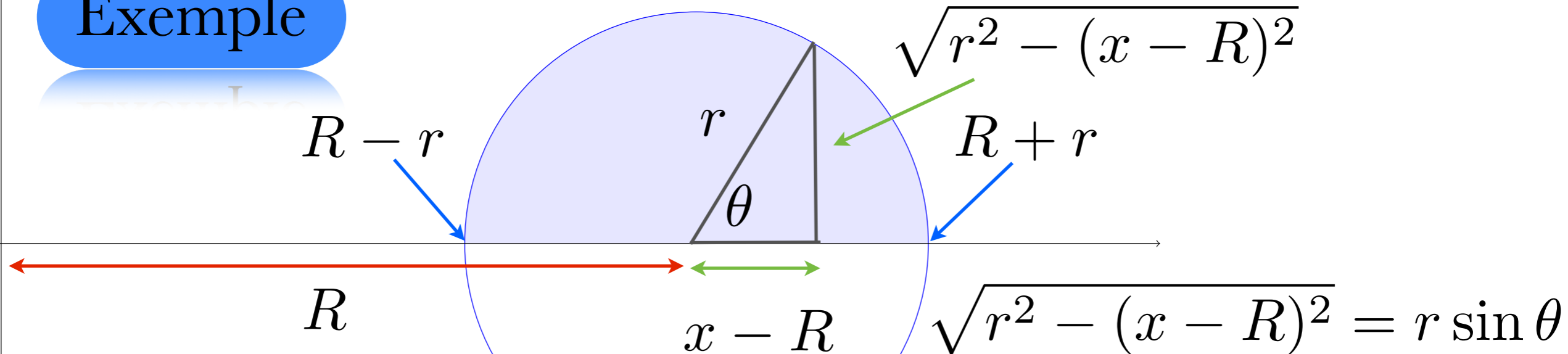
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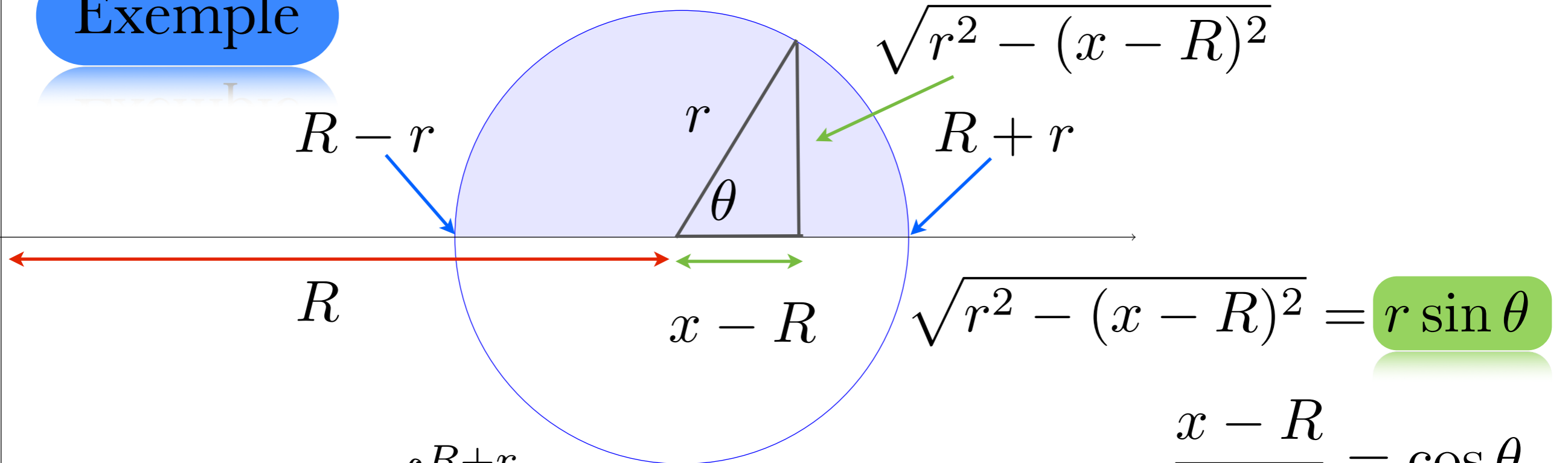
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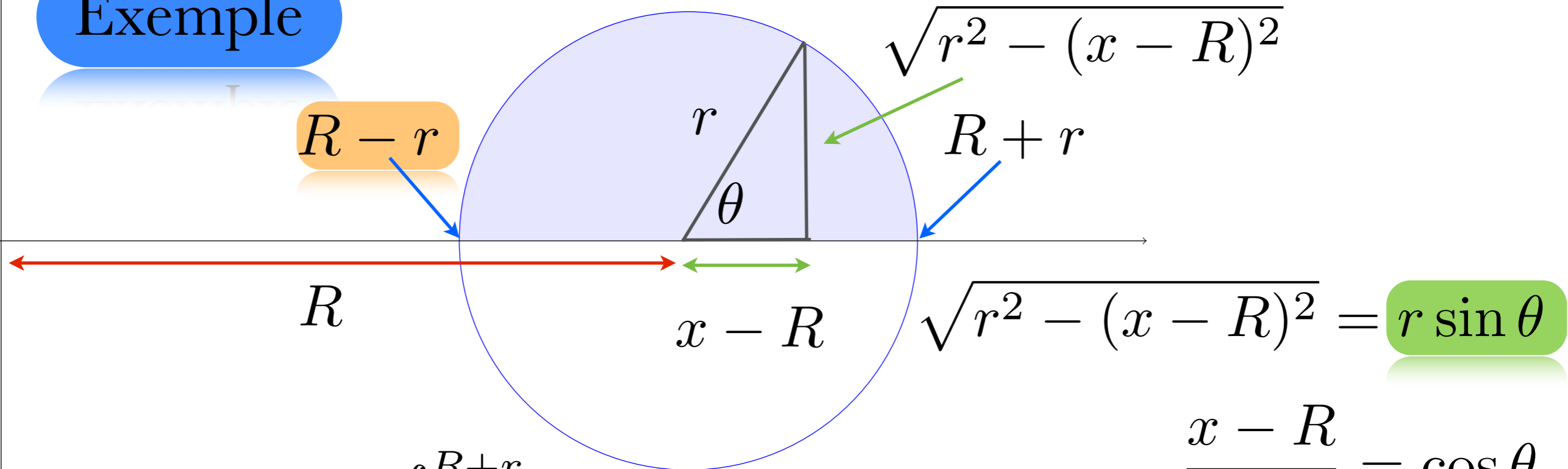
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Exemple



$$\sqrt{r^2 - (x - R)^2} = r \sin \theta$$

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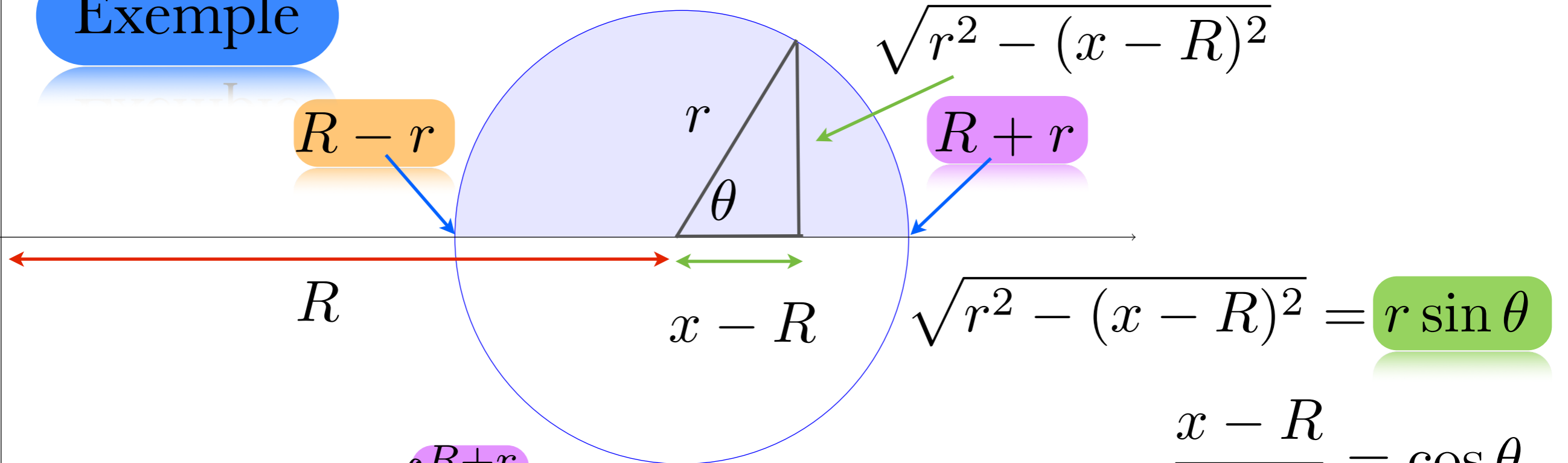
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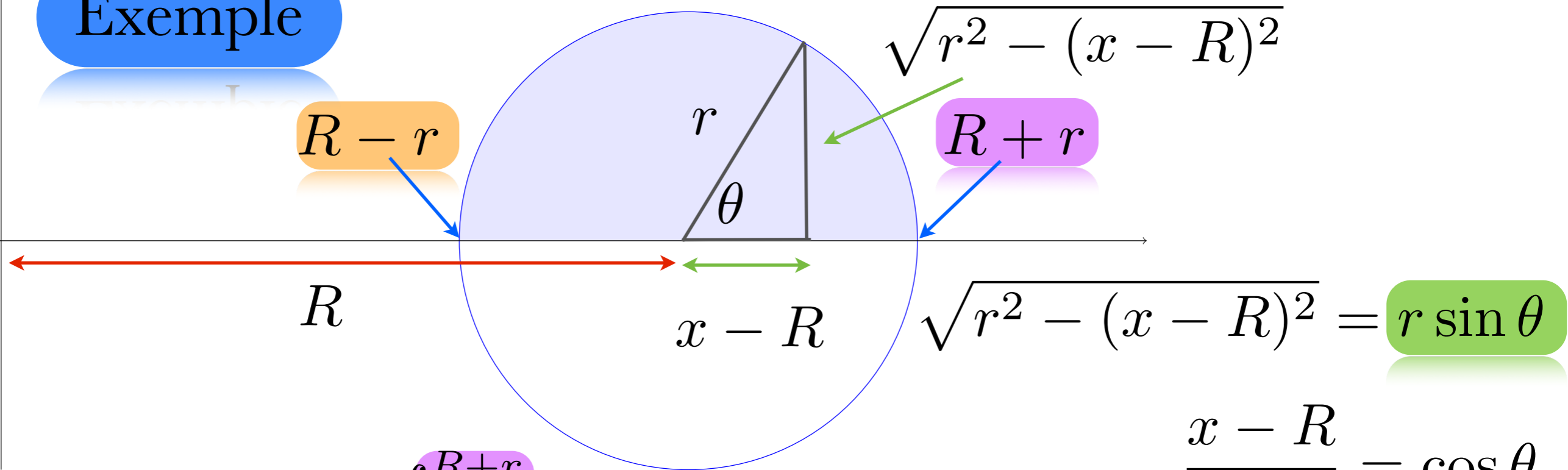
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Exemple



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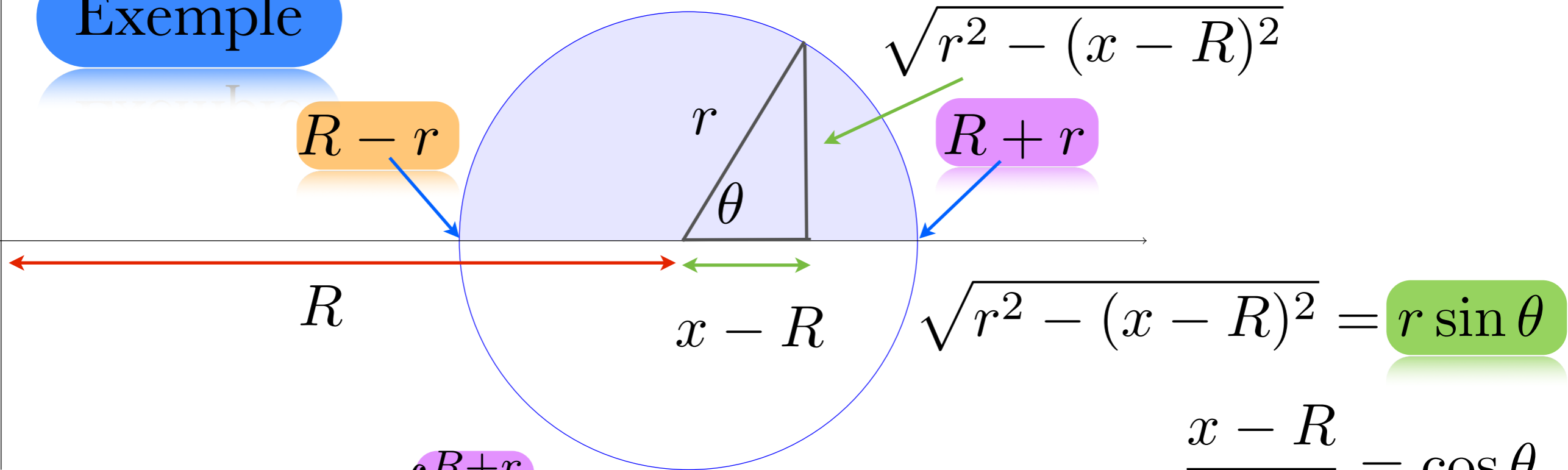
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$$= -2\pi \int_{\pi}^0 (r \cos \theta + R) r^2 \sin^2 \theta d\theta$$

Exemple



$$\sqrt{r^2 - (x - R)^2} = r \sin \theta$$

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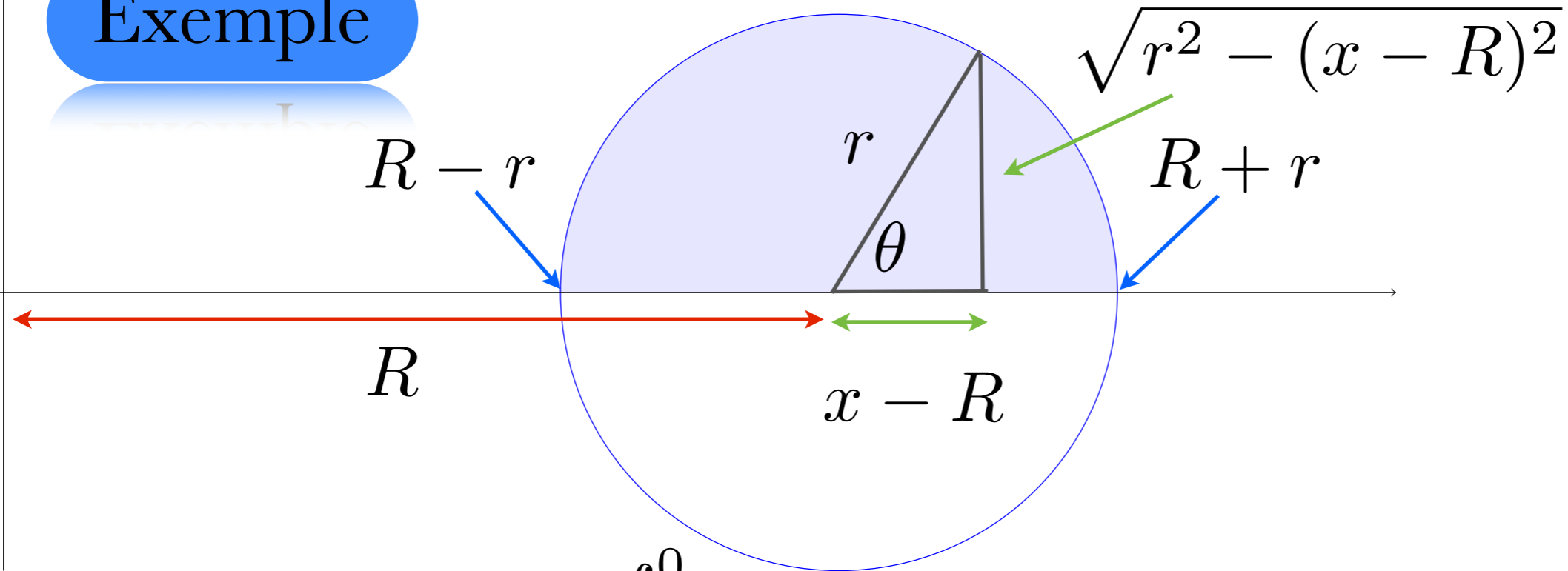
$$\text{Vol}_{\frac{1}{2} \text{ beigne}} = 2\pi \int_{R-r}^{R+r} x \sqrt{r^2 - (x - R)^2} dx$$

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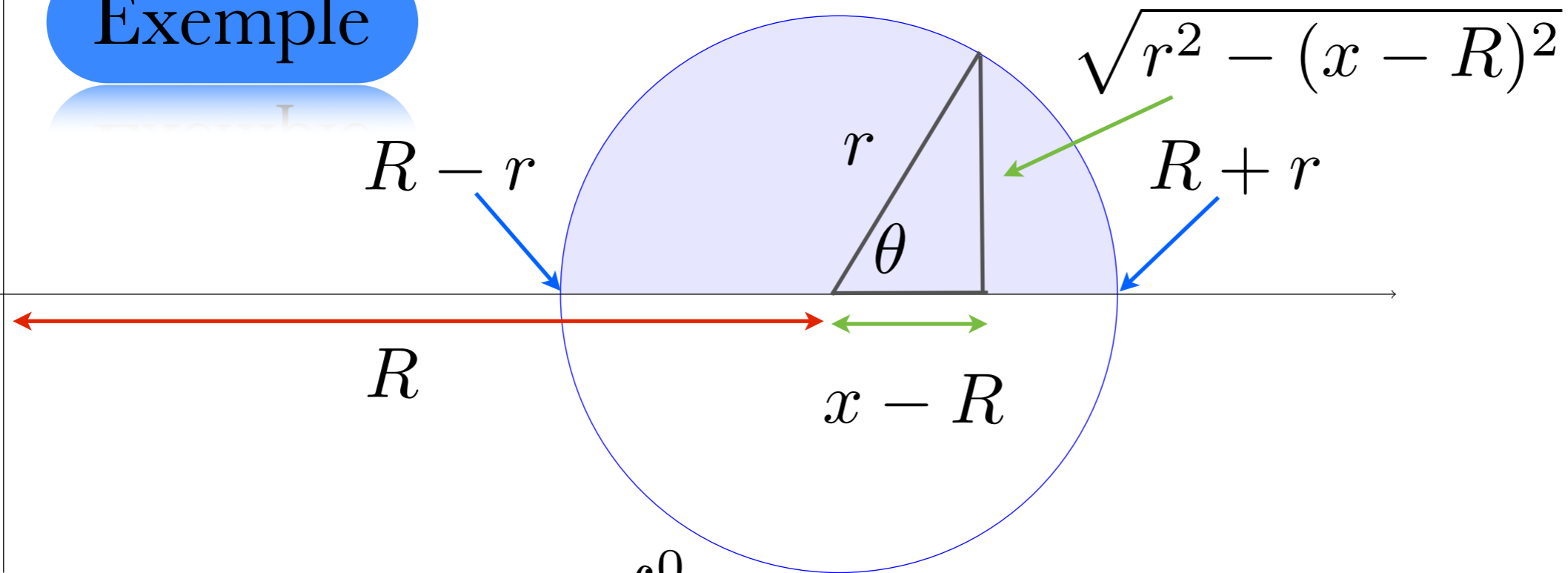
$$= -2\pi r^2 \int_{\pi}^0 r \cos \theta \sin^2 \theta + R \sin^2 \theta d\theta$$

Exemple



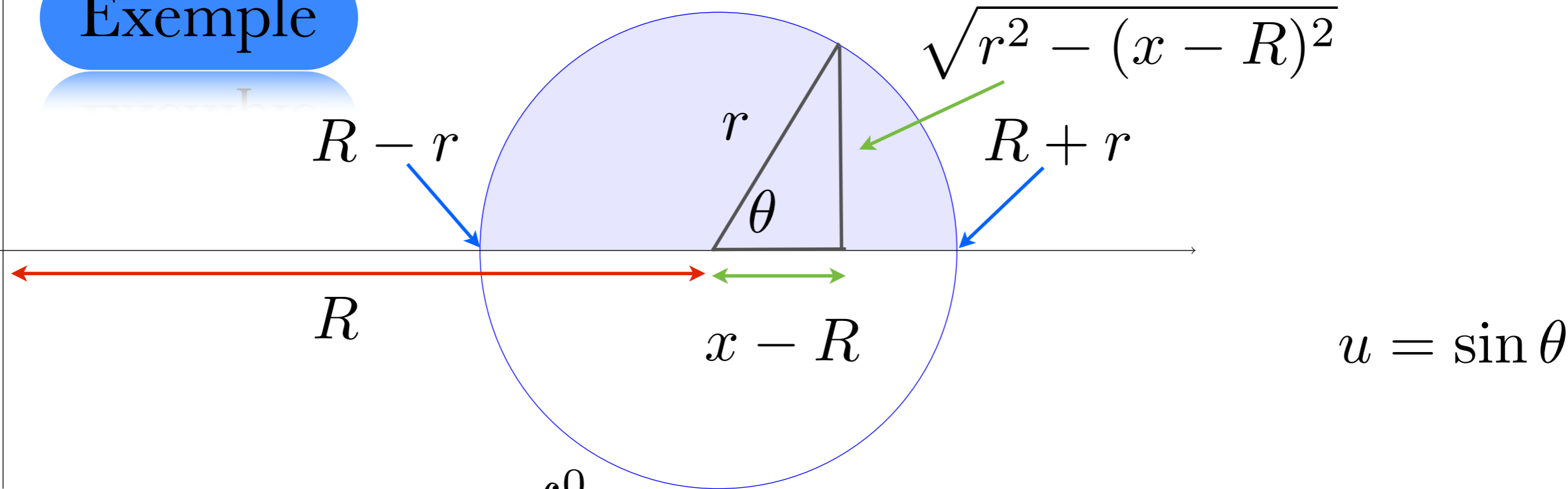
$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^2 \int_{\pi}^0 r \cos \theta \sin^2 \theta + R \sin^2 \theta d\theta$$

Exemple



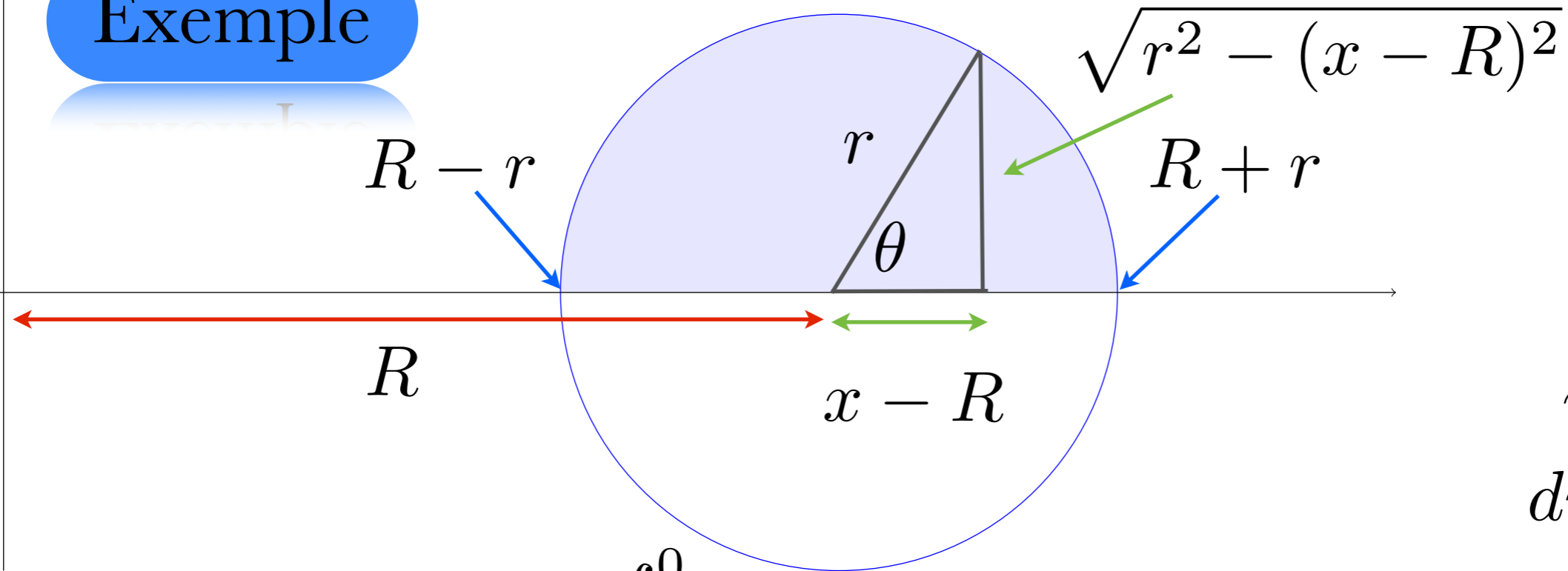
$$\begin{aligned} \text{Vol}_{\frac{1}{2}\text{beigne}} &= -2\pi r^2 \int_{\pi}^0 r \cos \theta \sin^2 \theta + R \sin^2 \theta \, d\theta \\ &= -2\pi r^3 \int_{\pi}^0 \cos \theta \sin^2 \theta \, d\theta - 2\pi R r^2 \int_{\pi}^0 \sin^2 \theta \, d\theta \end{aligned}$$

Exemple



$$\begin{aligned} \text{Vol}_{\frac{1}{2}\text{beigne}} &= -2\pi r^2 \int_{\pi}^0 r \cos \theta \sin^2 \theta + R \sin^2 \theta \, d\theta \\ &= -2\pi r^3 \int_{\pi}^0 \cos \theta \sin^2 \theta \, d\theta - 2\pi R r^2 \int_{\pi}^0 \sin^2 \theta \, d\theta \end{aligned}$$

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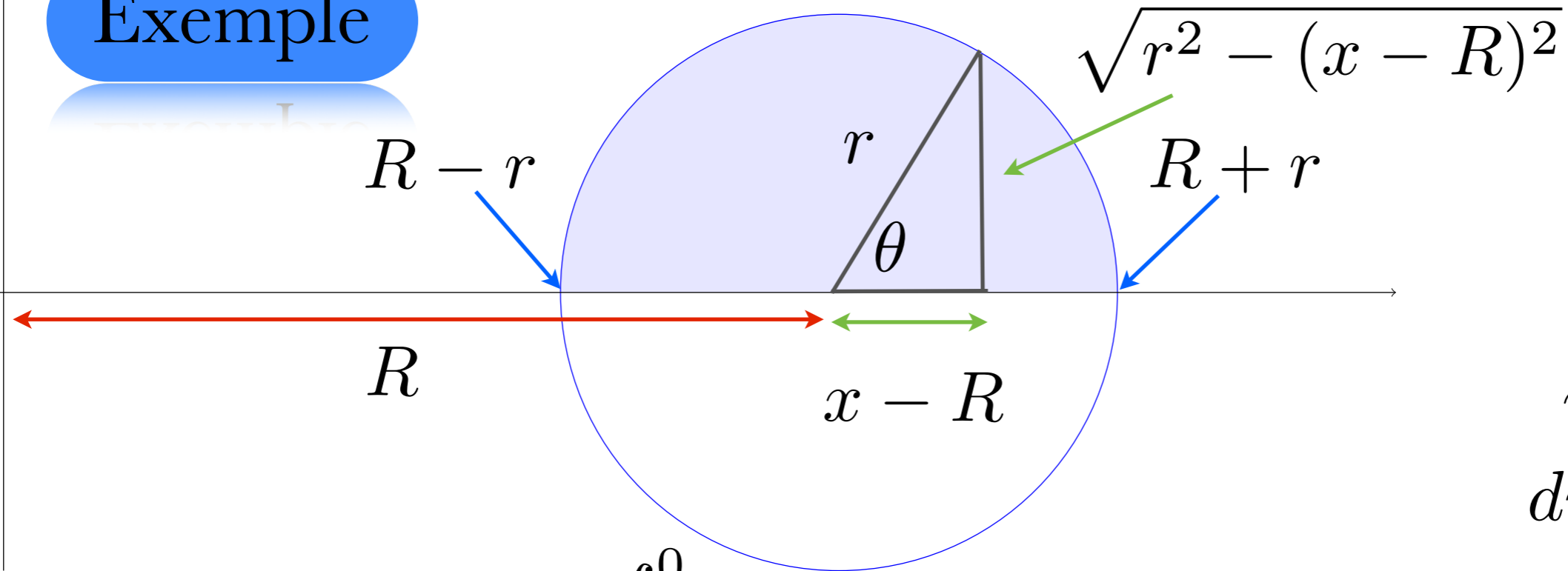
$$u = \sin \theta$$

$$du = \cos \theta \, d\theta$$

$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^2 \int_{\pi}^0 r \cos \theta \sin^2 \theta + R \sin^2 \theta \, d\theta$$

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Exemple



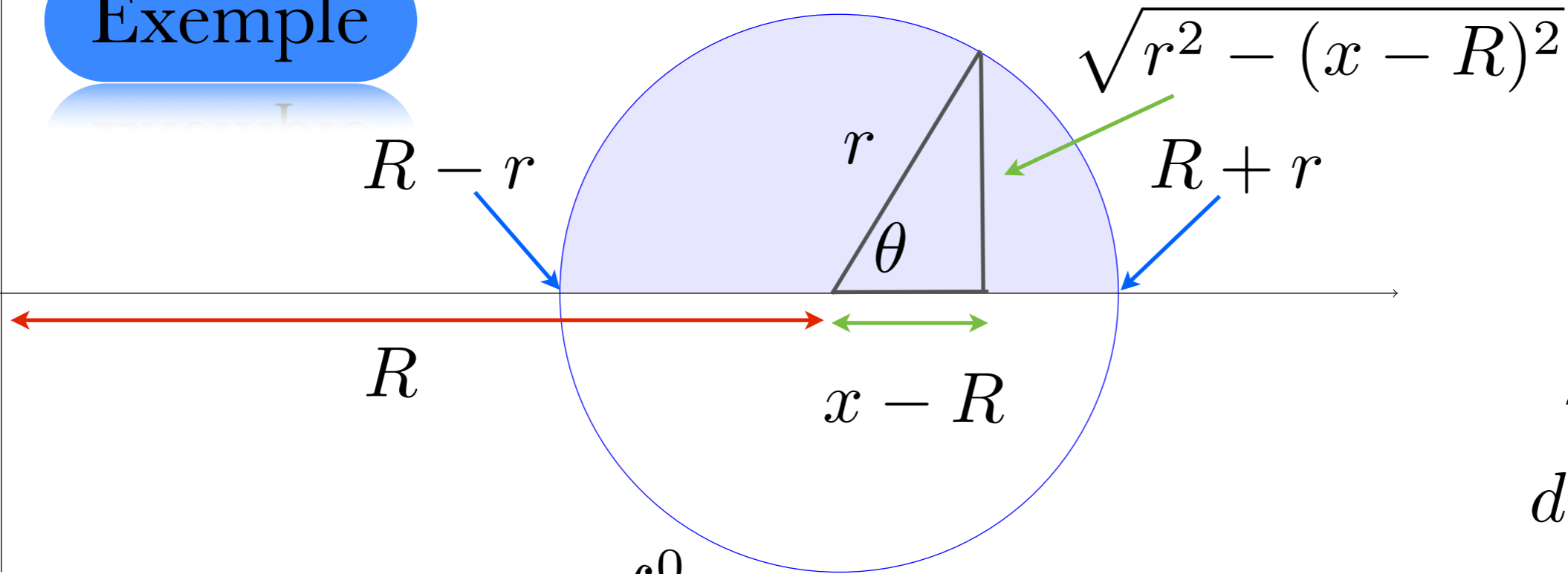
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Exemple



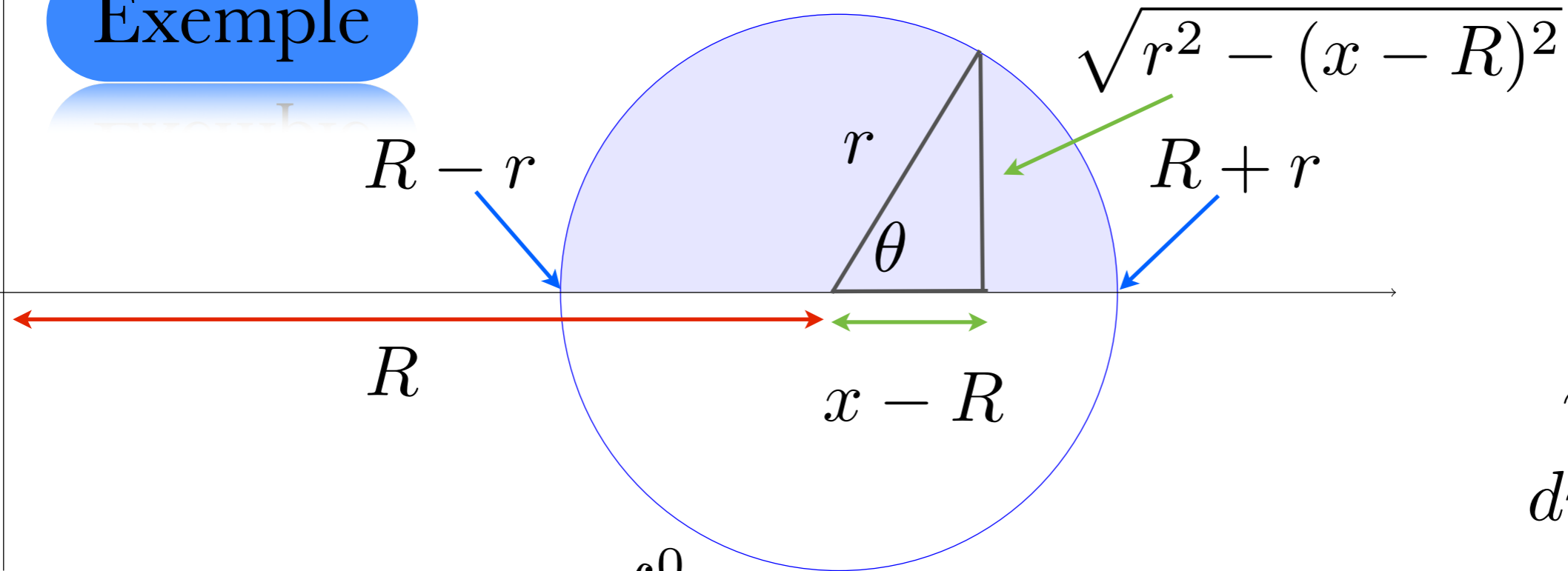
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Exemple



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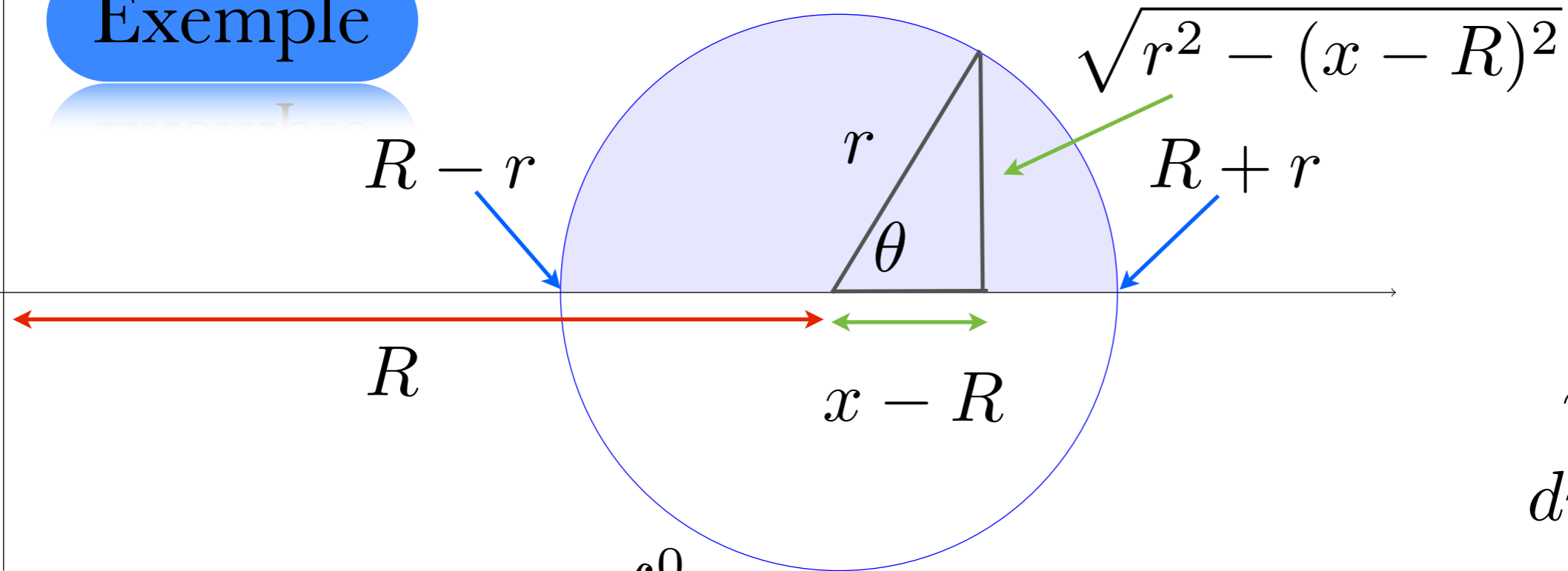
$$du = \cos \theta d\theta$$

$$\text{Vol}_{\frac{1}{2} \text{ beigne}} = -2\pi r^2 \int_{\pi}^0 r \cos \theta \sin^2 \theta + R \sin^2 \theta d\theta$$

$$= -2\pi r^3 \int_{\pi}^0 \cos \theta \sin^2 \theta d\theta - 2\pi R r^2 \int_{\pi}^0 \sin^2 \theta d\theta$$

$$= -2\pi r^3 \int_{?}^{?} u^2 du - 2\pi R r^2 \int_{\pi}^0 \frac{1 - \cos 2\theta}{2} d\theta$$

Exemple



$$u = \sin \theta$$

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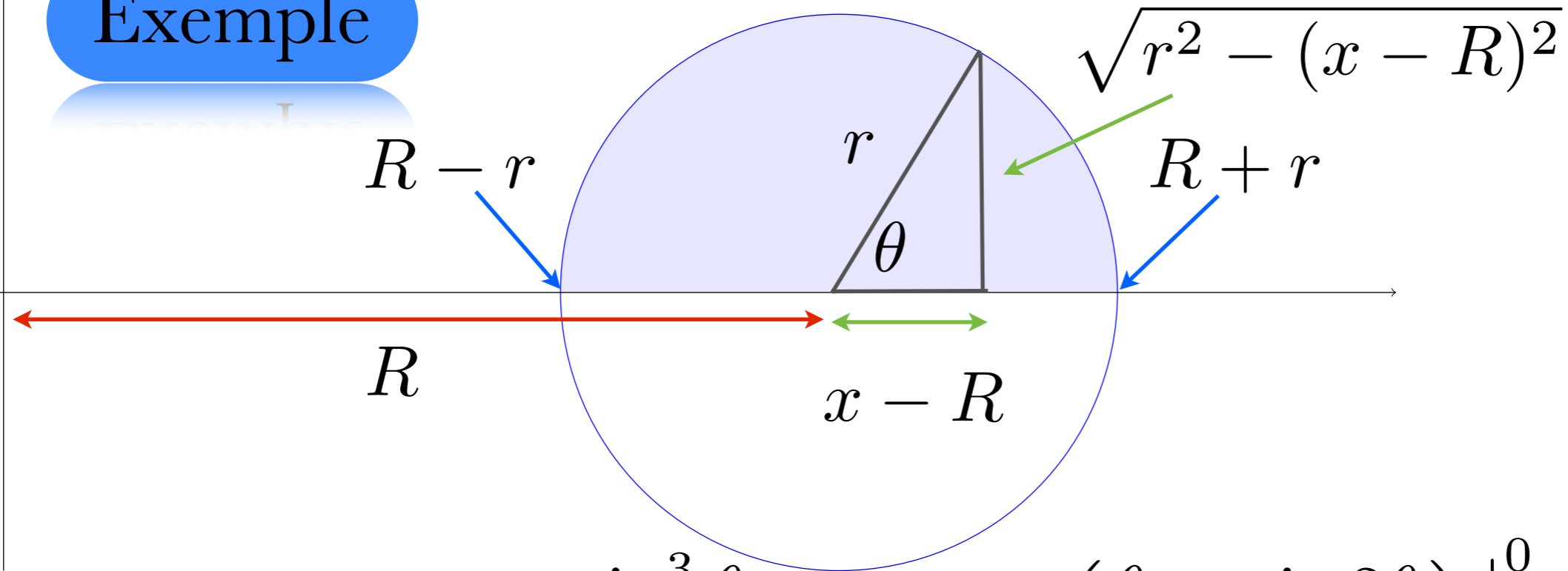
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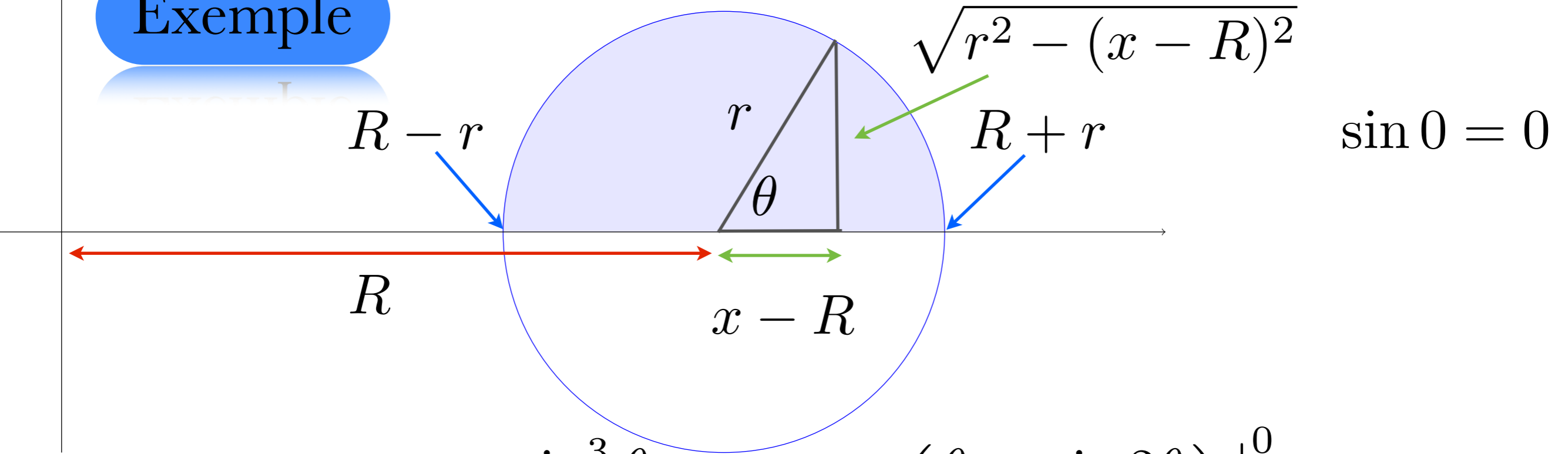
$$= -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

Exemple



$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

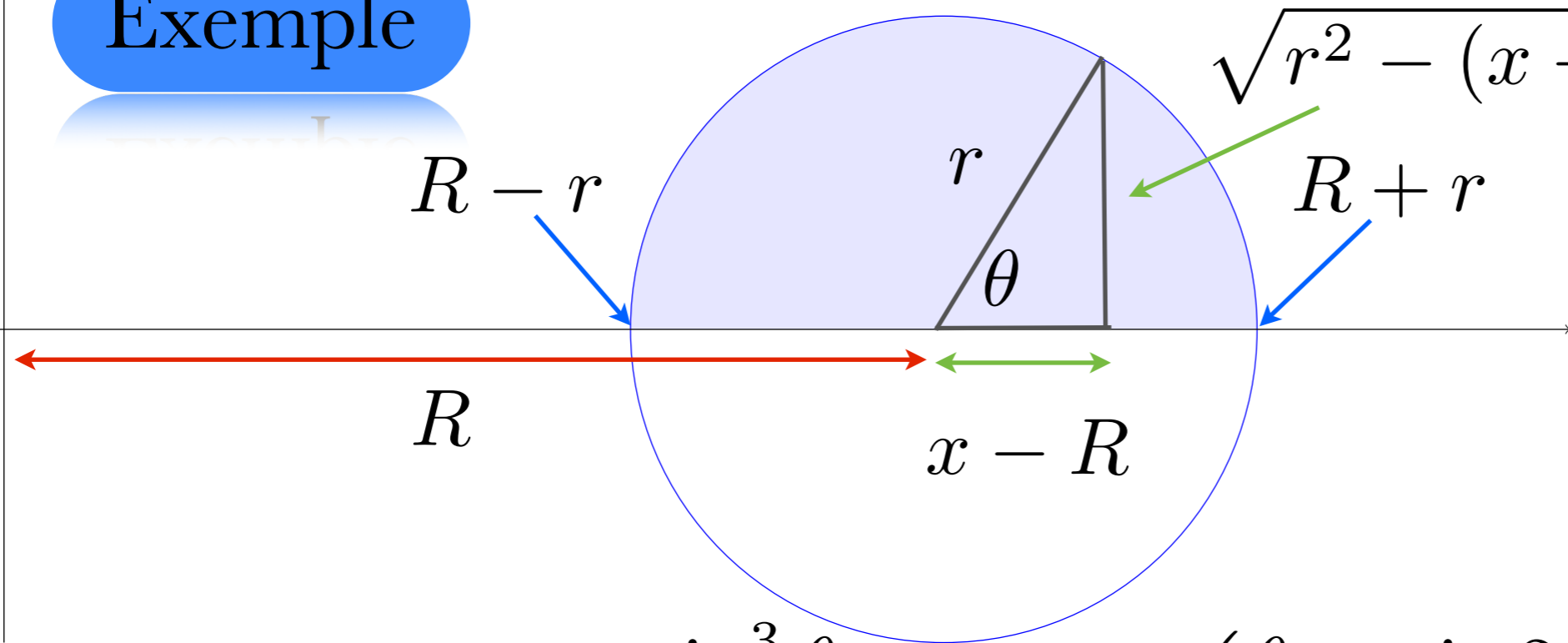
Exemple



$$\sin 0 = 0$$

$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

Exemple

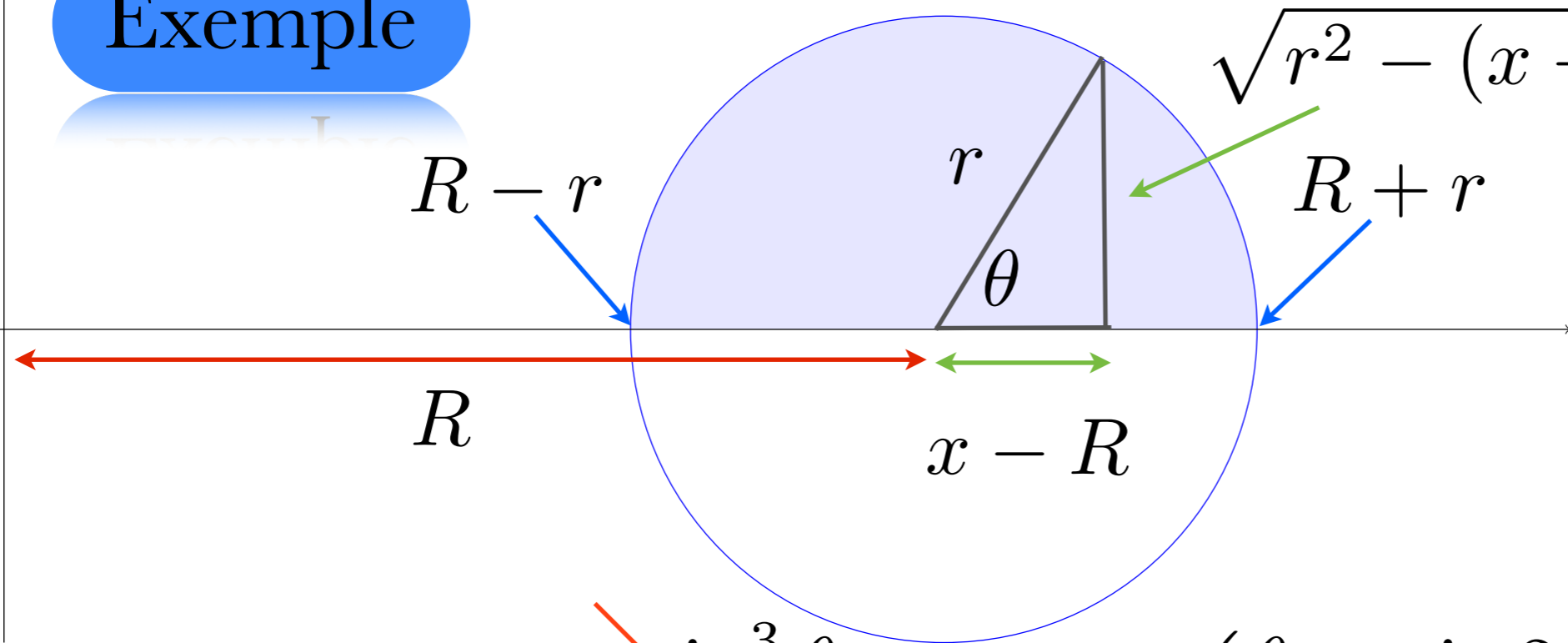


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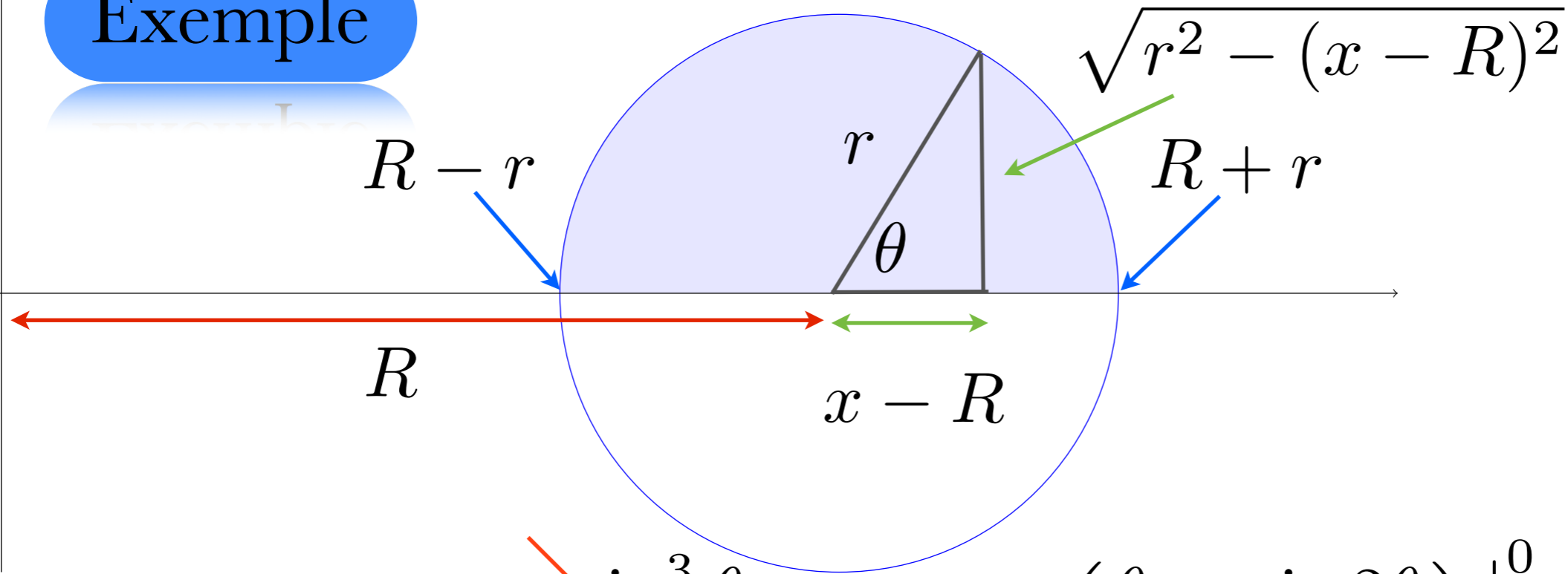


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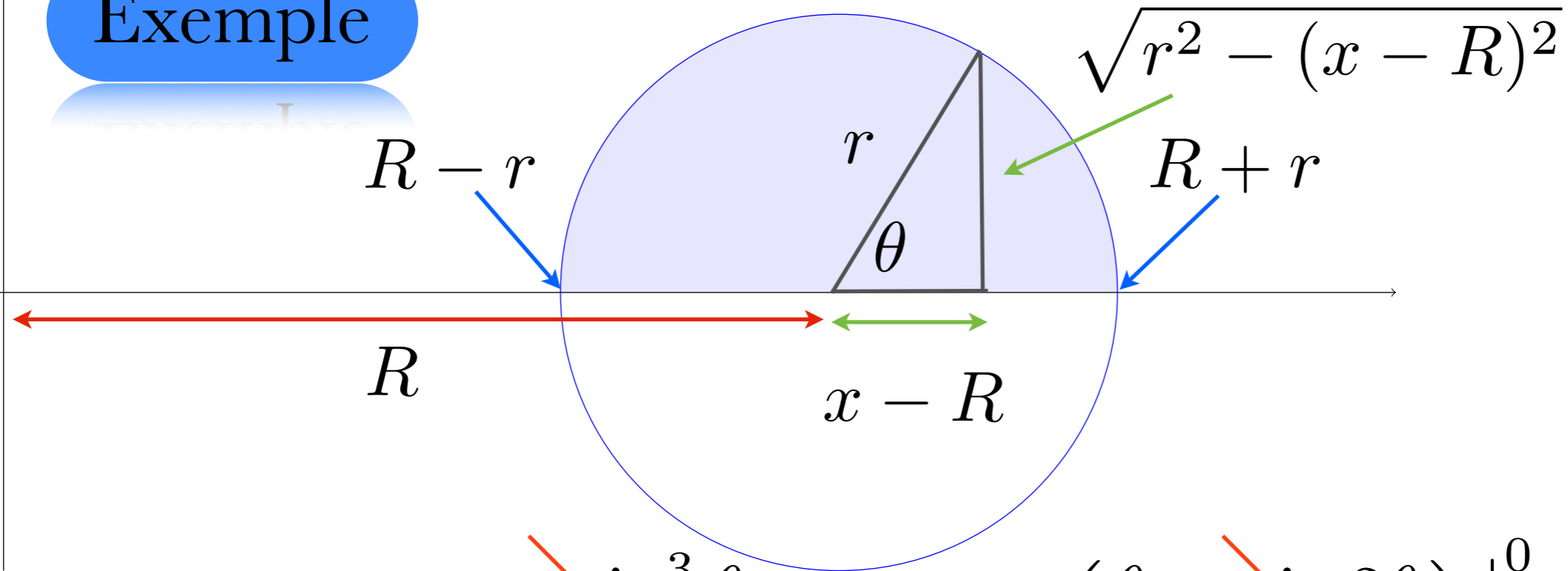
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$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

Exemple



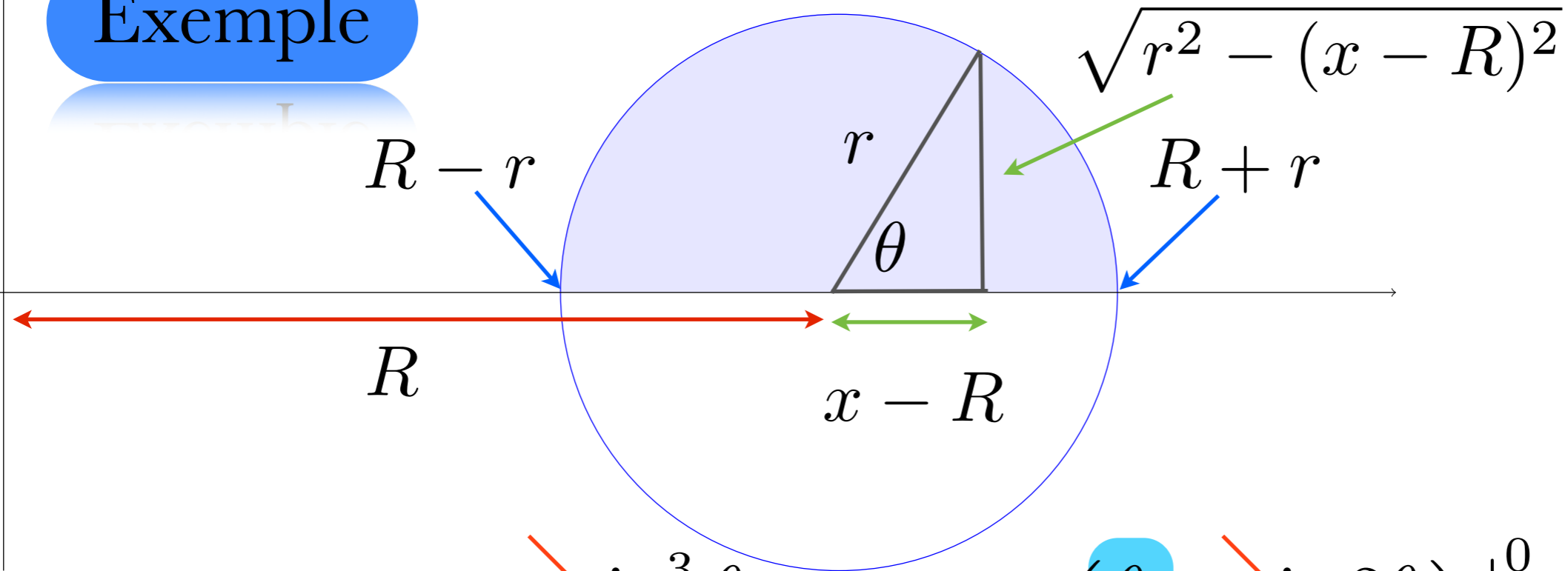
$$\sin 0 = 0$$

$$\sin \pi = 0$$

$$\sin 2\pi = 0$$

$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

Exemple



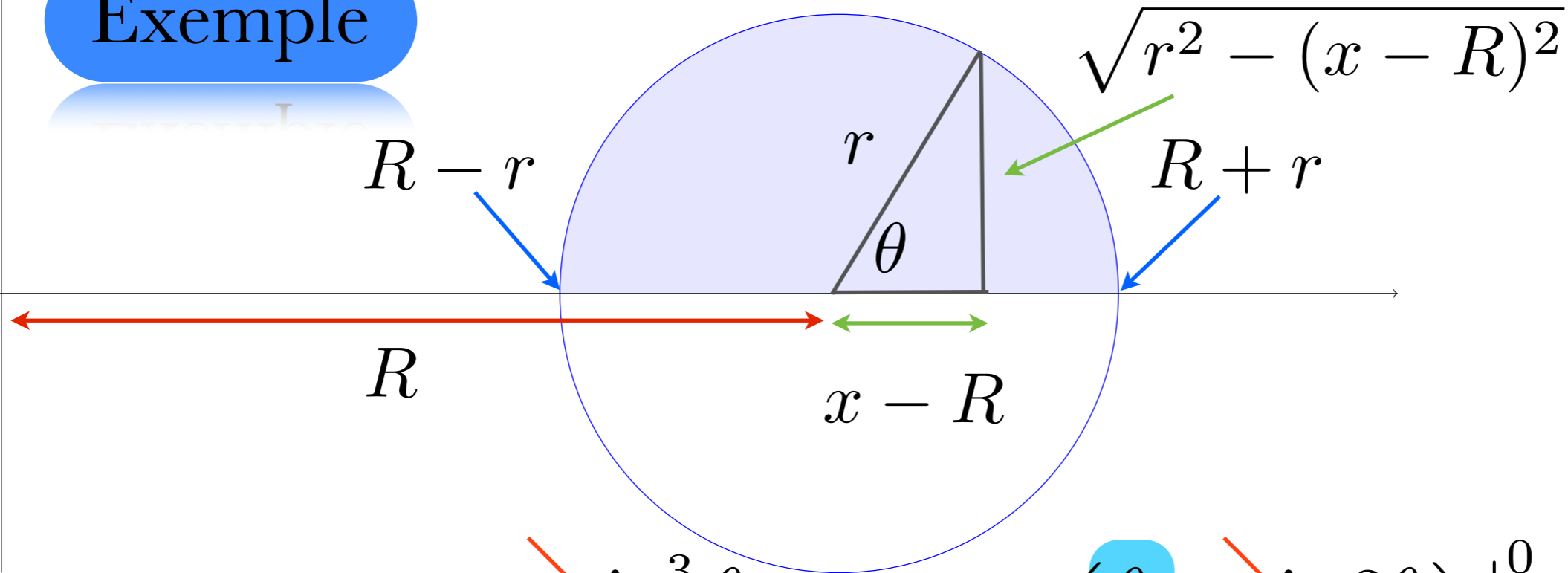
$$\sin 0 = 0$$

$$\sin \pi = 0$$

$$\sin 2\pi = 0$$

$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

Exemple



$$\sin 0 = 0$$

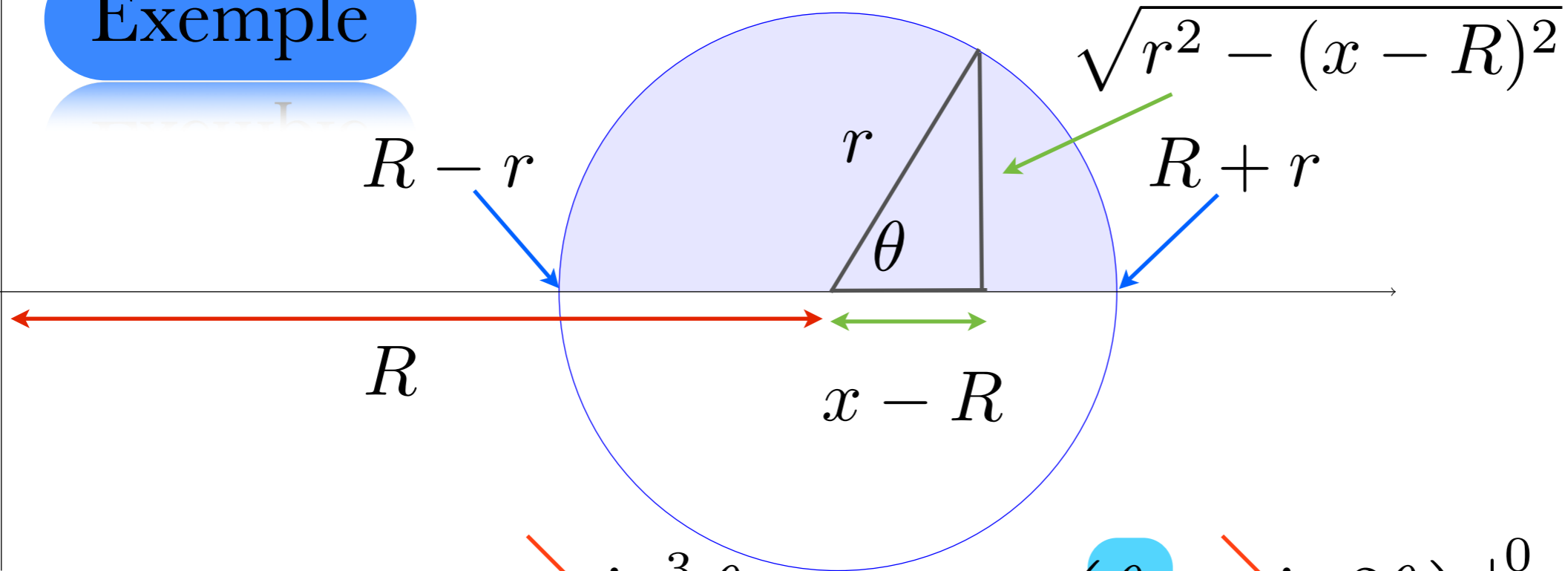
$$\sin \pi = 0$$

$$\sin 2\pi = 0$$

$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

$$= -2\pi R r^2 \frac{0}{2} + 2\pi R r^2 \frac{\pi}{2}$$

Exemple



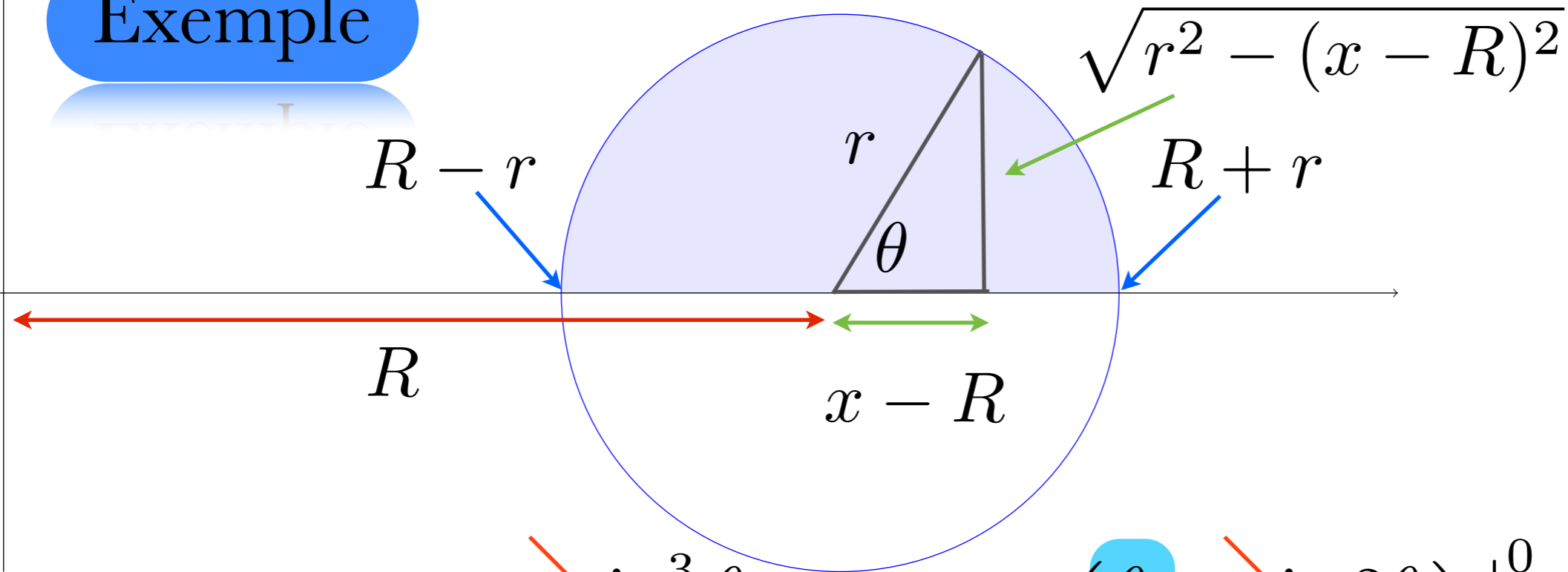
$$\sin 0 = 0$$

$$\sin \pi = 0$$

$$\sin 2\pi = 0$$

$$\begin{aligned} \text{Vol}_{\frac{1}{2} \text{ beigne}} &= -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0 \\ &= -2\pi R r^2 \frac{0}{2} + 2\pi R r^2 \frac{\pi}{2} \\ &= \pi^2 R r^2 \end{aligned}$$

Exemple



$$\sin 0 = 0$$

$$\sin \pi = 0$$

$$\sin 2\pi = 0$$

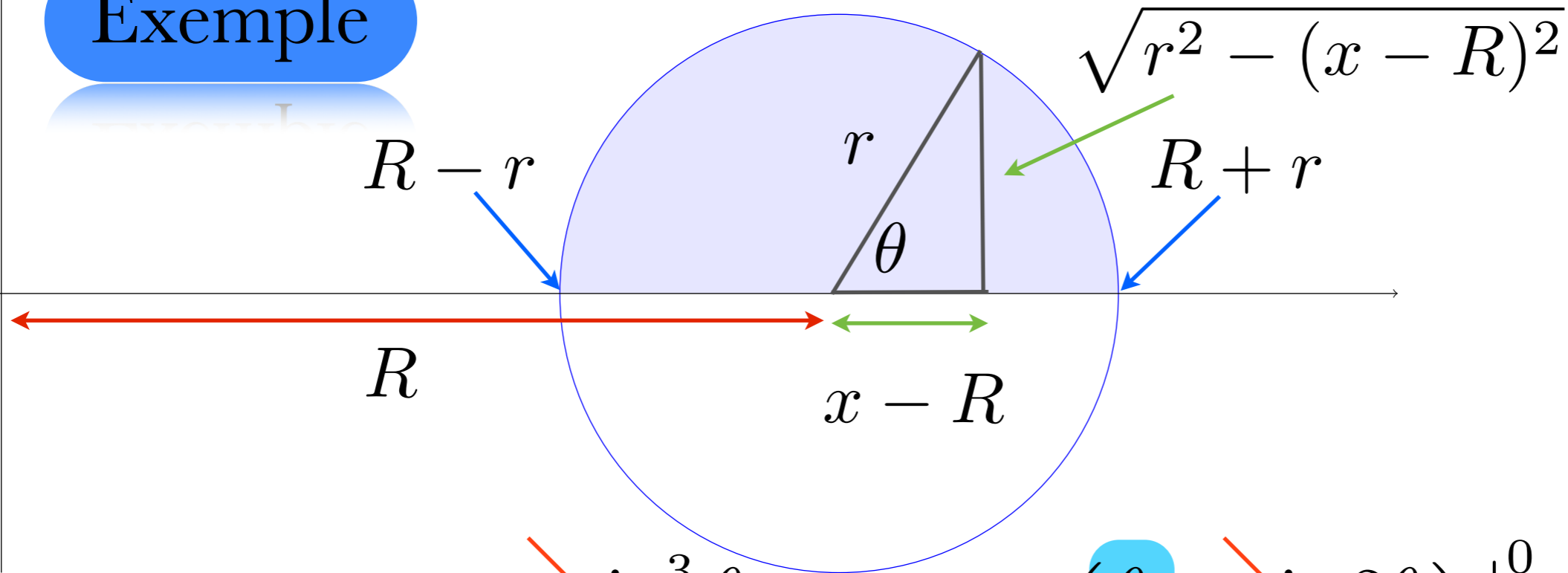
$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

$$= -2\pi R r^2 \frac{0}{2} + 2\pi R r^2 \frac{\pi}{2}$$

$$= \pi^2 R r^2$$

$$\text{Vol}_{\text{beigne}} = 2\text{Vol}_{\frac{1}{2}\text{beigne}}$$

Exemple



$$\sin 0 = 0$$

$$\sin \pi = 0$$

$$\sin 2\pi = 0$$

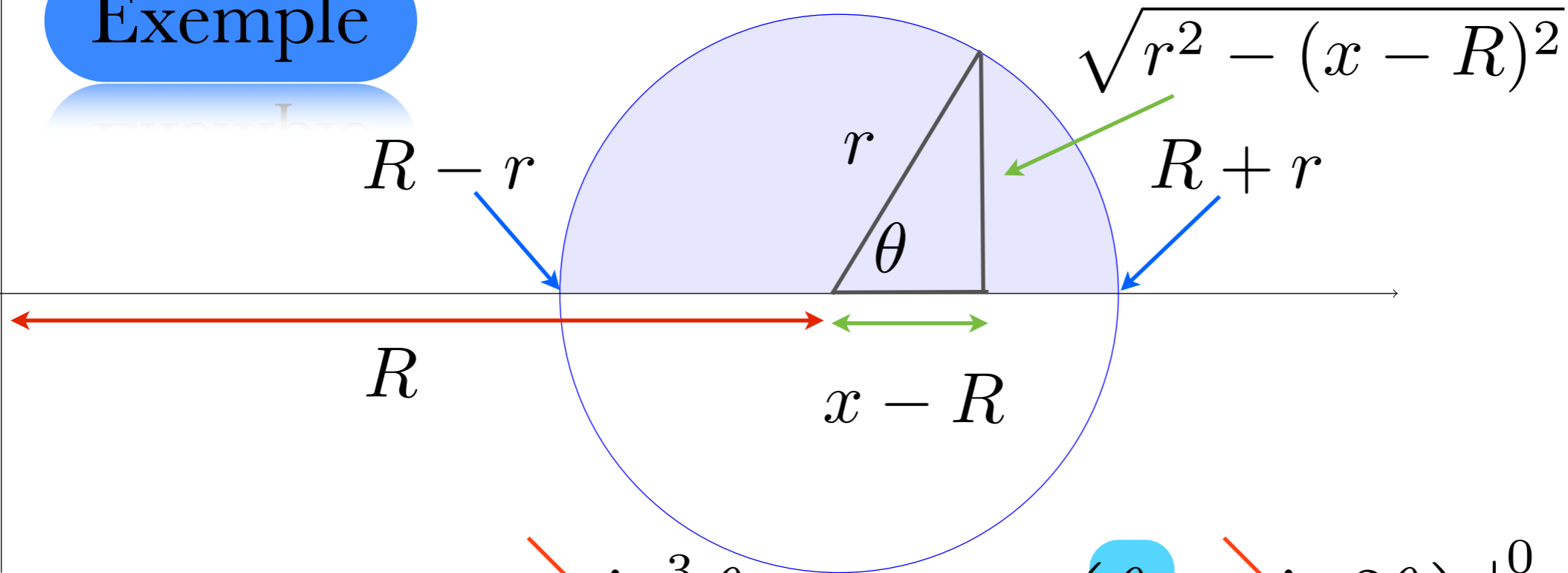
$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

$$= -2\pi R r^2 \frac{0}{2} + 2\pi R r^2 \frac{\pi}{2}$$

$$= \pi^2 R r^2$$

$$\text{Vol}_{\text{beigne}} = 2\text{Vol}_{\frac{1}{2}\text{beigne}} = 2\pi^2 R r^2$$

Exemple



$$\sin 0 = 0$$

$$\sin \pi = 0$$

$$\sin 2\pi = 0$$

$$\text{Vol}_{\frac{1}{2}\text{beigne}} = -2\pi r^3 \frac{\sin^3 \theta}{3} - 2\pi R r^2 \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^0$$

$$= -2\pi R r^2 \frac{0}{2} + 2\pi R r^2 \frac{\pi}{2}$$

$$= \pi^2 R r^2$$

$$\text{Vol}_{\text{beigne}} = 2\text{Vol}_{\frac{1}{2}\text{beigne}} = 2\pi^2 R r^2 = (2\pi R)(\pi r^2)$$

Faites les exercices suivants

Section 3 # 8 et 9

Aujourd'hui, nous avons vu

- ✓ Calcul de solide de révolution à l'aide de la méthode des tubes

Devoir:

Section 3.2