Examen 4 (solutions)
201-NYC Algèbre linéaire
Professeur : Dimitri Zuchowski

Question 1. (10%)

a) $\text{Im}(z_3) = 3$

b) $\text{Re}(z_4) = -5$

c) $\|z_2\| = \sqrt{5}$

d) $\text{Arg}(z_1) = \arctan\left(\frac{1}{1}\right) = \frac{\pi}{4}$

e) $\bar{z_3} = -2 - 3i$

Question 2. (5%)

a) $4 - 3i \longrightarrow 5e^{i \arctan\left(-\frac{3}{4}\right)}$.

b) $7e^{i\frac{\pi}{12}} \longrightarrow 7\cos\left(\frac{\pi}{12}\right) + 7i\sin\left(\frac{\pi}{12}\right)$

Question 3. (35%)

a) $14 - 17i$

b) $32 + 42i$

c) $-\frac{7}{4} + \frac{5}{4}i$

d) $2 + i$

e) $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

f) -15

g) $16i$

Question 4. (10%)

$\text{cis}\left(\frac{k2\pi}{8}\right), k = 0, 1 \ldots, 7$

Question 5. (10%)

$$\sqrt[3]{-3 - 3i} = \left(\sqrt{18}\text{cis}\left(\frac{5\pi}{4}\right)\right)^{\frac{1}{3}} = \left\{18^{\frac{1}{6}}\text{cis}\left(\frac{5\pi}{12}\right), 18^{\frac{1}{6}}\text{cis}\left(\frac{13\pi}{12}\right), 18^{\frac{1}{6}}\text{cis}\left(\frac{21\pi}{12}\right)\right\}$$

Question 6. (10%)

$iz^2 - 3iz - 1 + 3i = i(z - (1 + i))(z - (2 - i))$

Question 7. (10%)

$z^5 - 5z^4 + 13z^3 - 65z^2 + 36z - 180 = (z - 2i)(z + 2i)(z + 3i)(z - 3i)(z - 5)$ donc z=5.

Question 8. (10%)

$$\bar{z_1}z_2 = (a - bi)(c + di) = ac + adi - bci - bdi^2 = (ac + bd) + (ad - bc)i = (z_1 \circ z_2) + (z_1 \wedge z_2)i$$